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The geography of wealth: shocks, mobility, and precautionary savings*

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Abstract

The spatial distribution of wealth in the United States is very heterogeneous, with important differences within and across US states. We study the distribution of wealth in a country and how it is shaped by the characteristics earnings across regions, and by the frictions individuals face to move and reallocate across space. For this, we develop a tractable model of consumption, savings, and location choice with many regions, incomplete markets, and heterogeneous agents facing persistent and transitory income shocks. Our analysis focuses on the role of income shocks, precautionary savings, mobility, and sorting in shaping the geographic distribution of income and wealth over time. Our theory extends the workhorse macroeconomic model of consumption and savings under uncertainty and risk to an economy with multiple labor markets and costly mobility. Despite the complex spatial and individual heterogeneity, we can characterize the optimal consumption, savings, and mobility decisions of workers in closed form. Mobility frictions increase precautionary savings as workers hedge against sharp fluctuations in consumption generated by their mobility decisions. The spatial distribution of wealth is primarily driven by the interaction between persistent income shocks, saving behavior, and worker sorting across locations. The results highlight the importance of accounting for worker mobility and regional heterogeneity in earnings dynamics when studying the spatial distribution of wealth.

JEL Classification: R12, R23, E21, J61, F16

Keywords: Mobility, precautionary savings, spatial equilibrium, wealth, inequality.

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1 Introduction

Economic activity is unevenly distributed across space. Differences in the composition of industries across regions, the endowments of natural resources, the allocation of labor and the sorting patterns of workers across space, agglomeration and congestion forces, and historical circumstances shape local productivity and the distribution of income across individuals and regions. As households save part of their income for the future, these differences may also shape the distribution of wealth across regions, in particular since a substantial fraction of individuals' wealth is strongly connected with the location in which they choose to live and work.[1] Then, can differences in economic conditions across space help us understand regional differences in earnings and wealth?

Using microdata on earnings and wealth, we uncover a series of facts about the volatility of earnings and the distribution of wealth in the United States. First, the wealth-to-earnings ratio is higher in the East and West coasts and in the North, and lower in much of the South and the Rust belt. This is true for overall net wealth and for net wealth excluding housing. Moreover, there is a strong correlation between median earnings and median wealth between states. In addition, the volatility of wages also presents important differences across regions. The volatility of (residual) log-hourly wages is higher in most of the East Coast states, states in the South and in the West, including Alaska and Hawaii. With the exception of Illinois, wage volatility is lower in the Midwest and the North. These differences are substantial and wage volatility can be over 100% larger across different states. In this way, we note some similarities and some important differences across US states in average wages, residual log-wage volatility, and wealth.

Guided by these facts, we study the spatial distribution of wealth and how it is affected by the characteristics of workers' income in different regions, and by workers' mobility and reallocation decisions. Our analysis focuses on the role of income shocks, precautionary savings, sorting, and mobility in shaping the geographic distribution of income and wealth over time.

To do this, we develop a tractable heterogeneous agents model with incomplete markets that

[1] While housing is a salient category, the value of local businesses and investments in real estate are also strongly connected to individuals' location.

feature many sectors and regions. Our model has risk-averse workers facing uninsurable idiosyncratic income shocks. Workers are forward-looking and form rational expectations about future economic conditions. With convex marginal utility, agents accumulate savings as a precautionary measure against fluctuations in their earnings. In addition, individuals have idiosyncratic preferences over sectors and regions and can move, facing some costs, to a different labor market.

We study the problem of the worker taking prices as given. We show that, under reasonable assumptions, we can characterize the consumption, savings, and mobility decisions of workers in closed form. Our paper extends the seminal results in Caballero (1990) to an economy with many sectors and regions, where individuals can move and reallocate facing a cost in terms of their accumulated wealth. We show how savings are affected by differences in the characteristics of income across regions, such as the average level of wages and the volatility of the income process, and due to the option of moving, how mobility and sorting affect consumption-savings decisions.

Assuming an extreme value distribution for worker's preference shocks over labor markets, we characterize the optimal consumption and savings decisions of workers and the patterns of mobility. The optimal consumption satisfies a spatial Euler equation which takes into account that future consumption depends on future mobility and reallocation decisions. In this way, savings and wealth are influenced by mobility and reallocation frictions. As in standard models with a single labor market, or many markets but no mobility or reallocation, savings depend on the relative patience of workers and their future permanent income. Moreover, workers save more if the volatility of earnings is higher, as this allows for self-insurance and a smoother path of consumption. In this way, differences in the volatility of earnings across states can shape the distribution of wealth across US states. With mobility and preference shocks, savings respond to the economic characteristics of all other labor markets and to the frictions to move and reallocate that workers face. Our novel finding is that, in a spatial economy with preference shocks and mobility frictions, the expected value of future preference shocks affects the intertemporal consumption savings decision, and workers will self-insure through savings against adverse effects of preference shocks, a force that is absent when there are no frictions to move. In addition, we show that individuals with a high level of earnings, but which is expected to gradually decline in the future, save part of their high income. We show that, due to the complementarity between local wages and the persistent component of earnings, sorting across space will lead to differences in the pattern of savings across space. In sum, we show that consumption differs between individuals in the same labor market due to differences in the realization of idiosyncratic income shocks, the level of savings, and their mobility and reallocation choices.

As we stressed before, differences in the characteristics of the income process across states and the patterns of sorting of workers are key variables that determine savings. While there is a very large literature that estimates income processes, decomposing them into a transitory random component and a persistent random component, this literature abstracts from differences in the volatility of income across regions and from mobility and sorting of workers across space.[2] In contrast, in our work, we estimate the parameters that characterize the income process in different U.S. states, in particular parameters that govern the variance of persistent and transitory income shocks, accounting for the dynamic selection of workers due to unobservable characteristics. We find that there is considerable heterogeneity across space in the variance of transitory and persistent income shocks. While both these variances are positively correlated with average earnings across states, the correlation is stronger for the variance of persistent income shocks.

In a quantitative application using our parameter estimates, we find that different forces shape the consumption and savings decisions in standard and novel ways. For example, while higher volatility of income shocks in a region would imply higher levels of savings in that region, workers dislike income fluctuations and thus will avoid moving into these regions in the first place. This is important as the characteristics of regions affect the patterns of regional sorting and, as we show, sorting and persistent income shocks play a key role in explaining the spatial distribution of wealth. In a similar fashion, we study the role of mobility frictions and also find different forces at play. On the one hand, mobility frictions affect the distribution of preference shocks conditional on the labor market choice. Increased frictions lead to an adverse selection of these shocks and workers have an incentive to increase their savings. At the same time, frictions distort sorting of workers across space, and since workers with different characteristics

[2]This literature estimates random income processes and originate in the seminal work of Lillard and Willis (1978) and MaCurdy (1982).

have different incentives to save, frictions also affect the distribution of wealth.

Baum-Snow and Pavan (2013) and Gaubert et al. (2021) document important trends in earnings inequality over time that differ across U.S. cities and states. Our work complements these findings by establishing a series of facts about the wealth distribution and how it relates to the characteristics of economic conditions at the local level. Moreover, Diamond and Gaubert (2022) study how increased spatial sorting can shape earnings inequality. We show that sorting and differences in earnings risk (or earnings volatility) are forces that affecting the distribution of consumption, savings and wealth across regions.

Our paper contributes to an important recent literature on dynamic labor reallocation and migration. The seminal paper by Artuç et al. (2010) studies the mobility and reallocation decisions of workers across sectors with different characteristics, but workers are homogeneous and hand-to-mouth, with no wealth accumulation.[3] In a recent influential paper, Kleinman et al. (2023) incorporate investment and capital dynamics across different sectors and regions, but the savings/investment decisions are conducted by homogeneous (by region) and immobile rentiers, while mobile workers do not save.

Bilal and Rossi-Hansberg (2021) argue that workers sort over industries and regions to exploit their comparative advantage, but also trade static gains in terms of amenities and wages for future earnings potential and capital accumulation. Their model emphasizes borrowing constraints, and free mobility allows workers to increase effective consumption when they cannot borrow. In our setup, we abstract from binding borrowing constraints, but, in contrast to them, workers face frictions and preferences to move. We show that individuals in our economy increase their precautionary savings to hedge against the negative effects on wealth and consumption due to mobility. While workers in our economy also use the "location asset", frictions to move lead to a gradual adjustment in reallocation and consumption in response to an idiosyncratic income shock.

Our model links to the recent literature on heterogeneous agents and trade. Carroll and Hur (2020) and Waugh (2023) develop heterogeneous agents models with earnings risk, incomplete

[3]Recent important papers in this literature, such as Dix-Carneiro (2014); Caliendo et al. (2019); Traiberman (2019), make the same assumption.

markets, and trade, but abstract from labor reallocation and mobility. An important aspect of these papers is non-homoteticity in preferences, where workers with different levels of earnings and wealth have different patterns of consumption. Lyon and Waugh (2019) allow for labor reallocation across industries, but a labor market is defined at the level of each good variety in a trade model, which cannot be mapped to a specific region or industry. Moreover, when agents decide to move out of a labor market, they are randomly assigned to another labor market/variety. Ferriere et al. (2021), Giannone et al. (2020), and Greaney (2020) develop quantitative models of incomplete markets with heterogeneous agents and labor reallocation/migration decisions to study the effects of shocks with asymmetric impact over regions on workers dynamic decisions to move and attend college and how illiquid wealth shapes these decisions. As tends to be the case with traditional models of heterogeneous agents, the setup rapidly becomes intractable as the model scales up the number of regions. Our model is simpler in some dimensions as we abstract from binding borrowing constraints and illiquid assets, but our closed-form expressions allow us to obtain a sharp characterization of the forces driving individuals' precautionary savings, mobility, and wealth across space. Moreover, we can profit from these tractable expressions to obtain closed-form expressions that characterize the dynamics of wealth inequality.

Dvorkin (2023) recently developed a model of heterogeneous agents and incomplete markets where individuals accumulate human capital and assets. In his paper, workers have CRRA utility and face shocks with permanent effects on income and assets that also drive mobility decisions. We connect closely to this paper but focus, in addition, on transitory income shocks which drive a stronger motive for precautionary savings, and have separate shocks affecting the preferences over regions and sectors. In both of these works, the closed-form expressions for the consumption and savings rules allow for aggregation of consumption and wealth by region.

The paper is organized as follows. Section 2 documents some facts of the distribution of wealth across US states. Section 3 develops a dynamic model of consumption, savings, and mobility to help rationalize these facts. In Section 4 we estimate a very large set of parameters related to the characteristics of the income process in each state, mobility frictions, preference shocks, and selection. In Section 5 we use our model under the estimated parameter values to undertand the forces that shape the distribution of wealth across space. Section 6 concludes.

2 Earnings volatility and wealth across US states

U.S. states show important differences in per capita income (Gaubert et al., 2021; Diamond and Gaubert, 2022). This fact holds even when differences in the composition of the population, the level of education, and the cost of living are taken into account.

Using wage data from the Outgoing Rotation Groups of the Current Population Survey (CPS), we document a new fact: US states show important differences in the volatility of individual earnings and wages. We document this fact using information on wages for individuals between 25 and 60 that we can link one year apart for the years 2010 to 2019. We follow the usual approach in the literature and compute measures of volatility of individual log-hourly wages, log-weekly earnings, and individual arc percent changes in wages or earnings.[4] As is usual in the literature, we drop from our sample individuals with very low levels of weekly earnings.[5] Hourly wages are computed as weekly earnings divided by the usual weekly hours. In all cases, we deflate nominal wages and earnings using the Consumer Price Index.

As recently documented in Moffitt et al. (2022), imputed earnings data can artificially magnify the amount of wage volatility. In our sample we exclude observations with imputed earnings. In addition, Moffitt et al. (2022) recommend to trim the top and bottom 1% of the sample of wages when using survey data, as the sample from surveys can fail to properly capture the very low and very high levels of income and magnify measures of volatility. We proceed in this way, trimming the top and bottom 1% of the distribution of wages or earnings by state and year.

Figure 1 shows the distribution of the variance in the change in individual weekly earnings and hourly wages across states for different demographic groups. To construct each measure, first we compute the variance of wage or earnings changes across individuals in each state. Then we use each state as an observation and plot the distribution of the selected moment across states. Thus, each box in the figure displays the interquartile range and the median of the volatility of

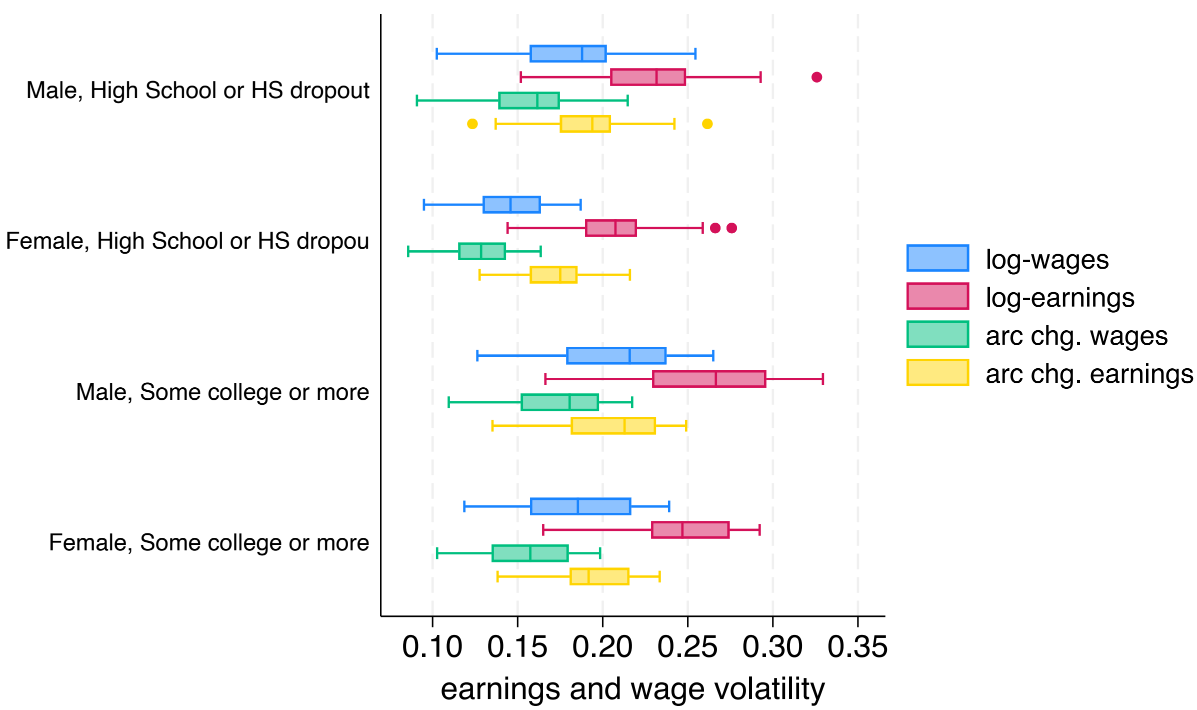
[4]The arc percent change in a variable x between two periods of time is defined as, arc chg.$(x_t) = \frac{x_t - x_{t-1}}{(x_t + x_{t-1})/2}$.

[5]In particular, we drop individuals with weekly earnings lower than five hours a week with an hourly wage of one half the the federal minimum hourly wage. In this way, we exclude individuals that work an equivalent of less than 260 hours a year, at one-half the minimum wage. These are sample restrictions similar to Heathcote et al. (2010).

wages, and the lines that extend out from each box are the lower or upper adjacent values.[6]

As the figure shows, there is an important dispersion in earnings and wage volatility across states, irrespective of the measured used or the demographic group, and the volatility of wages can be over 50% larger for a state at the 75% of the distribution than for a state at the 25%. The dispersion of the variance across states is larger for more educated individuals, and lower for females with a level of education equal to high school or less.

