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Abstract

We study climate policy in an economy with heterogeneous households, two types of goods (clean and dirty), and a climate externality from the dirty good. Using household expenditure and emissions data, we document that low-income households have higher emissions per dollar spent than high-income households, making a carbon tax regressive. We build a model that captures this fact and study climate policies that are neutral with respect to the income distribution. A central feature of these policies is that resource transfers across consumers are ruled out. We show that the constrained optimal carbon tax in a heterogeneous economy is heterogeneous: Higher-income households face a higher rate. Our main result shows that when the planner is limited to a uniform carbon tax, the tax follows the Pigouvian rule but is lower than the unconstrained carbon tax. Finally, we embed this model into a standard incomplete markets framework to quantify the policy effects on the economy, climate, and welfare, and we find a Pareto-improving result. The climate policy is welfare-improving for every consumer.

Keywords: carbon tax, inequality, consumption, welfare, climate change.

JEL Classification Codes: D62, H23, Q54.

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1 Introduction

Climate change is an increasingly important issue policymakers must address, with major international organizations labeling it "the single biggest health threat facing humanity" (World Health Organization) and "the defining issue of our time" (United Nations). Economists broadly support a carbon tax to combat climate change, as virtually all Economic Experts Panel members agree with the statement that carbon taxes "would be a less expensive way to reduce carbon-dioxide emissions than would a collection of [other] policies."[2] While many first generation climate-economy models used representative agent economies to determine the optimal carbon tax, a more recent line of research considers models with heterogeneity, especially along sectoral or geographic dimensions. Our paper contributes to this literature by studying the role that consumption inequality plays in the optimal design of carbon taxes and their unequal effects in the economy.

Using detailed household expenditure and embodied emissions data, we first document that the embodied carbon content of household expenditures is higher per dollar for low-income households relative to that of high-income households. Furthermore, the emission intensity of household expenditures also decreases with wealth. This suggests that a flat carbon tax would disproportionately affect low-income households. Based on this observation, we proceed to study whether a uniform carbon tax is optimal in an unequal world, and what climate policy could do to mitigate the unequal effects of such a tax.

To develop the analysis, we build a simple climate-economy model with clean and dirty goods and heterogeneous households. Consumption of dirty goods adds carbon to the atmosphere, generating a welfare loss ("a climate externality"). Households differ in their initial labor endowments and supply labor inelastically. Consumption goods are produced using a linear technology. The static nature of the model, where the dynamics are built exclusively on the stock of carbon that accumulates over time, lends tractability to the model and allows us to characterize the optimal climate policy in closed-form.

We start by establishing a result that resembles a representative agent framework. A utilitarian planner with access to a complete set of instruments can resolve any existing inequality, and a homogeneous carbon tax is optimal. This is the same tax that would prevail in a representative agent economy. In this economy, the preferences of the representative consumer determine the pricing of the climate externality. That is, the social cost of carbon

[2] See https://www.kentclarkcenter.org/surveys/carbon-tax/.

that measures the externality is priced at the marginal utility of the representative household.

The planner eliminates the existing inequality when unrestricted resource transfers are available. To study climate policy in an unequal world, we must limit the redistribution tools available to the policymaker. We proceed to study a constrained-optimal outcome where resource redistribution across households is ruled out. This centers the analysis around efficiency. We also restrict attention to climate policies that are neutral in terms of the initial distribution of income. This is important because a carbon tax could disproportionately hurt low-income households. Therefore, we are interested in climate policies that undo any distributional effects associated with the policy. The distinction we make is that while we want to prevent a planner from using climate policy to address existing inequality, we do want to consider climate policies that counteract any impact the climate policy itself might have on existing inequality.

The first main result of the paper shows that the constrained-optimal carbon policy is a set of carbon taxes. Each consumer pays the social cost of carbon priced at their private valuation (i.e., their marginal utility), leading to higher carbon taxes for high-income consumers. Thus, the constrained-optimal carbon tax of a heterogeneous economy is heterogeneous. Households get their tax payment rebated back as a lumpsum transfer. This household-specific tax-and-transfer policy, effectively preserves the initial distribution of resources across households. While some redistribution occurs through the implementation of differential tax rates, there are no direct transfers of resources between individuals. In the quantitative exercise, we address the effectiveness of this policy in controlling carbon emissions and fixing the climate externality.

We further study the optimality of a uniform carbon tax, the one considered in most policy proposals. As discussed above, a uniform carbon tax is not the outcome of an optimal policy design problem in an economy with heterogeneous households. Thus, we add uniformity of the tax rate as an additional constraint in the policy design problem. As a result, we find that the uniform constrained-optimal carbon tax equals the climate externality priced at a weighted average of individual marginal utilities. This is the central theoretical finding of the paper. When the coefficient of relative risk aversion is greater than one, the average marginal utility is higher than the marginal utility of average consumption, resulting in a lower carbon tax than the tax in a representative agent framework.

We show that these theoretical results extend to more general environments. Specifically, we show that the optimal carbon tax formulas—whether unconstrained, constrained-optimal, or uniform constrained-optimal—remain unchanged when we introduce stochastic shocks to labor productivity, endogenous labor and capital, and ad hoc borrowing constraints.

We then embed these features and those from the simple model into a standard incomplete market model to quantify the effectiveness of climate policies in reducing carbon emission and their distributional consequences. We calibrate the model's economic parameters to match features of the US income and wealth distribution and of US fiscal policy. We calibrate the model's climate parameters so that economic activities generate the same level of global emissions as in the data and contribute to temperature rises that are consistent with recent estimates.

We solve the model under several climate policy scenarios. First, we solve for the uniform carbon tax combined with a household-specific transfer that fully rebates each household's carbon tax payment. The equilibrium carbon tax schedule begins at \$41 per ton and rises gradually over time to a long run value of \$78 per ton as the social cost of carbon increases. Next, we solve for a proxy to the heterogeneous constrained efficient carbon tax where each household's carbon tax rate is function of their current labor productivity. The value of the initial carbon tax ranges from \$10 per ton for the poorest households to \$8,400 per ton for the richest. Just as in the uniform case, all tax rates rise over time with the social cost of carbon.

Both policies reduce the carbon emissions considerably and lower temperatures relative to business-as-usual (BAU), but the greatest effects appear far in the future. The heterogeneous carbon tax, which produces the strongest reduction in carbon accumulation relative to BAU, lowers long run temperatures by approximately 1.5 degrees Celsius relative to BAU. These carbon tax policies, whether uniform or heterogeneous, lead to a Pareto improvement relative to BAU. The welfare gains, measured from the date the carbon mitigation policy is enacted, are small on average (about 0.05 percent in consumption equivalence units), but they grow considerably over time with the accumulated climate improvement (relative to BAU).

Comparing the uniform and heterogenous tax cases, the latter leads to a somewhat lower temperature path and a slighter higher average welfare gain. In both cases, wealthier households realize a greater welfare benefit. Because these households have much lower marginal utilities of consumption, they place greater value on climate improvements than do the poor. With a uniform carbon tax, welfare gains increase monotonically in both wealth and in income (i.e., labor productivity). By taxing high-income households more, the heterogeneous carbon tax leads to more equal welfare gains across income conditional on having the same level of wealth.

Literature Review. This paper contributes to an expanding body of literature that delves into heterogeneous agent economies and incomplete markets, building upon the foundational work of Nordhaus and Boyer (2003) (and its contemporary version, Golosov et al. 2014) on representative agent neoclassical growth economies with climate dynamics. Recent notable contributions to this literature are Krusell and Smith Jr. (2022), Hillebrand and Hillebrand (2019), Fried et al. (2018), Douenne et al. (2023), Belfiori and Macera (2024), and Fried et al. (2023), and within a spatial economic framework Cruz and Rossi-Hansberg (2023) and Conte et al. (2022). The work most closely related to our paper is Fried et al. (2023), who build a life-cycle model with heterogeneous consumers to study the welfare and inequality implications of carbon taxation. Similar to our work, their model includes low-income households that spend relatively more on dirty goods, which is captured through Stony-Geary preferences. However, this paper is different as we take an optimal policy approach. While Fried et al. (2023) study the welfare consequences of alternative ways to rebate the revenue from an exogenously given carbon tax, we theoretically characterize the constrained-optimal climate policy (taxes and transfers) in the economy with heterogeneity and use these characterizations to inform the policies we study in the quantitative exercise. Furthermore, our paper distinguishes itself by focusing solely on carbon taxes and transfers, without considering their interaction with other distortionary taxes. In this regard, the paper differs also from Douenne et al. (2023) who study carbon taxation under a distortionary fiscal policy in a model with heterogeneous agents.

The study of constrained efficient allocations within climate-economy models with idiosyncratic risk and incomplete markets is also in Belfiori and Macera (2024) and Bourany (2024). This paper differs from them in focusing on household consumption inequality, while they both study regional heterogeneity across countries. We share with these papers the careful consideration of the tension between redistributive motives and efficiency in the policy design, as this tension naturally arises from a utilitarian social welfare function in a heterogeneous agent economy. Limiting the availability of resource transfers across agents, as in the pioneering work by Davila et al. (2012), is important in papers that study optimal policy in models with inequality and externalities.

This paper is also closely related to Jacobs and van der Ploeg (2019) who study optimal carbon taxes in an economy with clean and dirty consumption and heterogeneous households.

We share with Jacobs and van der Ploeg (2019) the optimal policy approach. However, our work differs from theirs because we do not explore the interaction of optimal carbon taxes with redistribution tools and other distortionary taxes. Instead, this paper restricts the analysis to optimal climate policy from a pure efficiency perspective. In particular, the goal of this paper is to look for climate policies that are neutral in terms of the existing income distribution (something not considered in either Jacobs and van der Ploeg 2019 or Fried et al. 2023).

The paper also connects to another strand of literature that emphasizes the distributional role of carbon tax revenue. Rausch et al. (2011) study the distributional impacts of carbon taxation using a static, large open-economy version of the MIT U.S. Regional Energy Policy (USREP) model, a multi-region and multi-sector general equilibrium model for the U.S. economy. Pizer and Sexton (2019) studies the distributional consequences of energy taxes using data from the 2014 U.S. Consumer Expenditure Survey. Fullerton and Monti (2013) build an analytical general equilibrium model with two agents and two goods. In the model, there is a two-sector economy with the production of a clean and a dirty good. The paper's central question is whether a rebate to low-income households can overcome the regressive effect of a carbon tax, and they find that it can not. Goulder et al. (2019) assess the impacts of a carbon tax across U.S. household income groups considering the supply and demand side effects of the tax. They find that the demand-side-effects of a carbon tax are regressive, while the supply-side impacts are progressive.

Our empirical work is closely related to Grainger and Kolstad (2010) and Sager (2019), who also document how emissions embodied in household expenditures vary with income. We focus on how embodied emissions intensities (emissions per dollar spent) vary with income, a relation that crucially informs our model calibration. We also show that these relations are robust to including variables such as education and wealth, which are also important determinants.

The paper is organized as follows. Section 2 uses data on household expenditure and embodied emissions to document how emissions intensities differ across income and wealth. Section 3 presents a simple model with unequal agents and climate change, and Section 4 provides the analytical characterizations. Section 5 presents the quantitative model, and Section 6 describes the calibration and quantitative results. Finally, Section 7 concludes with directions for future research.

2 Data

In this section, we document the embodied emissions content of household expenditures and how it differs across the income and wealth distribution. We combine data on household expenditures, income, and (liquid) wealth from the Consumer Expenditure Survey (CEX) with data on embodied emissions from the Environmental Protection Agency (EPA). The emissions data includes carbon dioxide (CO_2) emissions and other greenhouse gases such as methane and nitrous oxide, converted to CO_2 equilvalents using the Intergovernmental Panel on Climate Change (IPCC) Assessment Report's global warming potential over 100 years. It covers supply chain emissions (from cradle to factory gate) and margins (from factory gate to shelf, including transportation, wholesale, and retail).

To combine the expenditure data with the emissions data, we first construct a concordance to map 671 Universal Classification codes (UCC) for CEX expenditures to 394 North American Industry Classification System (NAICS) codes used in the emissions dataset (EPA). For example, the UCC code 100210 (cheese) is linked to the NAICS-6 code 311513 (cheese manufacturing), which is associated with 1.585 kilograms of CO_2-equivalent embodied emissions per 2018 dollar spent. As another example, the UCC code 560110 (physician services) corresponds to the NAICS-4 code 6211 (offices of physicians), which is associated with 0.082 kilograms of CO_2-equivalent embodied emissions per dollar.[3]

The CEX microdata consist of two surveys: The diary survey collects detailed expenditures on a subset of household expenditures (especially for groceries, such as flour, rice, and white bread) for two consecutive weeks and the interview survey collects more aggregated expenditures that cover most household expenditures (e.g. food at home, college tuition, camping equipment, and airline fares) for 1 year. Though the two surveys are not linked, we use the detailed food and beverage expenditures from the diary survey to estimate an embodied emission function and apply to the interview data on food and beverages at home. For all other interview expenditure categories, we use the constructed UCC-NAICS concordance to directly calculate embodied emissions. Finally, we include direct tailpipe emissions, first by dividing fuel and diesel expenditures by the average state price to calculate gallons, and then multiply by tailpipe emissions, about 9 kilograms of CO_2 per gallon driven (EPA).[4]

Using the constructed dataset, we document that the emission intensity of household ex-