Figure 1: Volatility of log-wages across U.S. states



Note: Distribution of the variance of (1) log-hourly wages, (2) log-earnings, (3) arc percent change in log wages, and (4) arc percent change in log earnings across states for different demographic groups. Measures computed using CPS data matched records over one year for individuals between 25 and 60 years old employed at the time of the survey. All in real dollars of 2021. Pooled 2010-2019 monthly data, trimmed at the top and bottom 1% of real hourly wages by state and year. Non-imputed earnings data only.

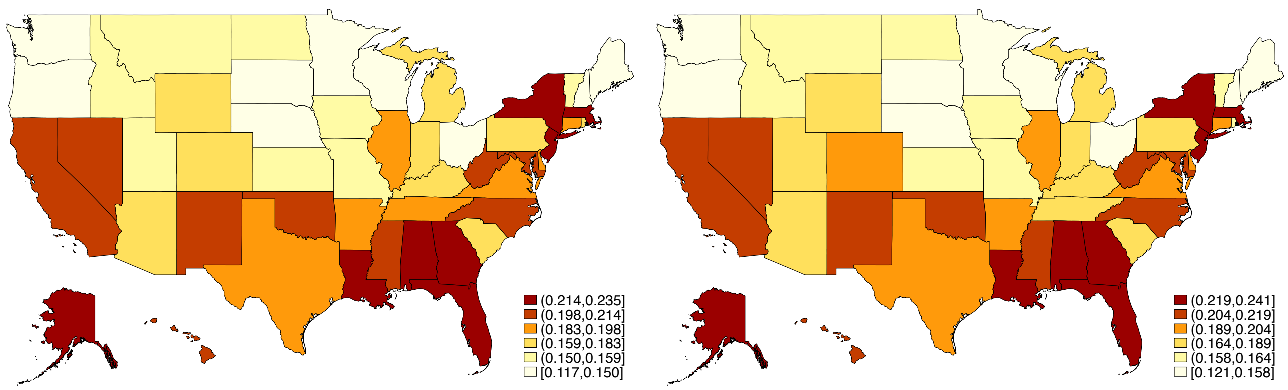
We follow standard practice in the literature and construct residual log-wages as the deviation of log-wages for an individual from the predicted value from a Mincer regression that uses a rich set of controls for demographic characteristics, industry, and state. We run this regression pooling

[6]The lower adjacent value is the maximum of the minimum point of the distribution or the 25th percentile minus 1.5 times the interquartile range minus. Similarly, the upper adjacent value is the minimum between the maximum value in the distribution and the third quartile plus 1.5 times the interquartile range.

all years and specify all of these controls as fixed effects that vary by year. That is, our regression has worker type-year fixed effects, state-year fixed effects, and industry-year fixed effects.[7]

Panel (a) of Figure 2 shows a map with the volatility of log-wages across U.S. states, while panel (b) shows the volatility of residual log-wages. As the figure shows, the volatility of (residual) log-hourly wages is higher in most of the East Coast states, states in the South and in the West, including Alaska and Hawaii. With the exception of Illinois, wage volatility is lower in the Midwest and the North. These differences are substantial and wage volatility can be over 100% larger across different states. Our findings align well with the evidence on earnings volatility in Lamadon et al. (2019), who use restricted access microdata from the Internal Revenue Service and show greater earnings volatility in the East and West coast, and in the South.

Figure 2: Volatility of log-wages across U.S. states



(a) Log-wages

(b) Residual log-wages

Note: Variance of log-hourly wages and log-residual wages for individuals between 25 and 60 years olf employed at the time of the survey. Residuals computed from a regression of log-wages on a rich set of demographic-year fixed effects, state-year fixed effects and industry-year fixed effects. CPS data matched records over one year. Pooled 2010-2019 monthly data, trimmed at the top and bottom 1% of real hourly wages by state and year. Non-imputed earnings data only.

Many economic theories predict that if earnings volatility originates primarily from uninsurable transitory or persistent (but not fully permanent) income shocks, individuals will self-insure against the effects of these shocks by increasing their precautionary savings (Leland, 1968; Caballero, 1990). We now explore the correlation between the volatility of earnings and wealth across space and produce a more in-depth analysis in the next sections.

[7] Our demographic groups (or worker type) results from the Cartesian product of gender dummies, race dummies (white, black, and other), education dummies, (high school dropout, high school, and college or more), and four age group dummies (less than 30, between 30 and and 39, between 40 and 49, and between 50 and 60). All of these dummies are interacted with a year dummy.

We use microdata on net worth to document several facts about the spatial distribution of wealth in the United States and its correlation to the volatility of earnings. For this, we use data from the Survey of Income and Program Participation (SIPP).

The SIPP is a household-based survey designed as a series of nationally representative panels. Each panel generally features a large sample of households that are interviewed multiple times over a three or four-year period, depending on the year. From 1996 to 2017, the survey is structured as non-overlapping panels, with surveys every 4 months. Beginning in 2018, the survey switched to having overlapping panels interviewed annually. The SIPP provides comprehensive information on individuals' earnings over time, government transfers, and participation in social programs, among other demographic, family, and work-related characteristics, since 1983. The wealth data is part of the "Assets and Liabilities" topical module available in different waves across the SIPP's panels over time.[8] The different asset and liability categories available in the SIPP are not as comprehensive and detailed as those in the Survey of Consumer Finances. However, in the appendix we show that the distribution of wealth in both surveys is very similar, except for the top percentiles of the wealth distribution.[9]

We use a similar sample selection criteria as before and our sample includes individuals between 25 and 60 years old. To increase the sample size, we pool data for the years 2014 to 2019.[10] In this way, we use a relatively similar period but abstract from the Covid pandemic years. In the SIPP (and the SCF) wealth is measured at the level of the household or the family, as it is non-trivial how to assign the value of different assets and liabilities to each family member. In our sample, we keep only the head of households and assign all of the family wealth to this individual.

We begin by examining two main indicators of wealth. First, we use total net worth, which comprises all assets of the household minus all liabilities. Since housing is an important component of wealth and house prices differ widely across regions, we construct a measure of non-

[8]There is a module with information about financial assets in every year from 1994 to 2021, excluding 2006, 2007, 2008, and 2012.

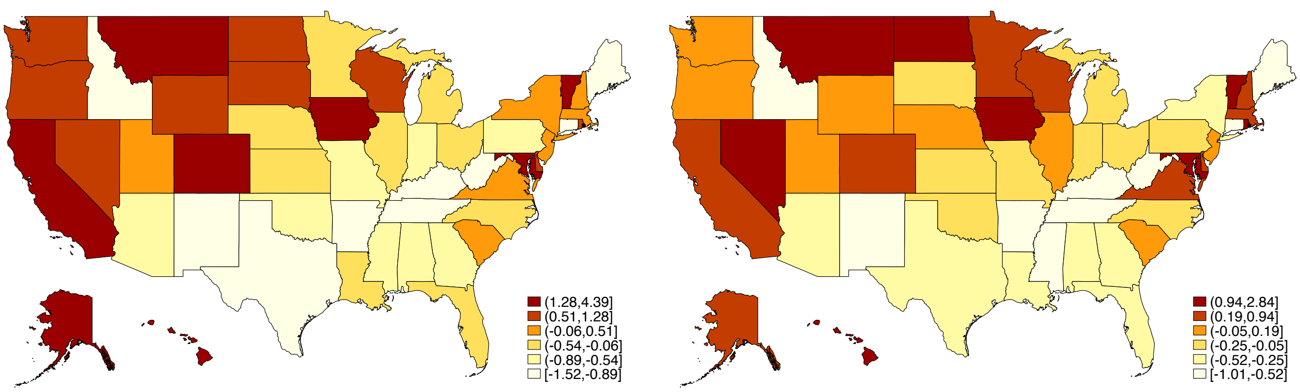
[9]The Survey of Consumer Finances is designed to oversample individuals at the top of the wealth distribution. Thus, it captures in great detail the different asset holdings of the wealthiest individuals in the United States.

[10]The SIPP was redesigned in 2014 and the measures of wealth before that year are not as comprehensive nor are comparable to the more recent periods.

housing net wealth, which excludes the value of real estate, mortgages, and equity lines of credit. As there may be important differences in the demographic composition of individuals across states, affecting earnings and wealth, we construct a measure of wealth to income ratio that removes the component coming from demographic characteristics. To do this, we run a regression of net-worth to earnigs and non-housing net-worth to earnings on indicators of gender, race and level of education, a polynomial of order four in age and state fixed-effects.

Figure 3 shows the state fixed effect of the ratio of net-worth and non-housing net worth of individuals to earnings at the individual level. Across U.S. states, wealth relative to earnings is higher in the East and West coast and in the North, and lower in much of the South and the Rust belt. In this way, the figure presents some similarities and some important differences relative to the map for log-wage volatility, leading to a low correlation across the two variables.

Figure 3: Wealth-to-earnings ratio across states



(a) Net worth

(b) Non-housing wealth

Note: The figure shows the state fixed effect from a regression of net worth over earnings and non-housing net worth over earnings on demographic characteristics and a polynomial of order four in age. Sample include head of households between 25 and 60 years old for the years 2014-2019. Sources: SIPP and authors' calculations.

Next, we develop a model to understand these features of the data and explore the link between wealth and wage volatility guided by our theory.

3 Model

We develop a model of consumption and savings in a spatial economy with many regions, where mobility across different labor markets is subject to individual preferences and frictions. To keep the analysis simple, we assume an exogenous path for prices, interest rates, and wages across regions, and characterize the dynamic problem of the household. We take this as a starting point to understand the main driving forces shaping consumption, savings, mobility, and wealth heterogeneity.

Our model combines the ideas of Caballero (1990) on precautionary savings in a context of volatile earnings shocks, with costly mobility and reallocation across space, as in Kennan and Walker (2011).

Time is discrete and runs forever. The economy is populated by a continuum of individuals of measure one facing a probability of death d, as in the Blanchard-Yaari perpetual youth model. Each period a new cohort of workers of measure d is born and the population is constant.

The economy consists of J labor markets, defined as a region-sector pair. Workers start each period attached to some labor market $j = \{1, 2, \ldots, J\}$ and have the option to reallocate to a different market at the beginning of the period if they choose, but mobility is costly.

Individuals derive utility from consumption of a final good each period. Following Caballero (1990), we assume that the utility for the period is CARA with parameter γ.[11] In addition, all individuals living in region j pay a rental cost $q_{j,t}$, which captures differences in housing costs between regions over time as in Bilal and Rossi-Hansberg (2021).

Workers are heterogeneous in the efficient units of labor they supply (inelastically) to their labor market. Although workers have on average one unit of labor to supply to the market, each worker's endowment is random and changes over time, which we interpret as idiosyncratic shocks to a worker's labor income. In terms of timing, we assume that innovations to a worker's labor supply are realized after making the mobility and reallocation decision, but before the consumption-savings decision.[12] Then, the total efficient units of labor of a worker i are $(1 +$

[11] This assumption, coupled with some others discussed next, allows us to derive closed-form expressions for the optimal decision rules of individuals.

[12] This assumption is convenient to get closed-form expressions that we can compare to the literature on pre-

$z_i' + \eta_i')$, where, $z_i' = \rho\, z_i + \epsilon_i$. ϵ_i' and η_i' are i.i.d. shocks distributed normally with mean zero and variance $\sigma^2_{\epsilon,\ell}$ and $\sigma^2_{\eta,\ell}$, respectively. Note that the autocorrelation parameter $\rho \in (0,1)$ is common across all labor markets, but the variances of the shocks can differ across regions. We label z_i the persistent income shock and ϵ_i the transitory income shock.

Markets are incomplete, and individuals can save (or borrow) for the future and self-insure against shocks using a risk-free real actuarial bond issued by an economy-wide government. Bonds can be purchased at the price of $1/(1+r_t)$ units of final consumption at date t and each bond pays one unit of the final consumption good the next period if the bondholder survives.[13]

Individuals have the option to move to a different labor market at the beginning of each period. Geographic mobility and reallocation across sectors are costly for workers, affecting their utility due to non-pecuniary factors.[14] We capture moving frictions with parameter $\psi_{j\ell}$, which scales utility due to non-pecuniary costs of relocating from labor market j to ℓ, with $\psi_{jj} = 0$, and $\psi_{j\ell} < 0$ for $j \neq \ell$.

In addition, mobility and reallocation decisions are influenced by idiosyncratic preference shocks for different labor markets. We denote these shocks by the random vector $\varepsilon \in \mathbb{R}^J_+$, where each element is distributed i.i.d. Weibull with shape parameter $\nu > 0$ and identical scale parameter $\lambda = \bar{\lambda}\, \frac{J^{1/\nu}}{\Gamma\left(1+\frac{1}{\nu}\right)}$.

Finally, we assume that a worker who dies at the end of the period is replaced by a newborn worker attached to the same labor market. Moreover, newborn workers start with no assets.[15]

Let $V_{j,t}(a, z, \varepsilon)$ denote the lifetime utility (or value function) of a worker with past labor market j (last period), persistent income shock z, real savings or bond-holdings a, and preference shocks ε. To simplify the notation, we omit the individual subscript i with the understanding that assets and shocks vary across individuals. We study the case of *perfect foresight about aggregate conditions* and use sub-index t to represent the aggregate state of the economy at that

cautionary savings, but can be relaxed with minimal changes.

[13]Note that we assume a common real interest rate for all regions. Borrowing implies that the worker issues these actuarial bond to the economy-wide government, and repays the next period if the worker survives.

[14]It is also possible to extend the model to have pecuniary costs of moving affecting the level of savings or wealth.

[15]These assumptions allow for a well-defined distribution of wealth over time if the death rate is sufficiently large. These assumptions can be relaxed at the cost of some cumbersome notation.

time. We write the problem of the worker recursively on individual state variables as,

$$V_{j,t}(a,z,\varepsilon) = \max_{\ell} \left\{ \varepsilon_\ell \, e^{\psi_{j\ell}} E_{\eta',z'|z,\ell} \left[\max_{c_{j\ell,t}, a_{j\ell,t+1}} \left\{ \left(-\frac{1}{\gamma} \right) e^{-\gamma \, c_{j\ell,t}} + \beta E_{\varepsilon'} \left[V_{\ell,t+1}(a_{j\ell,t+1}, z', \varepsilon') \right] \right\} \right] \right\} \quad (1)$$

$$\text{s.t.:} \qquad c_{j\ell,t} + \frac{1}{(1+r_t)} \, a_{j\ell,t+1} + q_{\ell,t} = w_{\ell,t}(1-\tau)\,(1+z'+\eta') + a \quad (2)$$

where (2) is the budget constraint linking consumption and savings of the worker after the mobility decision from j to ℓ. The discount factor β takes into account the discount rate and the survival probability. Individuals pay a proportional income tax τ that is common for the whole economy.[16] Given our timing assumption, workers make the reallocation decision before observing the realization of the labor supply shock η' and $z' = \rho\, z + \epsilon'$.

It is important to highlight that the persistent component of the labor supply, z' is complementary to the wages of the region. Thus, the value of z influences the mobility and reallocation decisions of individuals, as the expectation for an individual with high z is to have a high value of z' after reallocation.

Finally, note that non-pecuniary costs enter the problem multiplicatively with factor $e^{\psi_{j\ell}}$. Since both period utility and lifetime utility are negative, a higher value of non-pecuniary costs of moving from j to ℓ, decreases the value of making that transition.

3.1 Characterization of the worker's problem

We now discuss the optimal conditions that characterize the solution to the worker's problem in this economy. It is convenient to simplify the notation with the following definitions. Let $\tilde{y}_{\ell,t} = w_{\ell,t}(1-\tau) - q_{\ell,t}$ and $\tilde{w}_{\ell,t} = w_{\ell,t}(1-\tau)$. In addition, denote by $v_{j,t}(a,z) = E_\varepsilon[V_{j,t}(a,z,\varepsilon)]$ the ex ante lifetime utility.