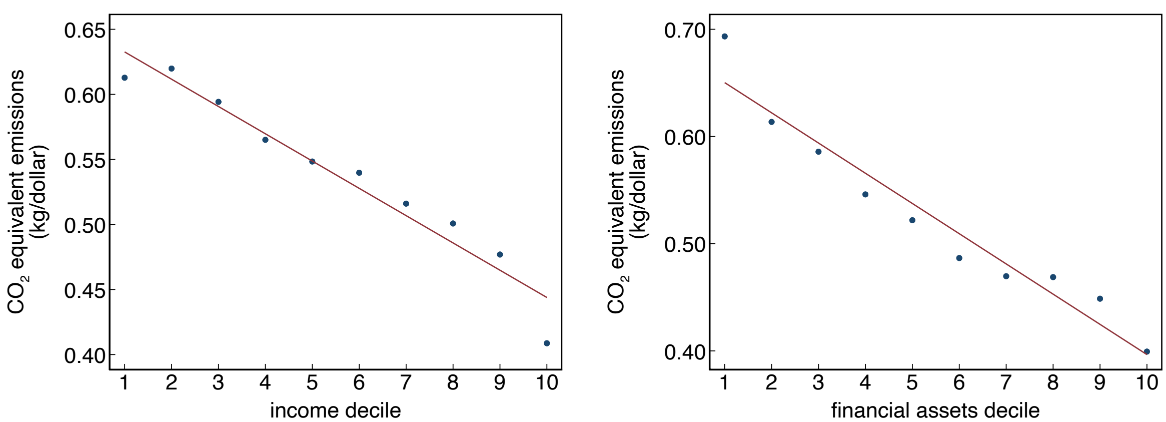
[3] The full concordance is provided online.

[4] See https://www.fueleconomy.gov/feg/label/calculations-information.shtml.

penditures decreases with income and with wealth. That is, the expenditures of lower income and lower wealth households are associated with higher embodied emissions per dollar spent. Figure 1 plots the average embodied emissions per dollar spent by income and (liquid) wealth decile.[5] Emission intensity is clearly decreasing in both income and wealth: Compared with the highest income and wealth households, the expenditure of the lowest income and wealth households is associated with about 25 additional kilograms of CO_2-equivalent emissions per \$100 spent.

Figure 1: Embodied emissions

(a) Income (b) Wealth



We further break down the source of this variation by broad expenditure categories. In Table 1, we can see that low-income households' expenditure baskets are more tilted toward expenditure categories with the highest emission intensities (utilities, transportation, and food and beverages at home), relative to high-income households. High-income households spend relatively more on all other expenditures, which are associated with lower emission intensities (including entertainment, education and child care, and health care).

To document the relationship between income and wealth and embodied emissions intensities more systematically, we regress the intensities on the natural logs of income and wealth in Table 2. Columns (1)–(2) demonstrate that wealth and income are negatively associated with embodied emission intensities, statistically significant at the 1 percent level. Column (3) shows that this result is robust to controlling for education, age, and family size fixed

[5]The CEX contains data on liquid wealth, containing only the value of checking, savings, money market accounts, and certificates of deposit. In Appendix C, we show that the results are robust to using the Panel Study of Income Dynamics, which contains a more complete representation of household wealth.

Table 1: Embodied emissions and expenditure shares

Expenditure category	Embodied emissions (CO_2 kg/dollar)	Expenditure shares (percent) Low income	Expenditure shares (percent) High income
Utilities	1.71	9.1	5.1
Transportation	1.16	18.4	16.0
Food and beverages at home	0.80	14.1	7.7
Other expenditures	0.11	58.4	71.2

High and low income correspond to the top and bottom deciles of income, respectively, conditional on working age.

effects. These effects are also economically significant: Using the coefficients in column (3), one standard deviation increases in log income and wealth are associated with 2.1 and 6.2 percentage point increases in the embodied emission intensities.

Table 2: Embodied emission intensity

	(1)	(2)	(3)
Wealth	-2.48^{***}		-1.87^{***}
	(0.131)		(0.175)
Income		-4.70^{***}	-2.14^{***}
		(0.243)	(0.612)
Observations	1488	5102	1488
Adjusted R^2	0.195	0.068	0.241

Standard errors in parentheses. (3) additionally includes college, age, and family size fixed effects. *** represents statistical significance at the 1 percent level.

3 A Simple Model

In the previous section, we documented that the emissions embodied in household expenditures substantially varied with income and wealth, suggesting that a carbon tax would have unequal consequences across households. In this section, we develop a simple model of unequal households and climate change to study how the optimal carbon tax depends on underlying inequality.

Consider an economy populated by a continuum of households, indexed by i with measure μ_i. There are two consumption goods, clean and dirty: c_{ct} and c_{dt}. Consumption of the dirty good adds carbon to the atmosphere, S_t, which evolves according to:

$$S_{t+1} = (1-\delta)S_t + \upsilon \sum_i \mu_i c^i_{dt} \tag{1}$$

where δ is the natural rate of carbon re-absorption and υ is the carbon content of dirty good consumption.

Households' preferences over consumption are given by:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_{ct}, c_{dt}) - x(S_{t+1})\right] \tag{2}$$

where $x(S)$ is the climate damage function with $x'(S) > 0$ and $x''(S) > 0$. The function x subsumes the welfare losses from the presence of carbon in the atmosphere, and we assume these losses take the form of a utility cost. In the quantitative exercise, we additionally consider the mapping from the carbon stock to the global temperature, and we include into x all climate-related welfare losses regardless of whether they are utility or output related. Therefore, x in the model represents global climate change.

The utility over consumption takes the following form:

$$u(c_{ct}, c_{dt}) = \frac{\left((c_{ct} + \bar{c})^{\gamma}\ c_{dt}^{1-\gamma}\right)^{1-\kappa}}{1-\kappa} \tag{3}$$

where γ represents preference over clean consumption and $\bar{c} > 0$ is the non-homotheticity parameter, which allows the model to match the differences in embodied emissions intensities across households documented in Section 2. Additionally, we assume that $\kappa > 1$.

Households are endowed with ε_i units of labor (inelastically supplied) and choose consumption to maximize utility (2) subject to the following set of budget constraints

$$p_t(1+\tau^i_t)c^i_{dt} + c^i_{ct} \leq w_t \varepsilon^i + t^i_t \tag{4}$$

for every period t where p_t is the relative price of dirty to clean consumption and w_t is the wage. Additionally, τ^i_t is a carbon tax on dirty good consumption and t^i_t is a lump-sum transfer.

A government collects carbon taxes and uses the proceeds to finance government spending and lump-sum rebates/taxes to households. There are no other distortionary taxes available.

The budget constraint of the government for every period t is:

$$\sum_i p_t \tau_t^i \mu_i c_{dt}^i = G_t + \sum_i \mu_i t_t^i \tag{5}$$

There are two production units, the clean and the dirty good producers indexed by j. A representative firm uses labor as the only input in each sector according to a linear technology. Thus, the aggregate production of the clean and the dirty good is given by $Y_{ct} = N_{ct}$ and $Y_{dt} = N_{dt}$, respectively.

Finally, market clearing for each period t requires that

$$N_{ct} + N_{dt} = \sum_i \mu_i \varepsilon^i \tag{6}$$

$$N_{ct} = \sum_i \mu_i c_{ct}^i + G_t \tag{7}$$

$$N_{dt} = \sum_i \mu_i c_{dt}^i \tag{8}$$

Definition 1 (Competitive Equilibrium with Carbon Taxes) *A competitive equilibrium with taxes* $\{\tau_t^i, t_t^i\}_{t=0}^{\infty}$ *is a sequence of prices* $\{p_t, w_t\}_{t=0}^{\infty}$ *and allocations* $\left\{\{c_{jt}^i, N_{jt}\}_{j=c,d}\right\}_{t=0}^{\infty}$ *such that* (i) *given prices and taxes, households choose* $\{c_{ct}^i, c_{dt}^i\}_{t=0}^{\infty}$ *to maximize (2) subject to (4) for all* i*;* (ii) *given prices, firms of sector* $j = \{c, d\}$ *choose* $\{N_{jt}\}_{t=0}^{\infty}$ *to maximize profits;* (iii) *the government budget constraint* (5) *is satisfied;* (iv) *the stock of atmospheric carbon evolves according to* (1)*, and* (v) *prices clear the markets.*

At an interior solution, household and firm optimality conditions imply:

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \tau_t^i \tag{9}$$

which states that the marginal rate of substitution between clean and dirty consumption equals the relative price for every period t. In this economy, profit maximization on the firm's side implies that $p_t = w_t = 1$ in every period t.

4 Analytical Results

4.1 Optimal Carbon Tax

We aim to study what is the optimal carbon tax in an economy with inequality. As a benchmark case, we first consider a framework that resembles a representative agent economy.

Specifically, we consider a government with access to a complete set of instruments, including type-specific taxes and lump-sum transfers, and no financing needs. The government collects carbon taxes and rebates the revenue to households as lump-sum transfers.

The optimal carbon-tax-and-transfer scheme arises from implementing the socially optimal allocation as a competitive equilibrium.

Definition 2 (Optimal Allocation) *Let $\{\alpha_i\}_{\forall i}$ be an arbitrary set of Pareto weights with $\sum_i \alpha_i = 1$. The socially optimal allocation is the sequence $\{c_{dt}^{i\star}(\alpha_i), c_{ct}^{i\star}(\alpha_i), S_t^{\star}\}_{t=0}^{\infty}$ that solves the social planner's problem, which is to maximize*

$$\sum_i \alpha_i \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_{ct}^i, c_{dt}^i) - x(S_{t+1}) \right) \right] \tag{10}$$

subject to the carbon cycle (1) and the resource constraint

$$\sum_i \mu_i c_{ct}^i + \sum_i \mu_i c_{dt}^i = \sum_i \mu_i \varepsilon^i \tag{11}$$

The first order conditions for this problem are:

$$(c_{dt}^i) : \alpha_i u_{dt}^i - \upsilon \mu_i \sigma_t - \mu_i \lambda_t = 0 \tag{12}$$

$$(c_{ct}^i) : \alpha_i u_{ct}^i - \mu_i \lambda_t = 0 \tag{13}$$

$$(S_{t+1}) : -x'(S_{t+1}) + \sigma_t - \beta \sigma_{t+1}(1-\delta) = 0 \tag{14}$$

where $\beta^t \sigma_t$ and $\beta^t \lambda_t$ are the Lagrange multipliers on the carbon cycle and resource constraint, respectively. Iterating forward from (14), we have:

$$\sigma_t = \sum_{j=1}^{\infty} [\beta(1-\delta)]^{j-1} x'(S_{t+j}) \tag{15}$$

The social cost of carbon is the discounted sum of climate-induced welfare losses associated with dirty consumption.

Notice that equations (12)–(13) hold for all i. Thus, for all i

$$\lambda_t + \upsilon \sigma_t = \frac{\alpha_i}{\mu_i} u_{dt}^i, \tag{16}$$

$$\lambda_t = \frac{\alpha_i}{\mu_i} u_{ct}^i. \tag{17}$$

That is, weighted marginal utilities are equated across agents. This implies that, for all i, j

$$\frac{u_{dt}^i}{u_{ct}^i} = \frac{u_{dt}^j}{u_{ct}^j}, \tag{18}$$

meaning that the marginal rate of substitution between goods are equated across agents.

Combine (16)–(17) to obtain:

$$1 + \frac{\upsilon \sigma_t}{\lambda_t} = \frac{u^i_{dt}}{u^i_{ct}} \tag{19}$$

The optimality condition says that the marginal utility must be equal across goods, after taking into account the climate externality.

Uniform Carbon Taxes. It follows from a simple observation of optimality conditions (19) and (9) that the optimal Pigouvian tax that implements the socially optimal allocation is

$$\tau^\star_t \equiv \frac{\upsilon \sigma_t}{\frac{\alpha_i}{\mu_i} u^i_{ct}} \tag{20}$$

Given a choice of Pareto weights, lump-sum transfers are equal to $t^i_t(\alpha_i) = (1+\tau^\star_t)c^i_{dt} + c^i_{ct} - \varepsilon^i$ so that weighted marginal utilities are equated as in equations (16)–(17).

Because weighted marginal utilities are equated across agents, the carbon tax in (20) is the same for all households and equals the social cost of carbon, valued in consumption units. Hence, a uniform carbon tax—consistent with the rule that prevails in a representative agent economy—is also optimal in economies with heterogeneous agents when lump-sum transfers are available. Of course, the actual tax rate depends on the allocation, which varies with alternative welfare weights. Importantly, the climate policy is not entirely uniform as it contains consumer-specific lump-sum transfers. Moreover, this uniform carbon tax entails a significant redistribution of resources across households implemented through these transfers.

Utilitarian Carbon Taxes. When the planner is utilitarian (i.e., $\alpha_i = \mu_i$), marginal utilities for both clean and dirty consumption are equalized across agents. In this case, the optimal allocation coincides with the one that prevails in a representative agent economy, and the optimal tax is the same. Using (20), the utilitarian carbon tax, $\tau^{\mathbf{U}}_t$ is equal to

$$\tau^{\mathbf{U}}_t = \frac{\upsilon \sigma_t}{u_{ct}}. \tag{21}$$

where u_{ct} indicates the marginal utility of clean consumption, which no longer depends on i. As a result, a dichotomy emerges in policy design regarding policy objectives and instruments: While the carbon tax addresses the climate externality, transfers effectively eliminate inequality within the economy.

Negishi Carbon Taxes. The planner can, of course, resolve the existing inequality when unrestricted lump-sum transfers are available. This is true in general, regardless of the presence of a climate externality. Because our primary interest is in understanding how the presence of inequality affects optimal climate policy, rather than how climate policy can be used to reduce inequality, a natural exercise is to restrict the planner's ability to redistribute resources. One way to do this is by considering a planner with Negishi Pareto weights as these rule out transfers across consumers. Negishi weights take the following form:

$$\alpha_i \equiv \frac{\frac{1}{u_\varepsilon^i}\mu_i}{\sum_j \frac{1}{u_\varepsilon^j}\mu_j}. \tag{22}$$

Here the welfare weights are equal to the inverse of the marginal utilities of individual's *total* consumption, which we denote by u_ε^i to indicate that each consumer's total consumption is equal to their endowment. Specifically, $u_\varepsilon^i = (\varepsilon^i + \bar{c})^{-\kappa}$. Using (20), the Negishi carbon tax, $\tau_t^{\mathbf{N}}$, is equal to

$$\tau_t^{\mathbf{N}} = \frac{\upsilon\sigma_t}{\sum_i \frac{\frac{1}{u_\varepsilon^i}\mu_i}{\sum_j \frac{1}{u_\varepsilon^j}\mu_j} u_{ct}^i}. \tag{23}$$

Each consumer receives a rebate with their tax bill so that individual transfers equal $t_t^i = \tau_t c_{dt}^i$ for every period t.