Proposition 1. *The optimal consumption-savings decisions of the worker, after choosing the*

[16] For the value function to be well-defined, the transversality condition $\lim_{t\to\infty} \frac{a_{j\ell,t}}{(1+r_t)^t} = 0$ must hold, which we impose.

location are characterized by,

$$c_{j\ell,t}(a, z', \eta') = \Xi_{\ell,t}(z') + \kappa_t \left[\tilde{y}_{\ell,t} + \tilde{w}_{\ell,t}\,(z' + \eta') + a\right], \tag{3}$$

$$a_{j\ell,t+1}(a, z', \eta') = (1 + r_t)(1 - \kappa_t)\left[\tilde{y}_{\ell,t} + \tilde{w}_{\ell,t}(z' + \eta') + a\right] - (1 + r_t)\,\Xi_{\ell,t}(z') \tag{4}$$

where κ_t is the marginal propensity to consume out of cash on hand and $\Xi_{\ell,t}(z')$ depends on the realization of the shock z', the selected labor market, and period. Moreover, the ex-ante lifetime utility and the share of individuals moving from labor market j to ℓ, $\mu_{j\ell,t}(z)$ are,

$$v_{j,t}(a, z) = \left(-\frac{1}{\gamma}\right)\frac{1}{\kappa_t}e^{-\gamma\kappa_t\, a}\,\bar{\lambda}\left(\frac{1}{J}\sum_{\ell=1}^{J} e^{\nu(\mathbb{B}_{1,j\ell,t}+\mathbb{B}_{2,\ell,t}z)}\,E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell,t}(z')}\right]^{-\nu}\right)^{-(1/\nu)}, \tag{5}$$

$$\mu_{j\ell,t}(z) = \frac{e^{\nu(\mathbb{B}_{1,j\ell,t}+\mathbb{B}_{2,\ell,t}z)}\,E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell,t}(z')}\right]^{-\nu}}{\sum_{m=1}^{J} e^{\nu(\mathbb{B}_{1,jm,t}+\mathbb{B}_{2,m,t}z)}\,E_{z'|z,m}\left[e^{-\gamma\Xi_{m,t}(z')}\right]^{-\nu}} \tag{6}$$

where $\mathbb{B}_{1,j\ell,t} = -\psi_{j\ell} + \gamma\kappa_{t+1}\tilde{y}_{\ell,t+1} - (\kappa_{t+1}\gamma\tilde{w}_{\ell,t+1})^2(\sigma^2_{\epsilon,\ell} + \sigma^2_{\eta,\ell})/2$, $\mathbb{B}_{2,\ell,t} = \gamma\kappa_t\tilde{w}_{\ell,t}\rho$, and $\Xi_{\ell,t}(z')$ and κ_t are defined recursively as,

$$\kappa_t = \frac{\kappa_{t+1}(1 + r_t)}{1 + \kappa_{t+1}(1 + r_t)} \tag{7}$$

$$\Xi_{j,t}(z) = \left(-\frac{1}{\gamma}\right)\left(\frac{1}{1 + \kappa_{t+1}(1 + r_t)}\right)\Bigg(\log\left(\beta(1 + r_t)\right) + \log(\bar{\lambda}) - \frac{1}{\nu}\log\left[\frac{1}{J}\sum_{m=1}^{J} e^{\nu(\mathbb{B}_{1,jm,t+1}+\mathbb{B}_{2,m,t+1}\,z)}\left(E_{z'|z,m}\left[e^{-\gamma\Xi_{m,t+1}(z')}\right]\right)^{-\nu}\right]\Bigg) \tag{8}$$

Proposition 1 extends the results in Caballero (1990) with persistent and transitory income shocks to an economy with many sectors and regions. Given the assumption of a common real interest rate across labor markets, the marginal propensity to consume out of cash-on-hand, κ_t, is identical for all individuals. However, Equation (3) shows that consumption differs between individuals in the same labor market ℓ due to differences in idiosyncratic shocks, assets, and their mobility and reallocation choices. For savings, it is similar, with wealth influenced by mobility and reallocation frictions through $\Xi_{j,t}(z)$.

It is easy to show that the consumption/savings decision satisfies a *spatial Euler equation* given by,

$$\frac{\partial u\left(c_{j\ell,t}(a,z',\eta')\right)}{\partial c} = \beta(1+r_t)\,\bar{\lambda}\,\left[\frac{1}{J}\sum_{m=1}^{J}\left(e^{\psi_{\ell m}}\,E_{\eta'',z''|z'}\left[\frac{\partial u\left(c_{\ell m,t+1}(a''_{j\ell,t+1},z'',\eta'')\right)}{\partial c}\right]\right)^{-\nu}\right]^{-1/\nu}. \tag{9}$$

As is usual, the Euler equation links consumption in period t to future consumption through savings and expected future income. The main difference here, is that consumption has a spatial component. That is, the consumption-savings decision in period t depends on the disposable income in region ℓ and the future expected disposable income in the future, which depends on next period's chosen region. However, next period's region has not yet been decided, and the right of (9) captures the expected marginal utility of future consumption across all possible future labor markets, which given the assumption on the distribution of preference shocks, is a generalized mean of the marginal utility of consumption in all labor markets. In this way, mobility frictions, preference shocks and future disposable income in different regions affect the expected value of next period (marginal) utility, thus affecting today's consumption and savings decisions.

Note that this condition is different from Bilal and Rossi-Hansberg (2021). On the one hand, we do not impose a borrowing limit other than the natural (the transversality condition), and our spatial Euler equation always holds with equality. On the other, our economy has mobility and reallocation costs, and our individuals may be "constrained in their location asset" ex-post.

However, similar to Bilal and Rossi-Hansberg (2021), workers sort across regions due to the complementarity between individual productivity and wages. In paricular, equation (6) shows that workers with high values of z will move with higher probability to regions with higher wages, as z is persistent expected to be high for many periods. Preference shocks affect the patterns of sorting, which is a different mechanism to the one in Bilal and Rossi-Hansberg (2021), in which borrowing constraints affect spatial sorting.

Similarly to them, in our model, the permanent component of income $\tilde{y}$, depends negatively on rents $q_{\ell,t}$. Thus, workers prefer locations with low rents, all else equal. However, workers dislike volatility of income, and migration and sorting will be infleunced by the characteristics of the income process in each region.

Note that, in the special case of symmetric and identical labor markets and no idiosyncratic income shocks, $\eta = z = 0$, such that individuals attain the same level of consumption irrespective of the labor market, equation (9) becomes,

$$\frac{\partial\, u\,(c_t(a))}{\partial c} = \beta(1+r_t)\,\left[\bar{\lambda}\,\left(\frac{1}{J}\sum_{m=1}^{J} e^{-\nu\,\psi_{\ell m}}\right)^{-1/\nu}\right]\frac{\partial\, u\,(c(a_{t+1}))}{\partial c},$$

and highlights a novel motive for saving in spatial economies: precautionary savings due to preference shocks. Assume that $\bar{\lambda} = 1$, then since $\psi_{\ell m} > 0$ for $\ell \neq m$, the term in squared brackets is greater than one and workers want to increase their savings and their future consumption, and reduce present consumption. The reason is that, due to frictions, workers are attached to their labor market, and are not able to easily move out if the realization of preference shock for their market in the future is bad. In this way, workers "escape the effect of adverse shocks by increasing their wealth and future consumption.[17] With no frictions, $\psi_{\ell m} = 0$, and the consumption-savings decision is standard, which in this simplified case implies a constant path of consumption over time.

The recent work of Mongey and Waugh (2024) and Donald et al. (2023) study the role of preference shocks and incomplete markets in economic environments characterized by static or dynamic, respectively, discrete choice decisions. As is usual in the literature, they assume that households do not have access financial markets and are hand-to-mouth, and argue that a planer intervention via transfers or the introduction of some securities increases workers welfare. Our results in this section highlight how access to a simple set of uncontingent financial securities, i.e. savings and borrowing, allow workers to self-insure for preference shocks.

[17]It is important to highlight that, the realization of preference shocks for the current period do not appear in our spatial Euler equation, since these shocks affect equally current and future utility. Thus, the savings motive we highlight are not due to temporary changes in patience or impatience, but rather about expected average realization of shocks conditional on the choice.

3.2 Risks, local characteristics, mobility, and precautionary savings

We now discuss how savings and mobility are affected by risk and labor market characteristics.

The savings policy rule (4) says that workers save a fraction of their current cash-on-hand, plus an additional term. Given our assumptions, the marginal propensities to consume and save out of current income or assets are identical across labor markets. Nonetheless, differences in the level of savings arise due to differences in the current cash-on-hand of workers, for example, due to differences in wages.

Additional effects related to expected future income and precautionary savings are captured by the last term in (4), leading to different levels of savings in different labor markets.

Equation (8) defines $\Xi_{\ell,t}(z)$ recursively. To make some progress, we use the equations in Proposition 1 assuming $z = \epsilon = 0$. In this case, there are no persistent income shocks, and $\Xi_{\ell,t}(z)$ and $\mu_{j\ell}$ do not depend on z. We can write (4) as,

$$-\Xi_{\ell,t} = \underbrace{\left(\frac{1}{\gamma}\right)\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\frac{\kappa_t}{\kappa_{t+s}}\log\left(\beta(1+r_t)\right)}_{\text{impatience}} - \underbrace{k_t\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\tilde{y}_{\ell\ell,t+s}}_{\text{permanent income}} \tag{10}$$
$$+\underbrace{\frac{\gamma\,\kappa_t}{2}\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\kappa_{t+s}\,(\tilde{w}_{\ell,t})^2\sigma^2_{\eta,\ell,t+s}}_{\text{precautionary savings}} + \underbrace{\left(\frac{1}{\gamma}\right)\sum_{s=1}^{\infty}\left(\frac{1}{\Pi_{k=0}^{s-1}(1+r_{t+k})}\right)\frac{\kappa_t}{\nu\kappa_{t+s}}\left[\log\left(\mu_{\ell\ell,t+s}\right)-\log\left(1/J\right)+\nu\log(\bar{\lambda})\right]}_{\text{mobility}},$$

where, to ease the exposition, we write the negative of $\Xi_{\ell,t}$. Equation (10) consists of four terms. The first term captures the effects of impatience in savings. If individuals are more impatient relative to the market, such that $\beta(1 + r_t) < 1$, then this first term is negative and savings would be lower. The second term captures future expected permanent income for a worker who does not move. With higher levels of expected permanent income, consumption increases and savings decline. The third term captures the effects of precautionary savings. Workers in sectors and regions that have more volatile earnings, leading to higher levels of $\sigma^2_{\eta,\ell,t}$, will save more to self-insure against these shocks. Note that, given our assumptions on preferences, sectors and regions with higher levels of wages would have higher levels of expected permanent income, but also higher levels of earnings volatility as the effective variance of earnings is $(w_{\ell,t}(1-\tau))^2\,\sigma^2_{\eta}$. Once again, this is the effect for non-movers. The last term captures how mobility affects consumption and savings. Clearly, $\mu_{\ell\ell,t+s}$ is endogenous and itself depends on the future values of Ξ across all locations. However, we can get some intuitions as mobility rates serve as a sufficient statistic for the (relative) expected value of labor markets. First, note that with only one labor market, the last term is zero and $\Xi_{\ell,t}$ would be the same as that in Caballero (1990) for i.i.d. income shocks. Related, if the probability of staying in a labor market is identical to the probability of moving to any other labor market, such that $\mu_{\ell\ell,t} = 1/J$, then the last term is zero. This would happen when all labor markets are identical and workers face no moving frictions, which is essentially a single labor market economy.

Even when all markets are identical, moving and reallocation frictions increase the probability of staying in a labor market and the last term is positive. In this way, frictions to move and reallocate increase savings. With identical labor markets, this is only driven by the i.i.d. preference shocks. Workers may find another labor market more preferable for idiosyncratic reasons, but since moving is costly in terms of assets and lower out-mobility rates (higher values for $\mu_{\ell\ell}$) imply higher moving costs, workers have incentives to save more in economies with higher frictions. In this way, the last term in equation (10) captures precautionary savings due to mobility (preference) shocks. Higher values of ν translate into a lower variance of preference shocks, which reduces this precautionary savings motive.

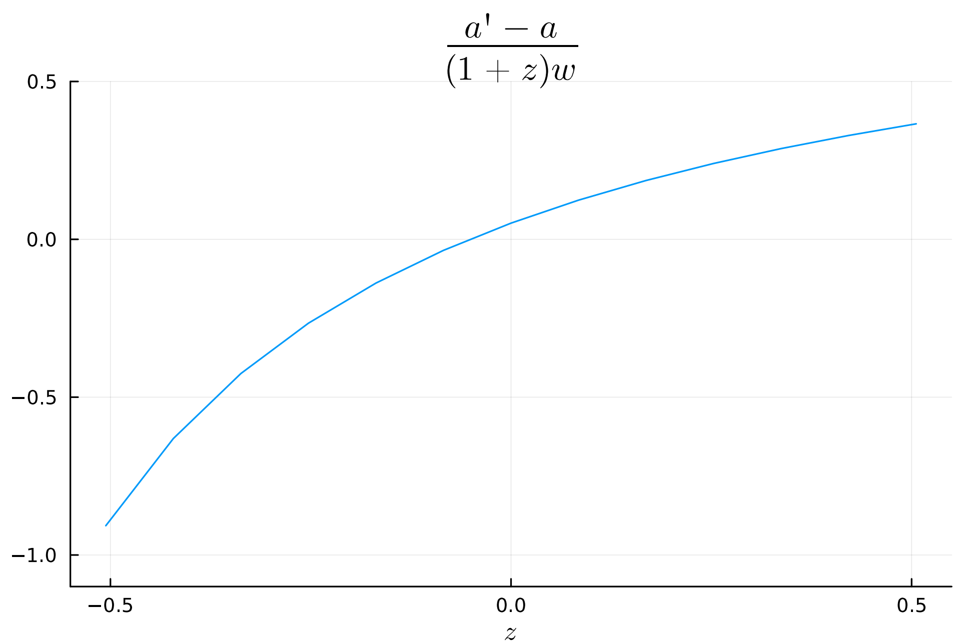
In a stationary equilibrium and under $z = \epsilon = 0$, equation (10) simplifies to,

$$-\Xi_{\ell,ss} = \left(\frac{1}{\gamma}\right)\left(\frac{1}{r_{ss}}\right)\left[\log\left(\beta(1+r_{ss})\right) - \gamma\kappa_{ss}\tilde{y}_{\ell\ell,ss} + \gamma^2\kappa_{ss}^2(\tilde{w}_{\ell,ss})^2\frac{\sigma^2_{\eta,\ell,ss}}{2} + \frac{1}{\nu}\left[\log\left(\mu_{\ell\ell,ss}\right) - \log\left(1/J\right)\right] + \log(\bar{\lambda})\right],$$

where $\kappa_{ss} = \frac{r_{ss}}{1+r_{ss}}$.

In the general case with persistent income shocks, z, the volatility of this component will also induce savings for precautionary motives. However, there will be another reason to save: "saving for a rainy day". In this case, when z is high and is expected to decline only gradually, individuals will save and achieve a smoother path for consumption. Similarly for negative values of z, when individuals decrease their savings or borrow. In addition, differences across space in wages and the complementarity between wages and z implies that people in high-wage locations will save (borrow) more when their z is higher (lower). Without mobility and sorting, saving for a rainy

Figure 4: Saving for a rainy day



day would not lead to differences in the level of savings between regions. But with sorting, due to the complementarity between wages and z, individuals with high z, who are net savers, are more likely to move (or stay) in regions with high wages, leading to differences in the level of savings and wealth between regions due to dynamic sorting. For simplicity, in our model we abstracted from permanent differences in productivity. It is important to highlight that, with differences in a permanent component of labor supply individuals will sort across space but this will not generate differences in savings across regions as, through the lens of our model, workers with higher permanent income will have higher permanent levels of consumption, but no differences in savings. Then, it is the combination of persistent, but mean reverting, income shocks and dynamic sorting that contributes to differences in wealth accumulation across regions.

Figure 4 shows the behavior of the savings rate, that is, the change in assets relative to earnings, for different values of z and a particular set of parameter values in our model.[18] The savings rate function is concave and increasing in z, which means that individuals with above-average levels of z save positive amounts. As z gradually returns to the mean, savings decline.