The tax rate comes from solving the Lagrange multiplier on (17). Summing over i, we get

$$\sum_i \alpha_i u_{ct}^i = \sum_i \mu_i \lambda_t$$

and plugging in the Negishi weights in (22)

$$\sum_i \frac{\frac{1}{u_\varepsilon^i}\mu_i}{\sum_j \frac{1}{u_\varepsilon^j}\mu_j} u_{ct}^i = \lambda_t$$

Finally, replacing λ_t into (19) to get (23).

4.2 Constrained-Optimal Carbon Tax

A Negishi planner weighs the welfare of high-income households more heavily and avoids any redistribution, ruling out net transfers across households. In this subsection, we will restrict attention to a utilitarian planner who weighs consumers equally using the population measures, but is still restricted to choosing allocations that imply no resource transfers across households, thereby maintaining the heterogeneous nature of the economy. We will show that, when the $\kappa \neq 1$, the constrained efficient utilitarian carbon tax differs from both the utilitarian carbon tax and the Negishi carbon tax, with very different implications for the distribution of welfare.

The analysis establishes a nuanced distinction. While we want to prevent a planner from using climate policy to address existing inequality—something a utilitarian planner with unrestricted transfers will do—we also want to consider climate policies that counteract any impact the climate policy itself might have on existing inequality. As described in Section 2, carbon taxes can be regressive. This paper focuses on studying carbon taxes that are neutral in terms of their effect on the current income distribution.

4.2.1 A Climate Policy Neutral on the Income Distribution

Consider a transfer scheme in which the government rebates the proceeds from carbon taxation back to each household, effectively keeping the underlying distribution of resources across households unchanged. This climate policy takes inequality as given and preserves its initial level.

Specifically, transfers are equal to

$$t_t^i = \tau_t^i c_{d,t}^i \tag{24}$$

for all i and t. Plugging the transfer scheme (24) into the household budget constraint, the planner is now constrained to consider only allocations that satisfy the following implementability condition:

$$c_{c,t}^i + c_{d,t}^i \leq \varepsilon_i \tag{25}$$

for all i and t.

Condition (25) is certainly more restrictive than the feasibility condition (11) and prevents the utilitarian planner from pursuing further redistribution. The constrained-optimal carbon tax and transfer scheme arise from implementing the constrained-optimal allocation as a competitive equilibrium.

Definition 3 (Constrained-Optimal Allocation) *The constrained-optimal allocation is the sequence $\{c_{dt}^i, c_{ct}^i, S_t\}_{t=0}^{\infty}$ that solves the constrained-optimal utilitarian social planner's problem, which is to maximize social welfare (10), with $\alpha_i = \mu_i$ for all i, subject to the carbon cycle (1) and the implementability condition* (25).

The first order conditions for this problem are:

$$(c^i_{dt}) : u^i_{dt} - \upsilon\sigma_t - \lambda^i_t = 0 \tag{26}$$

$$(c^i_{ct}) : u^i_{ct} - \lambda^i_t = 0 \tag{27}$$

$$(S_{t+1}) : -x'(S_{t+1}) + \sigma_t - \beta\sigma_{t+1}(1-\delta) = 0 \tag{28}$$

where $\beta^t \mu_i \lambda^i_t$ is the Lagrange multiplier on the implementability condition (25).

Combine equations (26) and (27) to obtain:

$$u^i_{dt} = u^i_{ct}\left(1 + \frac{\upsilon\sigma_t}{u^i_{ct}}\right) \tag{29}$$

In contrast to the optimal allocation, the weighted marginal utilities in (26) and (27) and the marginal rate of substitution between consumption of clean and dirty goods in (29) are not necessarily equal across agents.

It follows that the constrained-optimal carbon tax is no longer uniform. The following proposition characterizes the constrained-optimal carbon tax. The proof is in Appendix A.

Proposition 1 (Constrained-Optimal Carbon Tax) *Suppose that the constrained-optimal allocation is $\{c^i_{dt}, c^i_{ct}, S_t\}^\infty_{t=0}$ for all i. Then, there exists a sequence of prices $\{w_t, p_t\}^\infty_{t=0}$ such that the allocation is a competitive equilibrium with taxes given by*

$$\tau^i_t = \frac{\upsilon\sigma_t}{u^i_{ct}} \quad \forall i. \tag{30}$$

The revenue is rebated back to the consumer with transfers equal to $t^i_t = \tau^i_t c^i_{d,t}$ for every period t and for all i.

The constrained-optimal carbon tax in (30) equates the social cost of carbon, valued in units of the consumption good for each consumer. Thus, the constrained-optimal carbon tax of a heterogeneous economy is heterogeneous itself. For each individual, the social cost of carbon is valued at their marginal utility. Because households with lower income have a higher marginal utility, it is easy to show that

$$\tau^j_t < \tau^k_t$$

for all j and k with $\varepsilon_j < \varepsilon_k$.

Therefore, the constrained-optimal carbon tax calls for a higher rate for households with higher incomes. Notice that, absent any transfer of resources across consumers, some redistribution still occurs through the differential tax rates. In this way, the carbon tax serves a dual role of achieving efficiency by correcting the externality and addressing equity through some redistribution.

4.2.2 A Uniform Carbon Tax

A uniform carbon tax is the rule considered in most policy proposals. However, it is interesting to notice that it is not the optimal tax rate in a heterogeneous economy when transfers across consumers are ruled out. To obtain a uniform carbon tax as the outcome of an optimal policy design problem in an economy with heterogeneous households, uniformity of the tax rate must be added as an additional constraint in the planning problem. From (9), this constraint expressed in terms of the allocation is given by:

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{u^j_{dt}}{u^j_{ct}}$$

for all i, j. Furthermore, for preferences of the form specified in (3), the constraint can be written as

$$\left(c^i_{ct} + \bar{c}\right) c^j_{dt} = \left(c^j_{ct} + \bar{c}\right) c^i_{dt} \tag{31}$$

for all i, j.

The following proposition characterizes the constrained-optimal climate policy in an economy where the planner is fully constrained from using climate policy to redistribute resources across households. In the model, this restriction implies no direct transfer of resources across individuals and uniform carbon taxes. The proof can be found in Appendix A.

Proposition 2 (Constrained-Optimal Uniform Carbon Tax) *Suppose that the allocation $\{c^i_{dt}, c^i_{ct}, S_t\}^\infty_{t=0,\forall i}$ solves the constrained-optimal planner's problem with the additional constraint (31). Then, there exists a sequence of prices $\{w_t, p_t\}^\infty_{t=0}$ such that the allocation is a competitive equilibrium with taxes given by*

$$\tau_t = \frac{\upsilon \sigma_t}{\sum_i \frac{\mu_i c^i_t}{\sum_j \mu_j c^j_t} u^i_{ct}} \tag{32}$$

with $c^i_t \equiv c^i_{ct} + \bar{c} + c^i_{dt}$. The revenue is rebated back with transfers equal to $t^i_t = \tau^i_t c^i_{dt}$ for every period t and for all i.

The constrained-optimal uniform carbon tax follows the Pigouvian rule, but uses a weighted average of marginal utilities to price the climate externality (as opposed to the marginal utility of the representative household in the utilitarian optimal carbon tax). For a given social cost of carbon, this will result in a lower carbon tax in an economy with inequality relative to a representative agent one, provided that the risk aversion κ is greater than one. In fact, when $\kappa = 1$, for a given social cost of carbon, the Negishi carbon tax, utilitarian optimal carbon tax, and uniform constrained optimal tax collapse to one value.

The actual value of the social cost of carbon can also differ in an economy with and without inequality, adding a source of potential differences in the tax rates. In the following section, we address these differences in the social cost of carbon numerically in the quantitative exercise.

Two central takeaways arise from the analysis. The first is that uniform carbon taxes, considered in most policy proposals, are not optimal in an unequal world. The constrained optimal planner understands that high-income individuals place more value on the climate relative to consumption (because the marginal utility of consumption is low for these individuals), and, thus, assign a higher tax relative to low-income households who place a higher value on consumption.

The second takeaway from our analysis highlights the implications of explicitly imposing uniform taxation as a policy restriction. In an economy with heterogeneity, the constrained-optimal carbon tax is likely lower than that in a representative agent world. This difference is primarily due to the observed distribution of consumption in the economy, which plays a vital role in the differential tax rates. In this case, the social cost of carbon is priced at a weighted average of consumers' marginal utilities, which is higher than the marginal utility of the representative agent when marginal utilities are sufficiently convex.

The implementation of the uniform-constrained carbon tax with individual lump-sum transfers can be challenging when the planner lacks enough information. In the next result, we characterize an alternative all-uniform climate policy that consists of a carbon tax, a clean subsidy, and transfers. In particular, consider an alternative market economy with taxes where households face a carbon tax on dirty consumption, τ_{dt}, a clean subsidy, τ_{ct}, and lump-sum transfers, t_t. The problem of the households is to maximize (2) subject to the following set of budget constraints

$$p_t(1 + \tau_{dt})c^i_{dt} + (1 - \tau_{ct})c^i_{ct} \leq w_t\varepsilon^i_t + t_t \tag{33}$$

for every period t. The first order conditions for this problem lead to the following optimality condition:

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{1 + \tau_{dt}}{1 - \tau_{ct}} \tag{34}$$

where the marginal rate of substitution between clean and dirty consumption equals the relative after-tax price of the goods. As before, optimality on the firm side implies that $p_t = w_t$.

Corollary 1 (Uniform Carbon Tax, Clean Subsidy, and Transfer) *Suppose that the allocation* $\{c^i_{dt}, c^i_{ct}, S_t\}^{\infty}_{t=0,\forall i}$ *solves the constrained-optimal planner's problem with the additional constraint* (31)*. Then, the constrained-optimal allocation* $\{c^i_{dt}, c^i_{ct}, S_t\}^{\infty}_{t=0,\forall i}$ *is also implementable as a competitive equilibrium with an all-uniform climate policy* $\{\tau_{dt}, \tau_{ct}, t_t\}$ *given by:*

$$\tau_{dt} = \gamma\mu^{\star}_t \quad ; \quad \tau_{ct} = (1-\gamma)\frac{\mu^{\star}_t}{1+\mu^{\star}_t} \quad ; \quad T_t = \tau_{ct}\bar{c} \tag{35}$$

The proof is in the appendix. This alternative policy can implement the constrained efficient allocation, providing an arguably more viable alternative to the uniform-constrained carbon tax with individual transfers. However, notice that this alternative decentralization, while being homogeneous across households, comes at the cost of adding an extra instrument (the clean subsidy) that can, as we show in the next section, add additional distortions to the economy.

In the next section, we do a quantitative exploration of these tax rates.

5 The Quantitative Model

For the quantitative exercise, we extend the simple model by including endogenous labor and savings decisions, borrowing constraints, and a richer fiscal policy. In this version of the economy, households face idiosyncratic labor productivity risk. We assume that ε^i_t follows a Markov process with transition matrix $\pi\left(\varepsilon^i_t, \varepsilon^i_{t+1}\right)$. Households supply $n^i_t\varepsilon^i_t$ efficiency units of labor, where n^i_t and ε^i_t denote hours supplied and labor productivity, respectively.

There are no state-contingent contracts but households can save in the form of real capital, k^i_{t+1}, which depreciates at a constant rate, δ_k. There is no aggregate uncertainty. Capital evolves over time according to the following law of motion

$$k^i_{t+1} = (1-\delta_k)k^i_{t+1} + x^i_t \tag{36}$$

where x is investment.

The household's problem is to choose consumption, labor, and savings to maximize

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c^i_{ct}, c^i_{dt}) - v(n^i_t) - x(S_{t+1})\right] \tag{37}$$

subject to the following set of budget constraints for every period t

$$p_t(1+\tau_t^i)c_{dt}^i + c_{ct}^i + k_{t+1}^i \leq (1-\tau_t^n)w_t\varepsilon_t^i n_t^i + (1-\tau_t^k)r_t k_t^i + (1-\delta_k)k_t^i + t_t^i \tag{38}$$

where (τ_t^n, τ_t^k) are labor and capital taxes and w_t is the wage per efficiency unit of labor. The price of the clean good is normalized to one. We assume households cannot borrow so

$$k_{t+1}^i \geq 0. \tag{39}$$

for every period t.

The production of the clean and the dirty consumption good uses labor and capital as inputs according to a constant return to scale technology. The aggregate production of the clean and the dirty good is given by $Y_{ct} = F(K_{ct}, N_{ct})$ and $Y_{dt} = F(K_{dt}, N_{dt})$, respectively. The problem of the producer is to choose $\{N_{jt}, K_{jt}\}_{t=0}^{\infty}$ to maximize profits.