[18] In the next section we discuss the calibration and estimation of the parameters in our model. To construct this figure, we use these values.

The opposite happens for low values of z.

In this economy, markets are incomplete and individuals are risk-averse. Equation (6) highlights the role of income risk, market incompleteness, and precautionary savings motives on mobility and reallocation decisions. Labor markets with more volatile income shocks will attract fewer individuals for two compounding reasons. First, as we previoulsy said, individuals dislike a volatile stream of consumption and directly discount the expected utility of these labor markets. Second, in these markets workers would increase their precautionary savings, indirectly leading to a lower level of expected consumption, which makes these markets less attractive for workers.[19]

Note that the level of income volatility in a labor market not only depends on the variance of idiosyncratic shocks to the effective units of labor, but also on the level of wages. Labor markets with higher wages increase the level of permanent income, making these markets more attractive for workers, but also increase the volatility of earnings, which serves as an offsetting force. In this way, our model implies that in labor markets with higher wages, savings due to precautionary motives will be higher.

3.3 Aggregation

Despite individual heterogeneity in asset holdings and earnings, we can characterize the evolution of aggregate consumption, earnings, and wealth for workers in labor market ℓ. The reason for tractability is that consumption and savings decisions are linear in cash-on-hand in each labor market for each worker as shown in Proposition 1. Thus, the average consumption depends linearly on the average cash-on-hand in each market, and similarly for savings.

As in Dvorkin (2023), we can use properties of the mixture of distributions to compute also the *evolution* of first (and higher-order) moments of the distribution of consumption and wealth.

Let $\Lambda_{\ell,t}(z')$ be the density (or measure) of workers with shock z' in labor market ℓ at the end of time t (joint distribution of z' and ℓ).

[19]Note that in our simple economy, the level of asset holdings of an individual does not directly influence the mobility decision. As we relax some of our assumptions, that would no longer be the case, but we lose the sharp characterization of some of our expressions and need to resort to a quantitative analysis.

Proposition 2. *If newborns start with zero assets and an exiting worker with persistent shock z in region j is replaced by a newborn with an identical z in the same region, then the evolution of average consumption, $\bar{c}_{\ell,t}$, and average asset holdings, $\bar{a}_{\ell,t+1}$, for workers with shock z' in labor market ℓ are characterized by,*

$$
\begin{aligned}
\bar{c}_{\ell,t}(z') &= \sum_{j=1}^{J} \int \frac{\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z) f_\ell(z' - \rho\, z)}{\bar{\Lambda}_{\ell,t}(z')} \left(\Xi_{\ell,t}(z') + \kappa_t \tilde{y}_{j\ell,t} + \kappa_t \tilde{w}_{\ell,t}\, z'\right) \mathrm{d}z + \\
&\quad \sum_{j=1}^{J} \int \frac{\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z) f_\ell(z' - \rho\, z)}{\bar{\Lambda}_{\ell,t}(z')} (1-\delta)\kappa\, \bar{a}_{j,t}(z) \mathrm{d}z, \qquad (11) \\
\bar{a}_{\ell,t+1}(z') &= \sum_{j=1}^{J} \int \frac{\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z) f_\ell(z' - \rho\, z)}{\bar{\Lambda}_{\ell,t}(z')} \left[(1+r_t)(1-\kappa_t)\left(\tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\, z'\right) - (1+r_t)\Xi_{\ell,t}(z')\right] \mathrm{d}z \\
&\quad + \sum_{j=1}^{J} \int \frac{\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z) f_\ell(z' - \rho\, z)}{\bar{\Lambda}_{\ell,t}(z')} (1-\delta)\,(1+r_t)(1-\kappa_t)\, \bar{a}_{j,t}(z) \mathrm{d}z, \qquad (12)
\end{aligned}
$$

where δ is the fraction of agents that die in a period (death probability), and $\bar{a}_{j,t}(z)$ is the average asset holding of workers in labor market j and shock z at the end of period $t-1$. Moreover,

$$
\bar{\Lambda}_{\ell,t}(z') = \sum_{j=1}^{J} \int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z) f_\ell(z' - \rho\, z)\, \mathrm{d}z, \qquad (13)
$$

where $f_\ell(x)$ is the density of a normal with mean zero and variance $\sigma^2_{\epsilon,\ell}$.

Proposition 2 is interesting as it shows in which way the model connects and departs from an economy with a single labor market. With a single market, the expressions simplify and average consumption depends linearly on average cash-on-hand, as in Caballero (1990). With many labor markets, the average consumption in region ℓ depends on the average income and wealth of all individuals moving into labor market ℓ. In other words, the distribution of cash-on-hand in labor market ℓ is the mixture of the distributions of cash-on-hand from all individuals that move into labor market ℓ from all other labor market j, conditional on z. Thus, the average cash-on-hand is just the weighted average of the average in each origin labor market. Since consumption is a linear function of cash-on-hand, the result follows. Similarly for savings.

In addition, equation (12) shows that the distribution of wealth in a region depends on the patterns of sorting of workers with different levels of persistent income across states and their consumption-savings decisions. As we study in detail in Section 5, if savings behavior does not differ between workers with different z, or if workers do not sort across space due to differences in z, there would be virtually no difference in wealth across regions.

4 Estimation of model parameters

We estimate a large set of parameter related to the variance of transitory and persistent income shocks, mobility costs, and elasticity ν. We show that it is feasible to estimate parameters in two steps. First, obtain estimates for the variances and persistence parameter ρ, and in a second stage, estimate parameters for mobility costs and elasticity ν.

The discount factor β and interest rates, we calibrate directly. We use data on rents to calibrate $q_{\ell,t}$.[20]

In our estimation we assume an economy in a stationary equilibrium.

4.1 Estimation of income process with mobility and selection

There is a very large literature that estimates random income processes originating in the seminal work of Lillard and Willis (1978) and MaCurdy (1982). In general, this literature abstracts from differences in the volatility of income across regions and from mobility and self-selection of workers across space. In this section, we contribute to this literature by estimating the parameters that characterize the income process in different U.S. states, ρ, $\sigma^2_{\epsilon,\ell}$, and $\sigma^2_{\eta,\ell}$, accounting for the dynamic selection of workers due to unobservable characteristics.

The intuition for estimation is simple. Our model tightly links the distribution of workers across labor markets and residual income over time. The joint distribution of workers that at

[20] It is important to highlight that, while differences in $q_{\ell,t}$ affect disposable income, they will not affect savings in our model since these are part of the "permanent" income and a decrease in rents will increase consumption by the same amount.

the end of period t were located in labor market ℓ and had a labor supply of z' is

$$\Lambda_{j\ell,t-1,t}(z,z') = \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)\, f_\ell(z' - \rho\, z), \tag{14}$$

where $\mu_{j\ell,t}(z)$ is the conditional probability that a worker in j with past shock z moves to ℓ in period t and $f_{\epsilon,j}(z' - \rho\, z)$ is the conditional density that a worker in ℓ with past shock z gets a labor supply shock of z', which given our assumptions, is just a normal density with mean zero and variance $\sigma^2_{\epsilon,\ell}$. Importantly, note that all the elements in the previous equation depend on the value of the parameters ρ and $\sigma^2_{\epsilon,\ell}$, for all ℓ. In addition, note that $\mu_{j\ell,t}(z)$ also depends on these parameters, plus additional parameters and endogenous variables in our model, as economic conditions in different locations influence the choices of workers.

Let $\zeta'_{i\ell t}$ be the residual log-earnings of individual i in labor market ℓ at time t, which in the model are equal to,[21]

$$\log(earnings_{i,\ell,t}) = \log(\tilde{w}_{\ell,t}) + \log\left(1 + z'_{i,\ell,t} + \eta'_{i,\ell,t}\right).$$

Assuming that earnings shocks are not very large, we can approximate the previous expression by

$$\log(earnings_{i,\ell,t}) \approx \log(\tilde{w}_{\ell,t}) + z'_{i,\ell,t} + \eta'_{i,\ell,t}$$

Since $\tilde{w}_{\ell,t}$ are common for all workers in a region, residual log-earnings in the model are $\zeta'_{i\ell t} = z'_{i,\ell,t} + \eta'_{i,\ell,t}$. Our estimation procedure uses moments of residual log-earnings in the data and those implied by the model, and picks the value of the parameters that minimize the distance between the two.

For this, note that the marginal density of z' in the model, integrating over j and z, is $\bar{\Lambda}_{\ell,t}(z')$. Although the unconditional mean of z in all locations is zero, due to selection, the mean of z conditional on the selected labor market can be different. Nonetheless, the mean and variance

[21] We also construct residual log-earnings in the data. As is usual in the literature, residual log-earnings are obtained as the difference between observed log-earnings for an individual and the projected log-earnings for this individual, where projected earnings come from a Mincer regression on workers' demographic characteristics, and industry and state fixed effects.

of residual earnings $z' + \eta'$ for labor market ℓ can easily be computed as,

$$E\left[z'_{i\ell t} + \eta'_{i\ell t}|\ell, t\right] = E\left[z'_{i\ell t}|\ell, t\right] = \int z' \,\bar{\Lambda}_{\ell,t}(z')\mathrm{d}z' \tag{15}$$

$$Var\left[z'_{i\ell t} + \eta'_{i\ell t}|\ell, t\right] = Var\left[z'_{i\ell t}|\ell, t\right] + Var\left[\eta'_{i\ell t}\right] = \int \left(z' - E\left[z'|\ell\right]\right)^2 \bar{\Lambda}_{\ell,t}(z')\mathrm{d}z' + \sigma^2_{\eta,\ell} \tag{16}$$

In addition, the covariance of residual earnings for two consecutive periods is,

$$\begin{aligned} Cov((z_{i\ell t-1} + \eta_{i\ell t-1}), (z'_{i\ell t} + \eta'_{i\ell t})|j, \ell, t-1, t) &= E\left[z\, z'|j, \ell, t-1, t\right] - E\left[z|j, t-1\right] E\left[z'|\ell, t\right] \\ E\left[z\, z'|j, \ell\right] &= \int\int z'\, z\, \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)\, f_\ell(z' - \rho\, z)\, \mathrm{d}z\, \mathrm{d}z'. \end{aligned} \tag{17}$$

Note that the empirical counterparts for all these moments can be easily computed using data on worker's residual earnings and labor market choices over two consecutive periods.

The difficulty in estimation is that the right-hand side of these moments and of (14), depends not only on the autocorrelation and variance of the income process, but also on a large number of model parameters and endogenous variables that define $\mu_{j\ell,t}(z)$ in the model. However, if $\mu_{j\ell,t}(z)$ was known, an estimation procedure that seeks to minimize the distance between model generated moments conditional on parameters and moments in the data would be easy to implement.[22]

Our estimation uses this idea. Our procedure uses the empirical $\mu_{j\ell,t}(z)$ (or conditional choice probabilities) , which can be estimated using data on workers' mobility and residual income under a reasonable parametric assumption. The empirical $\hat{\mu}_{j\ell,t}(z)$ is already (implicitly) evaluated at the population parameter values for ρ, $\sigma^2_{\epsilon,\ell}$ and $\sigma^2_{\eta,\ell}$. Taking logs on both sides of (6) and assuming a stationary equilibrium, we can approximate the difference in log-mobility in the model as

$$\log(\mu_{j\ell}(z)) - \log(\mu_{jj}(z)) = \mathbb{C}_{1,j\ell} + \mathbb{C}_{2,j\ell}\, z + \mathcal{O}_{\ell j}$$

where the expression for log-mobility is not exactly linear in z due to Jensens' inequality in the last term of (6) in the numerator and denominator, and thus we have an approximation error, $\mathcal{O}_{\ell j}$. Parameters $\mathbb{C}_{1,j\ell}$ and $\mathbb{C}_{2,j\ell}$ for all j and ℓ depend on the underlying model parameters and

[22] Appendix C contains additional details on how we link residual income in the data and the model.

the stationary values of endogenous variables. Note that the approximation error can be smaller if we used a higher order polynomial in z to approximate the expression.[23]

To obtain a data counterpart to this expression we proceed as follows. Using individual-level data on mobility and earnigns, we estimate a multinomial Logit model for each origin labor market with residual earnings and destination specific constants, $\tilde{\mathbb{C}}_{1,j\ell}$ and $\tilde{\mathbb{C}}_{2,j\ell}$, where the tilde denotes that these are estimates from the data. Using these estimates from the empirical multinomial Logit model, we can project mobility matrices conditional on residual earnings, $\tilde{\mu}_{j\ell}(\zeta)$ for different level of residual income. The estimated coefficients of the empirical Logit regressions satisfy,

$$\log(\tilde{\mu}_{j\ell}(\zeta)) - \log(\tilde{\mu}_{jj}(\zeta) = \tilde{\mathbb{C}}_{1,j\ell} + \tilde{\mathbb{C}}_{2,j\ell}\,\zeta,$$

where $\tilde{\mu}_{j\ell}(\zeta)$ are the constructed mobility matrices using the estimates of the empirical model conditional on different levels of residual log-earnings.

In the data, we do not observe z but rather residual earnings $\zeta = z + \eta$. Nevertheless, we can express mobility as a function of residual log-earnings in the model as,

$$\log(\mu_{j\ell}(\zeta,\eta)) - \log(\mu_{jj}(\zeta,\eta)) = \mathbb{C}_{1,j\ell} + \mathbb{C}_{2,j\ell}\,\zeta - \mathbb{C}_{2,j\ell}\,\eta + \mathcal{O}_{\ell j}.$$

Therefore, this expression says that a regression of the difference in log-mobility rates by origin and destination on an origin-destination fixed-effect and residual log-earnings in model-generated data would not consistently estimate parameters $\mathbb{C}_{1,j\ell}$ and $\mathbb{C}_{2,j\ell}$ since the error term in that regression is correlated with the regressor. Nonetheless, denote by $\tilde{\mathbb{C}}^m_{1,j\ell}$ and $\tilde{\mathbb{C}}^m_{2,j\ell}$, the estimates of that regression in model generated data.

Using standard results from linear estimation under measurement error in the regressor, we have that $\tilde{\mathbb{C}}^m_{1,j\ell}$ is a consistent estimator of $\mathbb{C}_{1,j\ell}$, and $\tilde{\mathbb{C}}^m_{2,j\ell} = \frac{\sigma^2_{z,j}}{\sigma^2_{z,j}+\sigma^2_{\eta,j}}\,\mathbb{C}_{2,j\ell}$, where we use $\sigma^2_{\eta,j}$ and $\sigma^2_{z,j}$ since mobility at time t depends on the permanent component of income of the previous period. Thus, for given values of $\sigma^2_{z,j}$ and $\sigma^2_{\eta,j}$, and using empirical estimates $\tilde{\mathbb{C}}_{2,j\ell}$, which are the data counterpart to $\tilde{\mathbb{C}}^m_{2,j\ell}$, we can obtain a consistent estimate of $\mathbb{C}_{2,j\ell}$. Finally, using $\mathbb{C}_{1,j\ell}$ and

[23] Under different parameter combinations, we computed this expression in our model and found that it is mostly linear around the state space with the highest probability

$\mathbb{C}_{2,j\ell}$, we can obtain model-consistent estimates of mobility, $\mu_{j\ell}(z)$.

In this way, we can construct a minimum distance estimator (or a generalized method of moments estimator) to minimize the distance between the moments of residual log-earnings in the data and the right-hand side of equations (15), (16) and (17), using values for $\hat{\rho}$, $\hat{\sigma}^2_{\eta,\ell}$, and $\hat{\sigma}^2_{\epsilon,\ell}$, and estimated values for $\hat{\mu}_{j\ell,t}(z)$ obtained using $\mathbb{C}_{1,j\ell}$ and $\mathbb{C}_{2,\ell}$, for all ℓ.

Finally, our estimation also uses the covariance of residual log-earnings between period t and $t-2$ in the model and the data to estimate a value for the persistence parameter ρ.

In sum, our estimation procedure uses the estimated conditional choice probabilities from the data, $\tilde{\mu}(\zeta)$, to account for dynamic selection of workers across space due to their persistent income shocks.[24]

To implement our estimation, we use data from different sources. First, we estimate the conditional choice probabilities using data from the American Community Survey for the years 2015 to 2019.[25] For this, we first construct a measure of residual log-earnings using the same controls and sample selection criteria as discussed in Section 2. In addition, we have data on individuals' state of residence in the current year and a year before. Using this information, we estimate the multinomial Logit model discussed before and obtain coefficients $\tilde{\mathbb{C}}^m_{1,j\ell}$ and $\tilde{\mathbb{C}}^m_{2,j\ell}$.

Conditional on a value for ρ we only need moments on the variance of residual log-earnings and the covariance of residual log-earnings with its first-order lag at the individual level by U.S. states. For this, we use data from the CPS outgoing rotation group from 2010 to 2019. As in Section 2, we use individuals that we can link from one year to the next. Since in the CPS we only observe individuals that live in the same residence one year apart, in our estimation procedure, we only use model counterparts for stayers when constructing the minimum distance objective function.[26]

Finally, our estimate for ρ uses data on the covariance of residual log-earnings with its second-order lag for the whole economy. In the data, we compute this moment using the Panel Study

[24] The idea of using of conditional choice probabilities in the estimation of a subset of model parameters goes back to Hotz and Miller (1993). Here, we extend this idea to the estimation of random income processes.

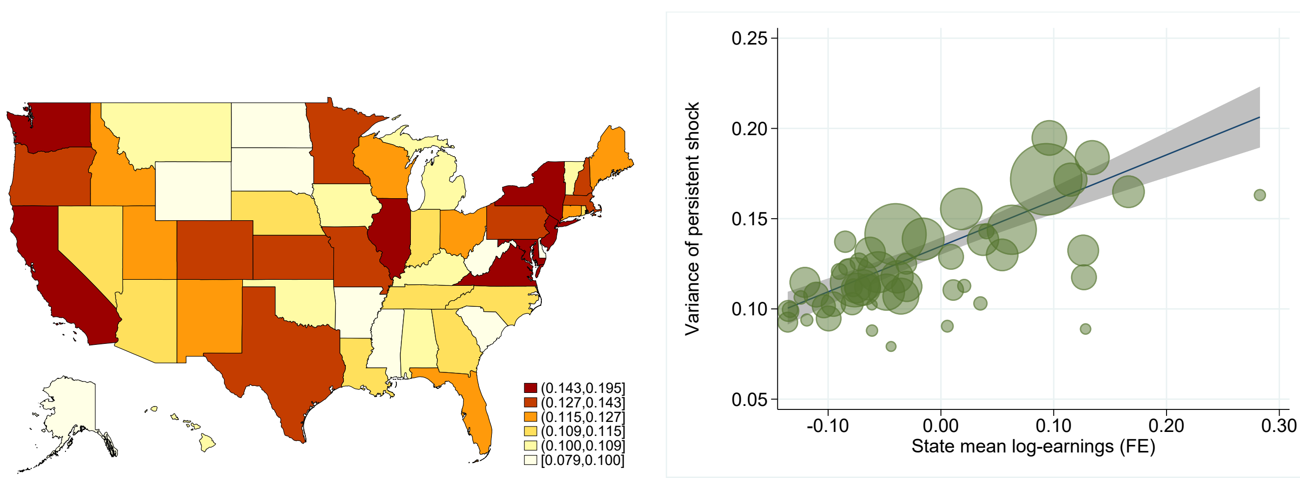
[25] We access the American Community Survey data via IPUMS, Ruggles et al. (2024).

[26] While other surveys offer a longer panel dimension, their sample size is small to adequately estimate properties of income process in states with few observations.

of Income Dynamics for the years 1969 to 1997. The estimated value of ρ is 0.914 at the annual frequency, which is close to the estimates by Floden and Lindé (2001) and Karahan and Ozkan (2013) which employ a similar random income process but without regions and migration.

Figures 5 and 6 show, respectively, the estimated value of the variance of the persistent and transitory component of residual log-earnings by U.S. state and their correlation with the state fixed effect from a Mincer regression of log-earnings that includes, in addition, demographic and industry controls. As Figure 5 shows, the variance of the persistent component is larger in the Northeast and West, and in some other states, such as Illinois, Texas, and Minnesota. At the extremes of the distribution, the variance of the persistent component can be more than one hundred percent larger. The correlation with the state mean log earnings (fixed effect), is quite strong and close to 0.8, and the variance of the persistent component is larger in states where earnings are, on average, higher.

Figure 5: Estimated Variance of Persistent Shock, $\sigma_\epsilon^2/(1-\rho^2)$



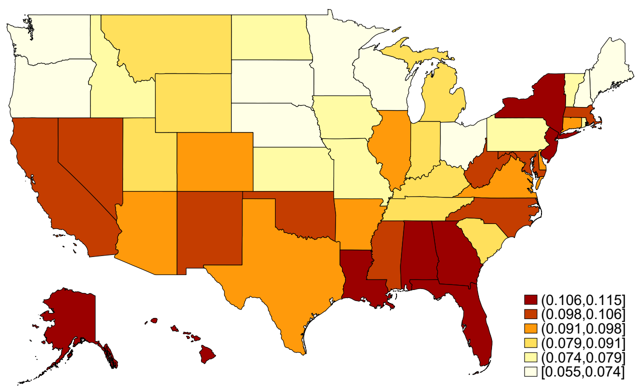
(a) Geographic distribution (b) Correlation with mean earnings

Note: Panel (a) shows the estimated variance of the persistent component of residual log-earnings, z, in each U.S. state, and panel (b) shows its correlation with the state fixed effect in a Mincer regression of log-earnings that includes also demographic and industry controls. See the text for estimation details and sample selection.

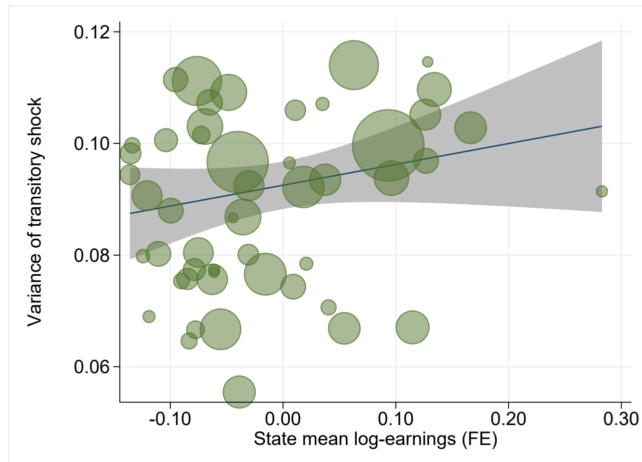
The variance of the transitory component presents some similarities and some striking differences. The variance of the transitory component is larger in the East Coast and parts of the West Coast, but also particularly large in the Southeast. While the dispersion across states is smaller, there is still an important heterogeneity. On the other hand, the correlation with the state mean log earnings (fixed effect) is weaker, at around 0.2.

As the figures implicitly suggest, the correlation between these variances across states is low, with a value slightly above 0.1.

Figure 6: Estimated Variance of Transitory Shock, σ_η^2



(a) Geographic distribution



(b) Correlation with mean earnings

Note: Panel (a) shows the estimated variance of the transitory component of residual log-earnings, η, in each U.S. state, and panel (b) shows its correlation with the state fixed effect in a Mincer regression of log-earnings that includes also demographic and industry controls. See the text for estimation details and sample selection.

One prediction of our model is that individuals living in states with higher variance of earnings shocks will have higher levels of savings, all else equal. We test this prediction by running a series of regressions of individual-level wealth measures on our estimated values for the variance of persistent and transitory shocks at the state level. Table 1 shows the results of these regressions. In columns (1) to (4), we follow the literature and transform wealth using the inverse hyperbolic sine. This function has the property that it approximates the logarithmic function well but can also be applied to negative values. Columns (1) and (2) show the results for total net worth, where the latter regression also includes controls for demographic characteristics, such as gender, race, education and a polynomial of order four in age. Since real estate represents a significant share of wealth which has a strong regional component but may not be necessarily related to precautionary savings, columns (3) and (4) show similar results but for non-housing net worth.

Table 1: Wealth and earnings volatility

	Asinh Total Net Worth		Asinh Non-housing Wealth		Non-housing Wealth / earnings	
	(1)	(2)	(3)	(4)	(5)	(6)
log-annual earnings	2.87***	2.22***	2.99***	2.34***		
	(0.08)	(0.10)	(0.09)	(0.11)		
$\sigma_\epsilon^2/(1-\rho^2)$	5.08*	2.66	8.59**	6.05*	9.83***	5.94*
	(2.34)	(2.68)	(2.58)	(2.82)	(2.64)	(2.25)
σ_η^2	−8.04	−1.01	−6.64	−0.58	−14.88**	−8.78*
	(5.64)	(5.85)	(6.16)	(6.12)	(4.58)	(4.00)
Demographic controls	no	yes	no	yes	no	yes
No. of Obs.	60877	60877	60866	60866	60866	60866

Note: Regression of individual-level wealth on state-level estimated variances of persistent and transitory shocks. SIPP data 2014-2019. Wealth measures in columns (1) to (4) are the inverse hyperbolic sine (asinh) of net-worth and non-housing net-worth. Columns (5) and (6) are the ratio of non-housing net-worth to earnings. Head of households only, 25 to 60 years old, where all family wealth is assigned to the head. We trim the top and bottom 1% of the distribution of wealth and earnings. Demographic controls include gender, race, education, and a polynomial of order four in age. Standard errors in parentheses clustered at state. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

Finally, columns (5) and (6) show similar regression, but using the ratio of wealth to earnings at the individual level. These last regresions do not have log-earnings as a control.

As the coefficient of log-earnings shows in columns (1) to (4), the wealth-to-earnings ratio is between two and three, which is consistent with estimates in the literature. The variance of persistent income shocks, $\sigma_\epsilon^2/(1-\rho^2)$ across all specifications is positive and, for the most part, statistically significative. In contrast, the coefficient for the variance of transitory shocks is estimated to be negative and, in most cases, not statistically significative.

Note that, since transitory shocks revert quickly to the mean, individuals do not need to accumulate an important amount of savings in order to effectively self-insure against these shocks. However, the effects of persistent shocks are long-lasting and the incentives to accumulate savings are stronger. Despite these patterns in the data, we highlight that these regression results should not be treated as causal. Our quantitative exercises in the nest section allows a closer inspection on the link between spatial differences and wealth.

4.2 Estimation of wages, mobility costs and the elasticity ν

The estimation of wages, mobility costs ψ and the elasticity ν uses the estimates of the variances and persistence parameters discussed in the previous subsection.

First, to estimate wages, note that, in the model,

$$E\left[\log(earnings_{i\ell t})|\ell,t\right] \;=\; \log(\tilde{w}_{\ell,t}) + E\left[z'_{i\ell t}|\ell,t\right].$$

The last term on the right is the mean value of z in region ℓ, which in general will not be zero due to selection. Nonetheless, given the estimates of mobility and the income process, we have the distribution of z between regions and can compute that expectation. Then, using the expectation of log-earnings from the data, we can recover $\log(\tilde{w}_{\ell,t})$.[27]

We estimate mobility costs and ν as follows. Recall that $\mathbb{B}_{1,j\ell} = -\psi_{j\ell} + \gamma\kappa\,\tilde{y}_{j\ell} - \frac{\kappa_t^2\gamma^2\tilde{w}_{\ell,t}^2}{2}(\sigma_{\epsilon,\ell}^2 + \sigma_{\eta,\ell}^2)$, with $\psi_{jj} = 0$ for all j. In this way, $\mathbb{B}_{1,jj}$ can be directly pinned down from the estimated values of wages, rents, interest rates, γ and estimates for the income process. In addition, let $\mathbb{B}_{2,\ell} = \gamma\kappa_t\tilde{w}_{\ell,t}\,\rho$, which is also pinned down from previously obtained parameter values.

Using the equilibrium condition for μ and Ξ from Proposition 1 and assuming a stationary equilibrium, we have,

$$e^{-\gamma\Xi_\ell(z')} = (\tilde{\beta}\bar{\lambda})^{\tilde{r}} e^{-\tilde{r}(\mathbb{B}_{1,\ell\ell}+\mathbb{B}_{2,\ell}z')} \left(E_{z''|z',\ell}\left[e^{-\gamma\Xi_\ell(z'')}\right]\right)^{\tilde{r}} (\mu_{\ell\ell}(z'))^{\frac{\tilde{r}}{\nu}}\; J^{\frac{\tilde{r}}{\nu}} \tag{18}$$

where $\tilde{\beta} = \beta(1+r)$ and $\tilde{r} = 1/(1+r)$. Expressed in logs,

$$-\gamma\Xi_\ell(z') = \tilde{r}\log(\tilde{\beta}\bar{\lambda}) - \tilde{r}(\mathbb{B}_{1,\ell\ell} + \mathbb{B}_{2,\ell}z') + \tilde{r}\log\left(E_{z''|z',\ell}\left[e^{-\gamma\Xi_\ell(z'')}\right]\right) + \frac{\tilde{r}}{\nu}\log\mu_{\ell\ell}(z') + \frac{\tilde{r}}{\nu}\log J,$$

which defines a fixed point for Ξ conditional on z given ν, $\mu(z)$ and the constant $\mathbb{B}_{2,\ell}$.

[27] Since worker's demographic characteristics and industry affect earnings, we use the average of the estimated state-time fixed effect as our left-hand side variable, which we obtain from the Mincer regression we use to obtain residual log-earnings, as discussed in Section 2.

Take the difference of log-mobility,

$$\begin{aligned}\log\left(\mu_{j\ell}(z)\right)-\log\left(\mu_{jj}(z)\right) &= \nu\left(\mathbb{B}_{1,j\ell}-\mathbb{B}_{1,jj}\right)+\nu\,\left(\mathbb{B}_{2,\ell}-\mathbb{B}_{2,j}\right)z \\ &\quad -\nu\left[\log\left(E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell}(z')}\right]\right)-\log\left(E_{z'|z,j}\left[e^{-\gamma\Xi_{j}(z')}\right]\right)\right]. \end{aligned} \tag{19}$$

The left-hand side (difference in log-mobility) is observed, and using the results from the previous section, we can approximate it as a linear function of z. Conditional on the value for ν, $\mathbb{B}_{1,jj}$, $\mathbb{B}_{2,j}$ and $E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell}(z')}\right]$, the right hand side has only $\mathbb{B}_{1,j\ell}$ as the unknown. Lets replace the left-hand side with the expressions from the previous section,

$$\begin{aligned}\mathbb{C}_{1,j\ell}+\mathbb{C}_{2,j\ell}\,z+\mathcal{O}_{j\ell}(z) &= \nu\left(\mathbb{B}_{1,j\ell}-\mathbb{B}_{1,jj}\right)+\nu\,\left(\mathbb{B}_{2,\ell}-\mathbb{B}_{2,j}\right)z \\ &\quad -\nu\left[\log\left(E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell}(z')}\right]\right)-\log\left(E_{z'|z,j}\left[e^{-\gamma\Xi_{j}(z')}\right]\right)\right]. \end{aligned} \tag{20}$$

where the coefficients in the left, $\mathbb{C}_{1,j\ell}$ and $\mathbb{C}_{2,j\ell}$ are estimated as discussed in the previous section and are also used to construct $\mu_{j\ell}(z)$. Then, taking expectations with respect to z conditional on j (as z is associated with the origin labor market), we have,

$$\begin{aligned}\mathbb{C}_{1,j\ell}+\mathbb{C}_{2,j\ell}\,E\left[z|j\right]+E\left[\mathcal{O}_{j\ell}(z)|j\right] &= \nu\left(\mathbb{B}_{1,j\ell}-\mathbb{B}_{1,jj}\right)+\nu\,\left(\mathbb{B}_{2,\ell}-\mathbb{B}_{2,j}\right)E\left[z|j\right] \\ &\quad -\nu E\left[\log\left(E_{z'|z,\ell}\left[e^{-\gamma\Xi_{\ell}(z')}\right]\right)-\log\left(E_{z'|z,j}\left[e^{-\gamma\Xi_{j}(z')}\right]\right)\Big|j\right]. \end{aligned} \tag{21}$$

Then, for a given value of ν, we can pick a value of $\mathbb{B}_{1,jj}$ that satisfies this equation under $E\left[\mathcal{O}_{\ell j}(z)|j\right]=0$. In this way, given a value of ν and solving a fixed point for the expected value of Ξ, we obtain all the needed values for $\mathbb{B}_{1,j\ell}$.