The government collects taxes and uses the proceeds to finance government spending and provide transfers, so that

$$\sum_i \mu_i \left(\tau_t^i p_t c_{dt}^i + \tau_t^n w_t \varepsilon_t^i n_t^i + \tau_t^k r_t k_{t+1}^i\right) = G_t + \sum_i \mu_i t_t^i. \tag{40}$$

Finally, market clearing for each period t requires that

$$N_{ct} + N_{dt} = \sum_i \mu_i \varepsilon_t^i n_t^i \tag{41}$$

$$K_{ct} + K_{dt} = \sum_i \mu_i k_t^i \tag{42}$$

$$\sum_i \mu_i \left(c_{ct}^i + k_{t+1}^i - (1-\delta_k)k_t^i\right) + G_t = F(K_{ct}, N_{ct}) \tag{43}$$

$$\sum_i \mu_i c_{dt}^i = F(K_{dt}, N_{dt}) \tag{44}$$

Definition 4 *A competitive equilibrium with taxes $\{\tau_t^i, \tau_t^n, \tau_t^k, T_t^i\}_{t=0}^{\infty}$ is a sequence of prices $\{p_{dt}, w_t, r_t\}_{t=0}^{\infty}$ and allocations $\left\{c_{jt}^i, n_t^i, k_t^i, N_{jt}, K_{jt}\right\}_{t=0,j=c,d}^{\infty}$ such that (i) given prices, households choose $\{c_{ct}^i, c_{dt}^i, n_t^i, k_t^i\}_{t=0,j=c,d}^{\infty}$ to maximize (37) subject to (38) and (39) for all i; (ii) Profit maximizing prices are $w_t = F_{Njt},\ r_t = F_{Kjt}$ for $j = c, d$ and $p_t = 1$ (iii) the stock of atmospheric carbon evolves according to (1), and (iv) prices clear the markets.*

At an interior solution, household and firm optimality conditions imply that (9) holds, together with the usual intratemporal margin on consumption and labor decisions, and the intertemporal Euler equation on capital accumulation:

$$\frac{v_{nt}^i}{u_{ct}^i} = (1-\tau_t^n) F_{Nt} \varepsilon_t^i \tag{45}$$

$$u_{ct}^i = \beta \mathbf{E}_t \left\{ u_{ct+1}^i \left[(1-\tau_{t+1}^k) F_{Kt+1} + 1 - \delta_k \right) \right] + \phi_{t+1}^i \right\} \tag{46}$$

where $\beta\phi_t^i$ is the Lagrange multiplier on the borrowing constraint (39).

The constrained-optimal allocation is the sequence $\left\{c_{jt}^i, n_t^i, k_t^i, N_{jt}, K_{jt}, S_t\right\}_{t=0,j=c,d}^{\infty}$ that maximizes social welfare

$$\sum_i \alpha_i \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{ct}^i, c_{dt}^i) - v(n_t^i) - x(S_{t+1}) \right] \tag{47}$$

subject to (38) and (39) for all i with pricing rules $w_t = F_{Njt},\ r_t = F_{Kjt}$ for $j = \{c, d\}$ and $p_t = 1$, the carbon cycle (1) and feasibility constraints (41)-(44).

To study optimal climate policy in the quantitative model, we follow the concept of constrained efficiency by taking the market structure and individual constraints as given and excluding net transfers across consumers. Thus, we look for climate policies that remain neutral regarding the initial income distribution, including tax bill rebates that undo the redistribution effects introduced by climate policy itself. Proposition 3 states that the theoretical characterizations of both the constrained-optimal and the uniform carbon tax hold in the quantitative economy. The proof is in Appendix B.

Proposition 3 *The constrained optimal carbon tax for the quantitative economy follows the rule* (30)*. Also, the uniform carbon tax takes the form of equation* (32)*. The tax revenue is rebated back lump-sum:* $t_t^i = \tau_t^i c_{dt}^i$.

This economy exhibits a pecuniary externality in addition to the climate externality. Due to market incompleteness, consumers' labor and savings decisions impact the equilibrium prices of labor and capital, subsequently influencing how labor income shocks affect consumers. The market allocation is typically not constrained-efficient, even without considering the climate, and can be improved by a utilitarian planner. This is studied in Dávila et al. (2012) and is not the focus of our paper. The policy in Proposition 3 takes as given that the market equilibrium is not efficient (from an ex-post redistributive point of view) and aims only at internalizing the climate externality, not the pecuniary externality.

As in Dávila et al. (2012), implementing the constrained-optimal allocation in this economy would call for capital and labor income taxes or subsidies. These income taxes capture the equilibrium effects of consumer's decisions on market prices and respond to ex-post redistributive motives. We fully characterize the constrained optimal allocation for the quantitative model economy in Appendix B, together with constrained-optimal income taxes that, together with the carbon tax and transfers, fully implement the constrained optimal allocation.

Implementing the actual tax rates is computationally challenging because taxes are history-dependent in this economy with idiosyncratic risk. This is also the case in Davila et al. (2012). In the quantitative exercise, we use the carbon tax formulas we have theoretically characterized in this section to inform the value and evolution over time of the carbon taxes we consider. The goal is to study the effects of carbon taxes in a model economy calibrated to match economic and climate targets, as well as features of the actual tax system.

We discuss the calibration strategy in the next subsection.

5.1 Calibration

We assume the disutility of labor takes the form

$$v(n) = \phi \frac{n^{1+\nu}}{1+\nu}, \tag{48}$$

where ϕ and ν govern the disutility of labor and the Frisch elasticity of labor, respectively.

We rewrite the budget constraint of the consumers (38) to include an earning tax function used in the literature (Heathcote et al. 2017) to approximate the US system of income-dependent taxes and transfers and match the progressivity of the tax system.

$$(1+\tau_t^i)c_{dt}^i + c_{ct}^i + k_{t+1}^i - k_t^i \leq w_t \varepsilon_t^i n_t^i + (1-\tau_{kt})(r_t - \delta_k)k_t^i - T_t(w_t \varepsilon_t^i n_t^i) \tag{49}$$

where

$$T_t(w_t \varepsilon_t^i n_t^i) = \tilde{T}(w_t \varepsilon_t^i n_t^i; \nu_y, \tau_y) + t_t^i \tag{50}$$

where t_t^i are the carbon tax rebates and $\tilde{T}_t$ is the earning tax function, that subsumes the labor income tax. Following Heathcote et al. (2017), the earnings tax bill for a household with pre-tax earnings $y = w_t n_t^i \varepsilon_t^i$ takes the form

$$\tilde{T}(y) = y - \tilde{y}^{\nu_y} \frac{1-\tau_y}{1-\nu_y} y^{1-\nu_y} \tag{51}$$

where $\tilde{y}$ denotes average earnings in the economy. The parameter τ_y shifts the average tax rate while ν_y controls the progressivity of the tax schedule. When $\nu_y = 0$, all earnings levels are taxed at the same flat rate of τ_y. As ν_y increases, the tax function becomes more progressive.

We calibrate the model's economic parameters to match standard moments, summarized in Table 3. Because high-income, high-wealth households account for the bulk of consumption and therefore also of emissions, it is important that the model generates a distribution that is skewed in both of these dimensions. To achieve this, we employ a common strategy from the literature and include a superstar state in the Markov chain for the productivity process (Castaneda et al., 2003). To calibrate this Markov chain, we first approximate an AR(1) process (in logs) using the Rouwenhorst method (Kopecky and Suen, 2010) with nine normal (i.e., non-superstar) states. The persistence of the process for these states is set to 0.94 as measured in the PSID. Next, we jointly calibrate the standard deviation of the normal process, the value of superstar productivity, and the persistence of the superstar state to target three moments from the data: a Gini coefficient of earnings of 0.47, a top 1 percent wealth share of 0.34, and a Gini coefficient of wealth of 0.83. The probability of becoming a superstar from any normal state is set so that superstars account for 0.1 percent of the population. When a household exits the superstar state, its new productivity level is drawn from the ergodic distribution over the normal states.