We estimate the Weibull shape parameter ν, which is closely related to the migration elasticity to changes in earnings, as equation (6) shows, to match observed spatial sorting patterns. Since there is complementarity between individual productivity z and wages, high-z households sort into high-wage locations. Higher values of ν correspond to less dispersion in idiosyncratic location preference shocks, so the strength of this sorting pattern is increasing in ν.[28] In other words,

[28] As $\nu \to 0$, location decisions are determined entirely by idiosyncratic preferences. In this case, all households choose each location with the same probability, regardless of z. As ν increases, pecuniary considerations become

the heterogeneity in persistent earnings shocks between individuals and the differences in their incentives to move across regions generates variation in the data that allows us to estimate parameter ν.[29]

We measure the strength of spatial sorting as follows. First, in each location we estimate the relationship between the probability of staying and z using the regression

$$\log(1 - \mu_{jj}(z)) = \mathrm{c} + \beta_j z + \varepsilon_z \tag{22}$$

where c is a constant and ε_z is an error term.[30] High-z households sort out of locations with $\beta_j < 0$. Since the gains from living in a high-wage location are increasing in z, there is a negative relationship between β_j and w_j. We measure the strength of sorting by estimating the regression

$$\beta_j = \mathrm{c} + \alpha \log(w_j) + \varepsilon_j \tag{23}$$

Intuitively, the more negative is α, the stronger is sorting by individual productivity. We choose ν such that the estimated coefficient from (23) in the model matches the empirically estimated coefficient $\hat{\alpha} = -1.068$. Figure 7 shows the relationship between β_j and w_j in both the model and the data. The estimated value of ν is 19.43.[31]

5 Regional differences and the geography of wealth

Using our model with the estimated parameters, we now evaluate the role of regional differences in explaining the spatial distribution of wealth. First, we examine the role of different parameters.

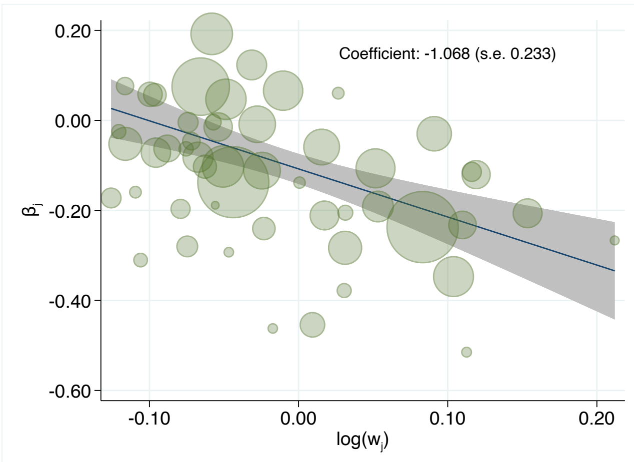
relatively more important and there is stronger sorting by z.

[29] In a model with homogeneous workers, Artuç et al. (2010) use changes in wages and mobility over time to identify the elasticity parameter (in their case, mobility was between industries). We do not use changes over time as heterogeneity in z between workers provides variability in earnings and mobility.

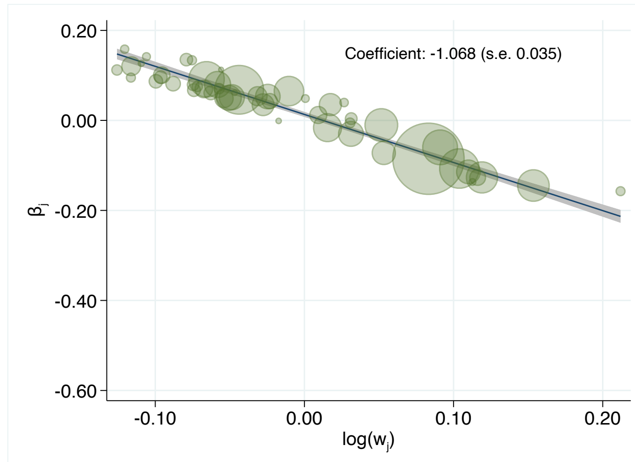
[30] The mobility matrices we construct from the data and for our numerical model are functions of z, which we discretize. We estimate (22) using weighted ordinary least squares, with weights equal to the probability distribution of z.

[31] Given our assumption for $\bar{\lambda} = (1/J)^{1/\nu}$, the mean for the distribution of the each Weibull shock is 1 and the standard deviation is 0.06. Higher values of ν compress the distribution around the mean and decrease the standard deviations.

Figure 7: Dynamic sorting: effect of residual earnings on the probability of moving



(a) Data



(b) Model

Note: Panel (a) shows the relationship between the estimates of β_j from regression (23) and log wages in the data. Panel (b) shows the same relationship in the model. β_j is a measure of the correlation between the probability of staying and individual productivity z in location j. The migration shares used for Panel (a) are computed using the procedure described in Section 4.1.

Then, we explain the role of sorting and differences in saving rates by people with different characteristics.

5.1 Persistence, frictions, and volatility

As discussed previously, saving for a rainy day affects the incentives to save for workers with different levels of persistent income z. In an economy with a single region, individuals with high and low levels of z (high and low levels of savings) will co-exist in the same region, with a limited effect on overall wealth. However, if individuals sort across regions based on the value of z, high savings individuals will be more numerous in some regions relative to others. Thus, an increase in the persistence parameter ρ will, all else equal, increase the incentives to sort across regions and increase the differences in wealth across regions. In addition, an increase in ρ leads to an increase in the effective variance of persistent income shocks, increasing precautionary savings.

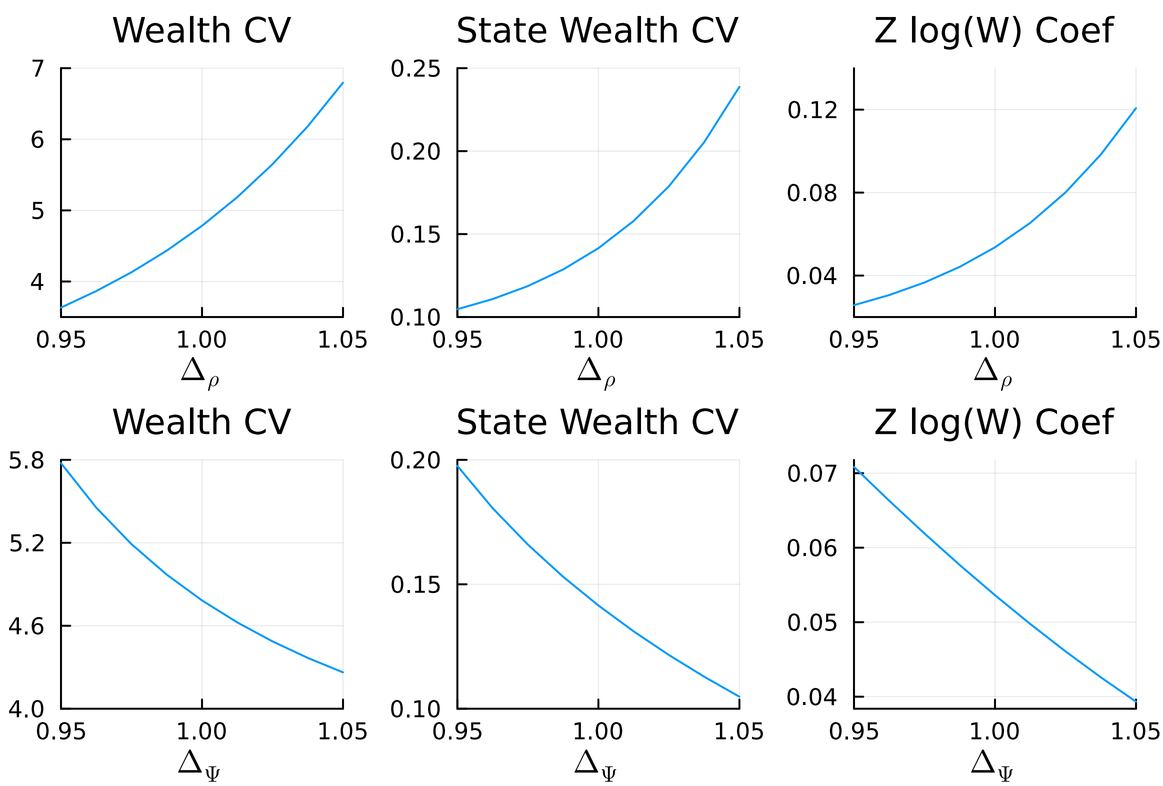
The top row of Figure 8 shows the effects of changing ρ in our model. We keep all parameters identical to the calibrated/estimated values and multiply ρ by $\Delta_\rho \in [0.95, 1.05]$. The left panel shows that overall wealth inequality, as measured by the coefficient of variation, increases with higher values of ρ. The middle panel shows that wealth inequality across states, as measured by the coefficient of variation of average state wealth, also increases with ρ. As previously discussed, inequality patterns are largely driven by spatial sorting. This is demonstrated by the right panel, which shows the intensity of sorting, as measured by the coefficient of a state-level regression of average z on log wage w. As ρ increases, z becomes more persistent and the correlation between z and w increases.

The bottom row of Figure 8 studies the effect of spatial frictions. As frictions distort how easy it is to move across regions and how much workers can profit from better sorting across space, an increase in the cost of moving will affect sorting and overall savings. We increase or decrease the cost of moving in all regions by multiplying the migration cost matrix Ψ by $\Delta_\Psi \in [0.95, 1.05]$. As the figure shows, an increase in the cost of moving in all regions leads to lower levels of wealth inequality and the dispersion of average wealth across states. While our analysis in Section 3 uncovered an additional motive to save due to preference shocks and mobility frictions in our model, which would increase savings in all regions, the figure shows that quantitatively, the distortion of sorting is a stronger force that ultimately leads to a lower level of wealth inequality. The right panel of the figure confirms this, as the correlation between z and state wages declines with higher values of mobility frictions.

We now evaluate how the dispersion in wages and wage volatility across states shape the distribution of wealth. Figure 9 shows the effects of reducing the dispersion in wages or the volatility of income shocks. To quantify these effects, we hold all parameters fixed at their calibrated/estimated values and set wages to $\Delta_w w_j + (1 - \Delta_w)\bar{w}$, where $\bar{w}$ is the population-weighted average wage and $\Delta_w \in [0.9, 1]$. Wages are less disperse than in the data when $\Delta_w < 1$, and equal to observed wages when $\Delta_w = 1$. We perform the analogous exercise for the volatilities of income shocks σ_ϵ^2 and σ_η^2.

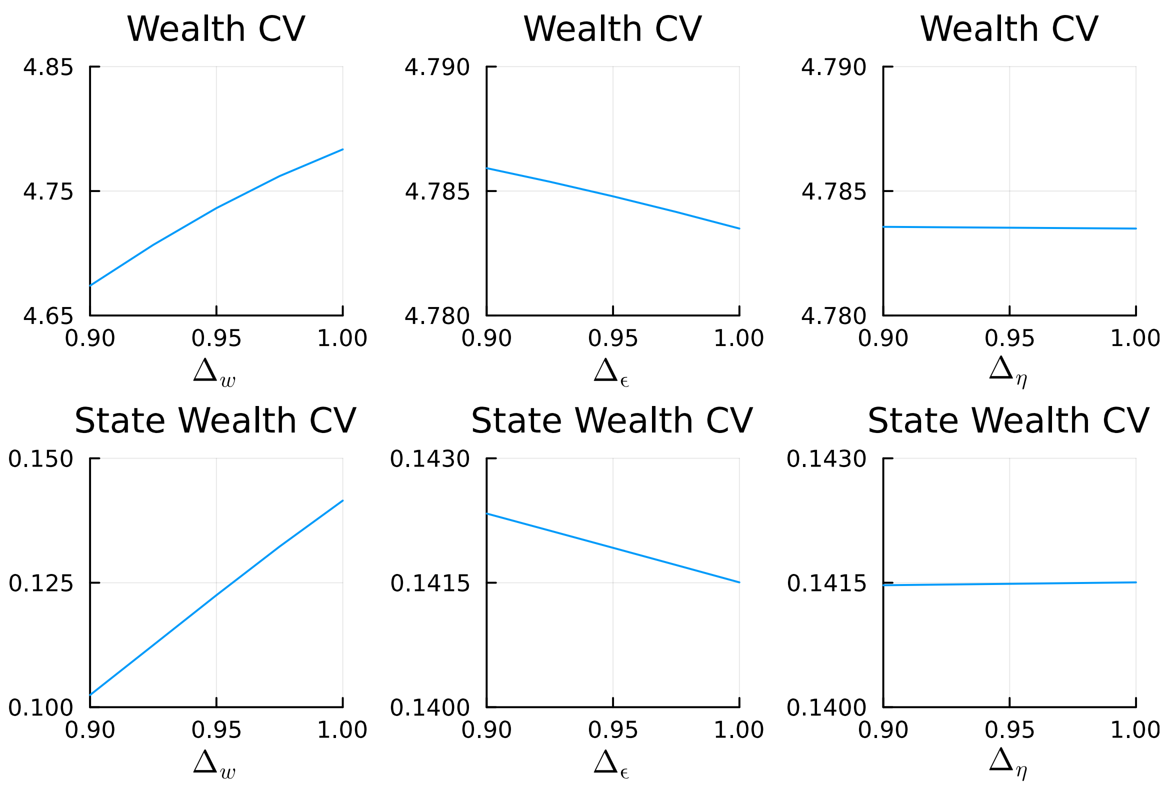
Decreasing wage dispersion reduces wealth inequality both in the aggregate and across space. This is driven in part by weaker sorting across space. The quantitative effects of the volatility of shocks is more limited, especially transitory shocks. In contrast to wage dispersion, increasing persistent volatility dispersion slightly lowers inequality. This occurs because there is a strongly

Figure 8: Effect of Parameters on Wealth Distribution



Note: The top row of figures shows how wealth inequality and spatial sorting vary with the persistence parameter ρ. In each figure, the z persistence parameter ρ is varied by plus or minus 5%. The left panel shows the coefficient of variation for the entire wealth distribution. The middle panel shows the coefficient of variation of average wealth across states. The right figure shows the coefficient of an across-state regression of average z on log wage. The bottom row of figures shows how outcomes vary when the migration cost matrix Ψ is varied by plus or minus 5%.

Figure 9: Effect of Dispersion on Wealth Distribution



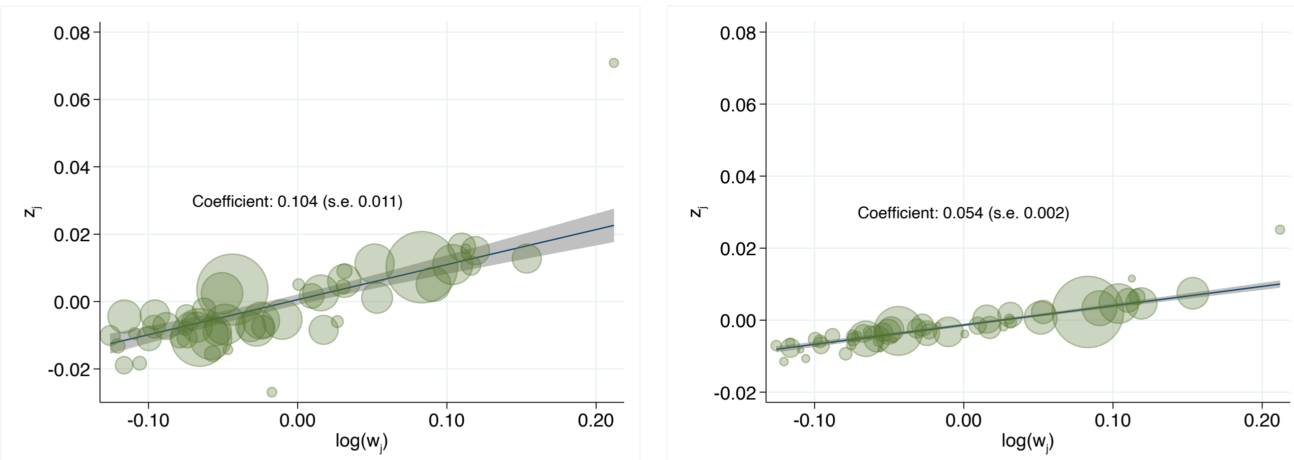
Note: The top row of figures shows how wealth inequality is affected by dispersion in wages, persistent income volatility σ_ϵ^2, and transitory income volatility σ_η^2 across space. The bottom row of figures shows how spatial inequality is affected by dispersion in these variables. We quantify the effects of dispersion in wages by setting wages equal to $\Delta_w w_j + (1-\Delta_w)\bar{w}$, where $\bar{w}$ is the population-weighted average wage. The last two columns show the analogous exercise for σ_ϵ^2 and σ_η^2.

positive correlation between wages and σ_ϵ^2 (see Figure 5). As such, increasing σ_ϵ^2 dispersion has the effect of making high-wage locations less attractive and low-wage locations more attractive. This reduces the incentive to sort by z, thereby lowering inequality.

5.2 Spatial sorting, savings and earnings volatility

Out theory and the previous quantitative exercises highlight the importance of persistent earnings shocks, sorting, and to a lesser extent, the volatility of income shocks, as forces shaping wealth. Figure 9 shows the strength of sorting in the model and our estimates in the data. In Section 4, we estimated the distribution of workers by z and labor market j, $\bar{\Lambda}_{j,t}(z)$, in the data. Since our model has fewer degrees of freedom, it cannot fit this distribution perfectly, but as Figure 9 shows, our model captures the relationship between average z and w by state quite well. In the model, this relationship is driven by the complementarity between wages and individual productivity, and the sorting of workers across space. In the data, the correlation between average z and $\log(w)$ is somewhat stronger than in our model, which implies that our model may underestimate the importance of sorting.

Figure 10: Sorting and the distribution of residual earnings across states



(a) Data (b) Model

Note: Panel (a) shows Panel (b) shows

In our model, different forces shape savings and wealth across regions. On the one hand, an increase in volatility of income, particularly of persistent income shocks, would lead to an increase in savings. Quantitatively, this effect tends to be small. However, note that, workers dislike income volatility, therefore, an increase in volatility in one region will lead to lower levels of inflows and higher outflows of population. Our estimates suggest that there is a positive correlation between the level of wages and the volatility of income. These two variables influence sorting in opposite ways. Then, while regions with high w attract workers with high z leading to an increase in savings and wealth in that regions, higher volatility pushes in the opposite direction, preventing a better sorting, despite an incentive to save in higher volatility regions.

In this way, both sorting, saving for a rainy day and precautionary savings will shape the

Table 2: Wealth and earnings volatility

	Data (1)	Model (2)
$E[z\|j]$	25.08*	5.84
	(13.17)	(7.08)
$\sigma_\epsilon^2/(1-\rho^2)$	5.81	3.38**
	(4.24)	(1.67)
σ_η^2	-11.96**	0.75
	(4.94)	(1.11)

Note: Regression of state-level wealth to earnings ratio on state-level estimated mean of persistent residual earnings, $E[z|j]$, and variances of persistent and transitory shocks. Standard errors in parentheses. $*p < 0.10$, $**p < 0.05$, $***p < 0.01$.

distribution of wealth in the data and the model. We investigate these reasons to save in our model and in the data using regression analysis. Table 2 shows how these features of the data explain the spatial distribution of wealth. We regress the average wealth to average income of each state on the mean of z for that state and our estimates of the volatility of income, both using real world data (column 1) and model-generated data (column 2). With only 51 data points and positive correlation across variables, it is difficult to obtain statistically significant coefficients in all variables. Nonetheless, point estimates are mostly as expected. Sorting is positively correlated with average wealth both in the data and the model. Persistent income shocks are also positively correlated with wealth, which implies that, at least to some extent, the volatility of persistent shocks affect precautionary savings. Similar to the regressions of the wealth to income ratio using microdata in Table 1, the volatility of transitory shocks has a negative sign in the data and is close to zero in the model, which at first sight may appear at odds with the theory, but our model can rationalize with the effect of the variance of these shocks affecting the pattern of sorting.

6 Conclusion

This paper provides a novel framework for understanding the spatial distribution of wealth in the United States by integrating regional differences in earnings volatility, mobility frictions, and precautionary savings motives. We develop a tractable model of consumption, savings, and location choice with incomplete markets and heterogeneous agents. Extending the seminal work of Caballero (1990) to an economy with many regions, mobility and sorting, we derive closed-form solutions to the consumption-savings problem that illuminate the complex interactions between the difference forces shaping wealth accumulation.

We first document important differences in the volatility of income and wealth across US states. In our model, standard forces shape the consumption savigs decision, such as impatience, precautionary savings, and saving for a rainy day. In the context of a spatial economy with preference shocks, mobility frictions, and incomplete markets, we identify an additional precautionary savings motive, as individuals save to hedge against potential adverse effects of future preference shocks. In addition, we highlight how the complementarity of wages in a region with individual-level productivity, generates selection of individuals with different savings incentives, which help explain the patterns of wealth and savings across regions.

These findings highlight the importance of considering regional heterogeneity and worker mobility when studying wealth inequality. Future research could extend this model to incorporate additional factors such as housing markets and life-cycle wealth dynamics to further enriching our understanding of the complex dynamics shaping the spatial distribution of wealth in modern economies.

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Appendix A Wealth data

The traditional source of micro-level wealth data for the United States is the Survey of Consumer Finances, produced by the Board of Governors of the Federal Reserve System. Unfortunately, the publicly available sample does not contain information on individuals' location, such as state or city. In our empirical analysis, we use wealth the Survey of Income and Program Participation, which has individual-level data on wealth and many other characteristics.

In this appendix, we compare these two surveys and find that they provide similar information on wealth for the overall U.S. economy, which gives us confidence about our state-level analysis.

A.1 Survey of Income and Program Participation

The Survey of Income and Program Participation (SIPP) is a household-based survey designed as a series of nationally representative panels. Each panel generally features a large sample of households that are interviewed multiple times over a three or four-year period, depending on the year. From 1996 to 2017, the survey is structured as non-overlapping panels, with surveys every 4 months. Beginning in 2018, the survey switched to having overlapping panels interviewed annually. The SIPP provides the most comprehensive information available on how the nation's economic well-being changes over time, which has been the SIPP's defining characteristic since 1983. The wealth data is part of the "Assets and Liabilities" topical module available in different waves across the SIPP's panels over time. There is a module with information about financial assets in every year from 1994 to 2021, excluding 2006, 2007, 2008, and 2012. The different asset and liability categories available in the SIPP are not as comprehensive and detailed as those in the Survey of Consumer Finances, as we discuss below.

A.2 Survey of Consumer Finances

The Survey of Consumer Finances (SCF) is administered by the Board of Governors and measures the financial characteristics of the U.S. population at the household level. There are two samples in the SCF- the first of which includes about 75% of the observations and is intended to provide good coverage of broad population characteristics such as home ownership. The second sample is made up of relatively wealthy households to help capture the top of the wealth distribution. Sampling is generally done at the level of a "Primary Economic Unit" (PEU) made up of an economically dominant individual or couple and the individuals who are financially dependent on them. Wealth data in the SCF is available every 3 years starting in 1989.

Since not all assets and liabilities in the SCF are included in the SIPP we construct our measures of net-worth and net-worth excluding housing using the following: Business assets; checking and savings, which includes bonds, CDs, money market accounts, and other cash; stock; private IRA; vehicle net worth; house value; residential debt; nonresidential assets; other assets.

A.3 Comparing the SIPP and SCF

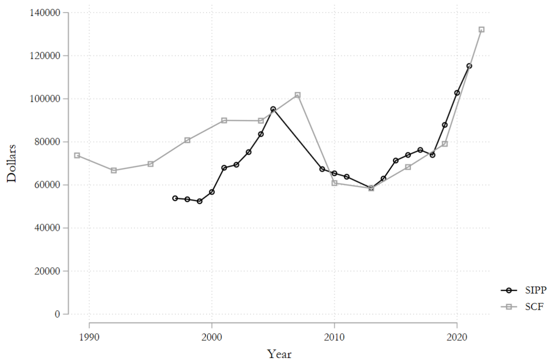
The two main differences between the SIPP and the SCF, oversampling of the top wealth individuals and larger asset and liabilities categories will translate into differences in the estimated wealth distribution across these surveys. As mentioned before, we construct comparable measures of net-worth across surveys to correct for one of these problems.

The figures below show the evolution over time of moments of wealth distribution across surveys. We focus on the median and the percentiles 75 and 90 to correct for the smaller sample of top-wealth individuals in the SIPP. Notable differences arise across surveys beyond the 90 percentile. Moreover, the mean is also affected, with lower values of average wealth computed using the SIPP. We deflate wealth by the CPI for that year.

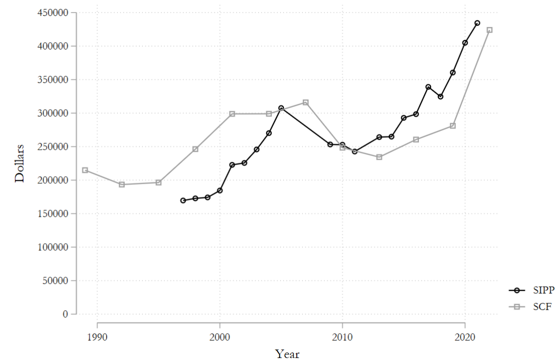
As Figure 11 shows, the moments of net-worth across surveys behave similarly over time and the levels are very close to each other, particularly since mid-2000.

Figure 11: Moments of the Wealth Distribution in the United States

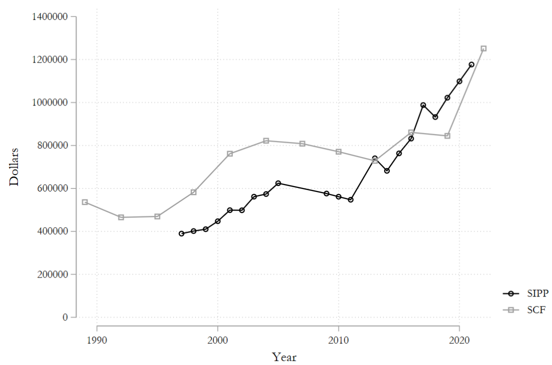
(a) 50th percentile of net worth



(b) 75th percentile of net worth



(c) 90th percentile of net worth



Note: 2007 Dollars. Deflated by CPI. Sources: SIPP, SCF, and authors' calculations.

Appendix B Proofs

B.1 Poposition 1

Equations (1) and (2) describe the problem of the worker.

$$V_{j,t}(a,z,\varepsilon) = \max_{\ell} \left\{ \varepsilon_\ell \, e^{\psi_{j\ell}} E_{\eta',z'|z,\ell} \left[\max_{c_{j\ell,t}, a^e_{j\ell,t+1}} \left\{ \left(-\frac{1}{\gamma}\right) e^{-\gamma \, c_{j\ell,t}} + \beta \left[E_{\varepsilon'} \left[V_{\ell,t+1}(a_{j\ell,t+1}, z', \varepsilon') \right] \right] \right\} \right] \right\}$$

$$\text{s.t.:} \qquad c_{j\ell,t} + \frac{1}{(1+r_t)} a_{j\ell,t+1} + q_{\ell,t} = w_{\ell,t}(1-\tau)\,(1+z'+\eta') + (a - \delta_{j\ell})$$

Begin with the optimal choice of consumption and savings after the reallocation decision has occurred. We can write the first-order condition for consumption and savings as,

$$e^{-\gamma\, c_{j\ell,t}(a,z',\eta')} = \beta(1+r_t)\, \frac{\partial E_{\varepsilon'}\left[V_{\ell,t+1}(a_{j\ell,t+1}(a,z',\eta'),z',\varepsilon')\right]}{\partial a_{j\ell,t+1}(a,z',\eta')}. \tag{24}$$

Since the realization of preference shock ε_ℓ for the chosen labor market multiplies the period and continuation utility, it does not affect differentially consumption and savings and does not show up in the first order condition.

We now use a guess-and-verify strategy. Assume that optimal consumption, savings and ex-ante value function, $v_{j,t}(a,z) = E_\varepsilon\left[V_{j,t}(a,z,\varepsilon)\right]$, take the following form,

$$\begin{aligned} c_{j\ell,t}(a,z',\eta') &= \Xi_{\ell,t}(z') + \kappa_t \left(a + \tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\, z' + \tilde{w}_{\ell,t}\, \eta'\right) \\ a_{j\ell,t+1}(a,z',\eta') &= -(1+r_t)\, \Xi_{\ell,t}(z') + (1+r_t)(1-\kappa_t)\left(a + \tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\, z' + \tilde{w}_{\ell,t}\, \eta'\right) \\ v_{j,t}(a,z) &= -\frac{1}{\gamma}\frac{1}{\kappa_t}\, e^{-\gamma\, \kappa_t\, a}\, \Omega_{j,t}(z) \end{aligned}$$

where, given the guess for consumption, next period assets are obtained using the budget constraint. Also, given the guess for the ex-ante value function, we have

$$\frac{\partial E_{\varepsilon'}\left[V_{\ell,t+1}(a_{j\ell,t+1}(a,z',\eta'),z',\varepsilon')\right]}{\partial a_{j\ell,t+1}(a,z',\eta')} = \frac{\partial v_{\ell,t+1}(a_{j\ell,t+1}(a,z'\eta'),z')}{\partial a_{j\ell,t+1}(a,z',\eta')} = -\gamma\, \kappa_{t+1} v_{\ell,t+1}(a_{j\ell,t+1}(a,z',\eta'),z'). \tag{25}$$

The unknown functions $\Xi_{\ell,t}(z')$ and κ_t can be obtained by matching coefficients after taking logarithm in the first order condition (24),

$$\begin{aligned} -\gamma\left[\Xi_{\ell,t}(z') + \kappa_t\left(a + \tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\, z' + \tilde{w}_{\ell,t}\, \eta'\right)\right] &= \log\left(\beta(1+r_t)\right) + \log\left(\Omega_{\ell,t+1}(z')\right) \\ &\quad -\gamma\kappa_{t+1}\left[-(1+r_t)\, \Xi_{\ell,t}(z') + (1-\kappa_t)(1+r_t)\left(a + \tilde{y}_{j\ell,t} + \tilde{w}_{\ell,t}\, z' + \tilde{w}_{\ell,t}\, \eta'\right)\right]. \end{aligned}$$

Then,

$$\begin{aligned}\kappa_t &= \frac{\kappa_{t+1}(1+r_t)}{1+\kappa_{t+1}(1+r_t)} \\ \Xi_{\ell,t}(z') &= -\frac{1}{\gamma}\left(\frac{1}{1+\kappa_{t+1}(1+r_t)}\right)\left[\log\left(\beta(1+r_t)\right)+\log\left(\Omega_{\ell,t+1}(z')\right)\right]. \end{aligned} \tag{26}$$

Using (24) and (25), we get that,

$$e^{-\gamma\, c_{j\ell,t}(a,z',\eta')} = \beta(1+r_t)\left(-\gamma\kappa_{t+1}\right)\, E_{\varepsilon'}\left[V_{\ell,t+1}(a_{j\ell,t+1}(a,z',\eta'),z',\varepsilon')\right]. \tag{27}$$

Substituting into the value function of the worker under optimal consumption/savings choices, and taking expectations over ε, we have,

$$v_{j,t}(a,z) = E_\varepsilon\left[\max_\ell\left\{\varepsilon_\ell\, e^{\psi_{j\ell}} E_{\eta',z'|z,\ell}\left[\left(-\frac{1}{\gamma}\right)\frac{1}{\kappa_t}e^{-\gamma c_{j\ell,t}(a,z',\eta')}\right]\right\}\right]. \tag{28}$$

Then, replacing consumption for the guess,

$$v_{j,t}(a,z) = \left(\frac{1}{\gamma}\right)\frac{1}{\kappa_t}\, e^{-\gamma\kappa_t\, a}\, E_\varepsilon\left[\max_\ell\left\{-\varepsilon_\ell\, e^{\psi_{j\ell}} E_{\eta',z'|z,\ell}\left[e^{-\gamma\left(\Xi_{\ell,t}(z')+\kappa_t\left(\tilde{y}_{j\ell,t}+\tilde{w}_{\ell,t}\, z'+\tilde{w}_{\ell,t}\,\eta'\right)\right)}\right]\right\}\right]. \tag{29}$$

Finally, given the guess for the ex-ante value function,

$$\begin{aligned}\Omega_{j,t}(z) &= -E_\varepsilon\left[\max_\ell\left\{-\varepsilon_\ell e^{\psi_{j\ell}} E_{\eta',z'|z,\ell}\left[e^{-\gamma\left(\Xi_{\ell,t}(z')+\kappa_t\left(\tilde{y}_{j\ell,t}+\tilde{w}_{\ell,t}\, z'+\tilde{w}_{\ell,t}\,\eta'\right)\right)}\right]\right\}\right] \\ &= \bar{\lambda}\left[\frac{1}{J}\sum_{\ell=1}^{J} e^{\nu A_{\ell,t}-\nu\psi_{j\ell}+\nu\,\gamma\kappa_t\tilde{y}_{j\ell,t}}\left(E_{\eta',z'|z,\ell}\left[e^{-\gamma\,\Xi_{\ell,t}(z')-\gamma\kappa_t\tilde{w}_{\ell,t}\,(z'+\eta')}\right]\right)^{-\nu}\right]^{-1/\nu}, \end{aligned} \tag{30}$$

where the last line uses properties of the Weibull distribution discussed next. In this way, the guess for consumption, savings and ex-ante value function are verified. Moreover, replacing this last expression one period forward in (26), defines the recursion for $\Xi_{\ell,t}(z')$ in the proposition.

Spatial Euler equation. Note that we can write (24) using (25) and (28) as,

$$e^{-\gamma\, c_{j\ell,t}(a,z',\eta')} = \beta(1+r_t)\left(-E_\varepsilon\left[\max_\ell\left\{-\varepsilon_\ell\, e^{\psi_{j\ell}} E_{\eta'',z''|z',m}\left[e^{-\gamma c_{\ell m,t+1}(a',z'',\eta'')}\right]\right\}\right]\right),$$

and using properties of the Weibull distribution we have the spatial Euler equation described in the text.

We already used a property of the Weibull distribution to compute the expectation of the maximum over ε. We also show that properties of the distribution allows us characterize the share of movers. For this we simplify the notation using the following lemma.

Lemma 1. *Let ε be a vector of size J of i.i.d. random variables distributed Weibull with identical shape parameter $\nu > 0$ and scale parameter $\lambda = \bar{\lambda} \frac{J^{1/\nu}}{\Gamma\left(1+\frac{1}{\nu}\right)}$, and let $\mathbb{X} \in \mathbb{R}^J_{(-)}$ be a vector of strictly negative scalars that may vary by labor market ℓ, then the expectation of the maximum of $\varepsilon \, \mathbb{X}_t$ and the share of draws for which element ℓ is maximum are,*

$$
\begin{aligned}
E_\varepsilon\left[\max_\ell \varepsilon_\ell \, \mathbb{X}_\ell\right] &= \left(\frac{1}{J}\sum_{\ell=1}^{J}(-\mathbb{X}_\ell)^{-\nu}\right)^{-1/\nu}, \\
\mu_{j\ell,t} = Pr\left(\varepsilon_\ell \, \mathbb{X}_\ell \geq \varepsilon_k \, \mathbb{X}_k;\, \forall k\right) &= \frac{(-\mathbb{X}_\ell)^{-\nu}}{\sum_{\ell=1}^{J}(-\mathbb{X}_\ell)^{-\nu}}.
\end{aligned}
$$

First, note that $E_\varepsilon\left[\max_\ell \varepsilon_\ell \, \mathbb{X}_\ell\right] = -E_\varepsilon\left[\min_\ell \varepsilon_\ell \, (-\mathbb{X}_\ell)\right]$. Since all elements $\mathbb{X}_\ell$ are negative, the terms $(-\mathbb{X}_\ell)$ are positive. Let $F(x)$ and $f(x)$ be the distribution and density functions of a Weibull random variable, respectively, then,

$$
\begin{aligned}
E_\varepsilon\left[\min_\ell \varepsilon_\ell \, (-\mathbb{X}_\ell)\right] &= \sum_{\ell=1}^{J}\int_0^\infty \varepsilon_\ell \, (-\mathbb{X}_\ell) \prod_{k\neq\ell} Pr_{\varepsilon_k}\left(\varepsilon_k(-\mathbb{X}_k) \geq \varepsilon_\ell(-\mathbb{X}_\ell)\right) f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\
&= \sum_{\ell=1}^{J}\int_0^\infty \varepsilon_\ell \, (-\mathbb{X}_\ell) \prod_{k\neq\ell}\left[1 - F(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\mathbb{X}_k))\right] f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\
&= \sum_{\ell=1}^{J}\int_0^\infty \varepsilon_\ell \, (-\mathbb{X}_\ell) \prod_{k\neq\ell} e^{-(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\lambda\mathbb{X}_k))^\nu} \frac{\nu}{\lambda}\left(\frac{\varepsilon_\ell}{\lambda}\right)^{\nu-1} e^{-(\varepsilon_\ell/\lambda)^\nu}\mathrm{d}\varepsilon_\ell \\
&= \sum_{\ell=1}^{J}\int_0^\infty (-\mathbb{X}_\ell)\nu e^{-\left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \sum_{k=1}^{J}\left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu} \left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \mathrm{d}\varepsilon_\ell
\end{aligned}
$$

define $z = \left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \sum_{k=1}^J \left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu$, then using a change of variables,

$$\begin{aligned} E_\varepsilon\left[\min_\ell \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\right] &= \sum_{\ell=1}^J(-\mathbb{X}_\ell)\lambda\left(\sum_{k=1}^J\left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu\right)^{-1-\frac{1}{\nu}}\left[\int_0^\infty z^{1/\nu}e^{-z}\mathrm{d}z\right] \\ &= \lambda\left(\sum_{\ell=1}^J(-\mathbb{X}_\ell)^{-\nu}\right)^{-1/\nu}\Gamma\left(1+\frac{1}{\nu}\right). \end{aligned}$$

Then, given the assumption for λ,

$$E_\varepsilon\left[\max_\ell \varepsilon_\ell\,\mathbb{X}_\ell\right] = -E_\varepsilon\left[\min_\ell \varepsilon_\ell\left(-\mathbb{X}_\ell\right)\right] = -\bar{\lambda}\left(\frac{1}{J}\sum_{\ell=1}^J(-\mathbb{X}_\ell)^{-\nu}\right)^{-1/\nu}.$$

Finally, letting $(-\mathbb{X}_\ell) = e^{\psi_{j\ell}}E_{z'|z}\left[e^{-\gamma\left(\Xi_{\ell,t}(z')+\kappa_t\left(\tilde{y}_{j\ell,t}+\tilde{w}_{\ell,t}\,(z'+\eta')\right)\right)}\right]$, then Lemma 1 shows that (30) holds.

To find the mobility probabilities, we follow similar steps.

$$\begin{aligned} Pr\left[\varepsilon_\ell\,\mathbb{X}_\ell \geq \varepsilon_k\,\mathbb{X}_k;\quad \forall k\right] &= Pr\left[\varepsilon_\ell\left(-\mathbb{X}_\ell\right) \leq \varepsilon_k\left(-\mathbb{X}_k\right);\quad \forall k\right] \\ &= \int_0^\infty \prod_{k\neq\ell} Pr_{\varepsilon_k}\left(\varepsilon_k(-\mathbb{X}_k) \geq \varepsilon_\ell(-\mathbb{X}_\ell)\right) f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\ &= \int_0^\infty \prod_{k\neq\ell}\left[1 - F(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\mathbb{X}_k))\right] f(\varepsilon_\ell)\mathrm{d}\varepsilon_\ell \\ &= \int_0^\infty \prod_{k\neq\ell} e^{-(\varepsilon_\ell(-\mathbb{X}_\ell)/(-\lambda\mathbb{X}_k))^\nu}\frac{\nu}{\lambda}\left(\frac{\varepsilon_\ell}{\lambda}\right)^{\nu-1} e^{-(\varepsilon_\ell/\lambda)^\nu}\mathrm{d}\varepsilon_\ell \\ &= \int_0^\infty e^{-\left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu\sum_{k=1}^J\left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu}\frac{\nu}{\lambda}\left(\frac{\varepsilon_\ell}{\lambda}\right)^{\nu-1}\mathrm{d}\varepsilon_\ell \end{aligned}$$

define $z = \left(\frac{\varepsilon_\ell}{\lambda}\right)^\nu \sum_{k=1}^J \left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu$, then using a change of variables,

$$\begin{aligned} Pr\left[\varepsilon_\ell\,\mathbb{X}_\ell \geq \varepsilon_k\,\mathbb{X}_k;\quad \forall k\right] &= \left(\sum_{k=1}^J\left(\frac{(-\mathbb{X}_\ell)}{(-\mathbb{X}_k)}\right)^\nu\right)^{-1}\left[\int_0^\infty e^{-z}\mathrm{d}z\right] \\ &= \frac{(-\mathbb{X}_\ell)^{-\nu}}{\sum_{k=1}^J(-\mathbb{X}_k)^{-\nu}}. \end{aligned}$$

Letting $(-\mathbb{X}_\ell) = e^{\psi_{j\ell}} E_{z'|z}\left[e^{-\gamma\left(\Xi_{\ell,t}(z')+\kappa_t\left(\tilde{y}_{j\ell,t}+\tilde{w}_{\ell,t}\,(z'+\eta')\right)\right)}\right]$, we have the expression (6) defining mobility.

B.2 Proposition 2

Let, $\bar{\Lambda}_{j,t-1}(z)$ be the distribution of workers with characteristics e that at the end of period $t-1$ were located in labor market j and had a labor supply shock z. We take this distribution to be conditional on period, such that integrating over labor market and z adds to one. Then, the joint distribution of those workers that at the end of period t located in labor market ℓ and had a labor supply of $1+z'+\eta'$ is $\bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)\, f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}$, where $\mu_{j\ell,t}(z)$ is the conditional probability that a worker in j with a past shock z moves to ℓ in period t, $f_{\epsilon',\ell}(z'-\rho\, z)$ is the conditional density that a worker in labor market ℓ with past shock z gets a labor supply shock of z', and $f_{\eta',\ell}$ is the density that the worker gets a shock η'. Therefore, equation (13) is the marginal of this joint distribution, integrating over j, z and η',

$$\bar{\Lambda}_{\ell,t}(z') = \sum_{j=1}^{J} \int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z) f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')\, \mathrm{d}z\, \mathrm{d}\eta'.$$

This expression uses the fact that workers who die are replaced by a newborn worker with identical z in the same labor market. Note that the realization for η' is observed after reallocation, thus there is no selection on this shock, and the previous equation simplifies to

$$\bar{\Lambda}_{\ell,t}(z') = \sum_{j=1}^{J} \int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z) f_{\ell}(z'-\rho\, z)\, \mathrm{d}z.$$

Define total consumption in labor market ℓ at the end of period t by workers with labor supply z' to be $\bar{c}_{\ell,t}(z')\,\bar{\Lambda}_{\ell,t}(z')$. Similarly, denote $\bar{a}_{\ell,t+1}(z')\,\bar{\Lambda}_{\ell,t}(z')$ as the total asset holdings of the workers, implicitly defining the average asset holdings.

Total consumption and assets are the result of the aggregation of consumption and savings decisions of workers who moved from j to ℓ and face a new realization of z'. Using the optimal decisions from Propostion 1, we have,

$$\bar{c}_{\ell,t}(z')\,\bar{\Lambda}_{\ell,t}(z') = \sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')\,(\Xi_{\ell,t}(z')+\kappa_t\tilde{y}_{j\ell,t}+\kappa_t\tilde{w}_{\ell,t}\,(z'+\eta'))\,\mathrm{d}z\,\mathrm{d}\eta' + (1-\delta)\sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')\,\bar{a}_{j,t}(z)\mathrm{d}z\,\mathrm{d}\eta', \quad (31)$$

$$\bar{a}_{\ell,t+1}(z')\,\bar{\Lambda}_{\ell,t}(z') = \sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')\,[(1+r_t)(1-\kappa_t)\,(\tilde{y}_{j\ell,t}+\tilde{w}_{\ell,t}\,(z'+\eta'))-(1+r_t)\Xi_{\ell,t}(z')]\,\mathrm{d}z\,\mathrm{d}\eta' + (1-\delta)\sum_{j=1}^{J}\int\int \bar{\Lambda}_{j,t-1}(z)\mu_{j\ell,t}(z)f_{\epsilon',\ell}(z'-\rho\, z)\, f_{\eta',\ell}(\eta')\,(1+r_t)(1-\kappa_t)\,\bar{a}_{j,t}(z)\mathrm{d}z\,\mathrm{d}\eta' \quad (32)$$

Where the second line in each of this expressions use the fact that newborns start with an identical value for assets equal to zero. Note that this expressions are equal to (11) and (12) after simplifying the integrals over η' as the density integrates to one and the mean is zero. In addition, aggregation over past value of assets is simply the average value of assets at the end of period $t-1$ times the number of individuals in that labor market. As optimal consumption and savings decisions are linear in assets and shocks, aggregates and averages are straightforward to calculate.

Appendix C Estimation of income process: additional details

Compute the expectation of log-earnings for the cross-section of workers conditional on labor market,

$$E\left[\log(earnings_{i\ell t})|\ell,t\right] = \log(\tilde{w}_{\ell,t}) + \underbrace{E\left[z'_{i\ell t}|\ell,t\right]}_{func(\rho,\sigma^2_{\epsilon,\ell,t},\mu_{j\ell}(z))}$$

where the expectation for the persistent shock conditional on labor market and workers' characteristics would, in general, not be zero due to selection, and is a function of parameters ρ, $\sigma^2_{\epsilon,\ell,t}$, and the distribution (see equation 15).

Define residual log-earnings as log-earnings minus the conditional expectation. Then, the variance of residual log earnings conditional on labor market, time, is

$$Var\left[\left(\log(earnings_{i\ell t}) - E\left[\log(earnings_{i\ell t})\right)|\ell, t\right]|\ell, t\right] = \underbrace{Var\left[z'_{i\ell t}|\ell, t\right]}_{func(\rho, \sigma^2_{\epsilon,\ell,t}, \mu_{j\ell}(z))} + \underbrace{Var\left[\eta'_{i\ell t}\right]}_{\sigma^2_{\eta,\ell,t}},$$

where the variance of the persistent shock is a function of parameters ρ, $\sigma^2_{\epsilon,\ell,t}$, and the distribution (see equation 16).

In this way, by regressing log-earnings on labor market-time fixed effects, we obtain residual log-earnings, and we use these in our estimation procedure to estimate the variances of the shocks.

In addition, using the estimated joint distribution of workers in labor market ℓ with shock z', we can easily calculate the conditional mean $E\left[z'_{i\ell t}|\ell, t\right]$, and remove this from the estimated conditional mean of log-earnings (fixed effect), obtaining estimates for $\log(\tilde{w}_{\ell,t})$ that we can use in our calibrated model.

Appendix D Additional Tables and Figures

Figure 12: Saving for a rainy day: All locations