Climate Parameters. We follow Golosov et al. (2014) in assuming that the stock of atmospheric carbon affects temperature changes according to:

$$T_t = \frac{\lambda}{\log(2)} \log\left(\frac{S_t}{\overline{S}}\right), \tag{52}$$

where $\lambda = 3$ and $\bar{S} = 581$ represents the pre-industrialization carbon stock (in gigatons). This parametrization implies that, for each doubling of the carbon stock, the temperature increases by 3 degrees (Celsius). We set $S_{2023} = 785$ to match the temperature rise of 1.3 degrees from the pre-industrial mean.

Our carbon disutility takes the form:

$$x(S) = \frac{\Psi}{2} S^2. \tag{53}$$

Table 3: Calibration

Parameters	Values	Targets / Source
Preferences		
Discount factor, β	0.96	capital-to-output: 4.8
Risk aversion, κ	2	standard value
Disutility from labor, ϕ	20.3	average hours: 30 percent
Frisch elasticity, $1/\nu$	0.50	standard value
Climate parameters		
Carbon absorption, δ	1/300	average life of carbon: 300 years
Carbon intensity, υ	326.4	1.4 degree increase by 2100 under BAU
Utility loss, ψ	0.03	welfare loss from 2.5 degree increase $\approx$ 1.74 percent output reduction
Clean share, γ	0.98	\$50/ton carbon tax leads to 0.8 degree reduction from BAU
Nonhomotheticity, $\bar{c}$	0.33	emissions intensity 30 percent higher for low-income than high-income households
Fiscal parameters		
Average, τ_y	0.25	average net tax rate: 13 percent
Progressivity, ν_y	0.16	37.9 percent marginal tax rate on top 1 percent earner
Capital, τ_k	0.27	Carey and Rabesona (2002)
Technology and shocks		
Capital weight, α	0.36	capital income share: 36 percent
Capital depreciation rate, δ_k	0.05	standard value
Persistence of wage process, ρ	0.94	author estimates
Standard deviation, σ_ε	0.24	Gini coefficient of earnings: 0.47
Superstar productivity, ε_{sup}	162.6	wealth share top 1%: 34%
Persistence of superstar state, $\pi_{10,10}$	0.94	Gini coefficient of wealth: 0.83
Probability of becoming a superstar, $\pi_{1:9,10}$	6e-5	fraction of superstars: 0.1%

We calibrate Ψ so that the welfare loss associated with a 2.5-degree temperature increase is equivalent to that from a 1.74 percent decline in output, which combines the production and utility damages used in Barrage (2020).

We calibrate υ so that under a business-as-usual (BAU) scenario, there is an additional 1.4 degree increase in temperature from 2023 to 2100 (for a total of 2.7 degree increase from pre-industrial levels).[6] We set the rate of natural reabsorption to 1/300 so that the average life cycle of carbon is 300 years (Archer 2005). The dirty share, $1-\gamma$, is set such that a \$50/ton carbon tax leads to 0.8 degree reduction from BAU, consistent with Krusell and Smith Jr. (2022). The nonhomotheticity parameter, $\bar{c}$, is calibrated so that the emissions intensity is 31 percent higher for households in the bottom 10 percent of income relative to those in the top 10 percent.

6 Quantitative Results

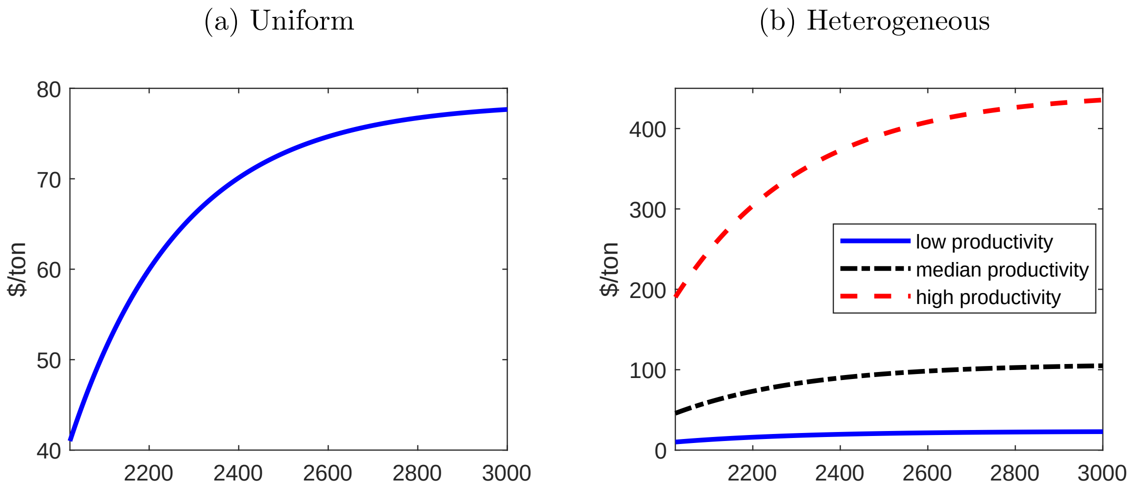
We use the calibrated model to measure the distributional effects of climate policy. We begin by contrasting the outcome of two carbon tax policies: one in which the government levies the same flat rate, τ_t, on all households according to the uniform constrained efficient carbon tax formula in (32), and a second one, in which the carbon tax schedule places higher tax rates on more productive households, consistent with the constrained efficient carbon tax in (30). In the latter case, carbon tax rates, $\tau_t(\varepsilon)$, are determined by the tax formula in (30), where the marginal utility of clean consumption is evaluated using the average consumption within a productivity (ε) group.[7]

Notice that in both cases, the optimal tax schedule is a function of endogenous variables, since both the present discounted social cost of carbon and the marginal utility of consumption depend on the taxes households face. Therefore, as part of the solution to this exercise, we must find a fixed point in the space of proportional carbon tax sequences. To do this, we first solve the business-as-usual transition (i.e., no carbon taxation). We then feed the equilibrium paths for the carbon stock, the distribution of wealth, and household consumption decisions in the optimal tax formula to compute a new sequence of τ_t and solve

[6] See `https://climateactiontracker.org/global/cat-thermometer`.

[7] The constrained optimal carbon tax from Section 4.2 also depends on household wealth, or equivalently the entire history of a household's productivity shocks. Indexing by the history of shocks would be computationally infeasible, while indexing by wealth introduces a distortion to the savings decision since a household understands how its future wealth will affect its carbon tax rate in future periods. The productivity-specific carbon tax does not have this problem since it depends only on exogenous shocks.

Figure 2: Constrained-efficient carbon tax



for the transition associated with that sequence. We repeat this process, updating carbon taxes after each iteration, until the path of carbon taxes converges.

As in Section 4.2, we assume that a household's carbon tax payments are exactly offset by a lumpsum transfer, which the households take as given. Because the household's choice set is unaltered by this tax and transfer scheme, we are able to isolate the effect of climate policy from the alternative ways of redistributing tax revenue.

Figure 2 plots the time path of the uniform and heterogeneous optimal carbon tax schedules. In both cases, the tax rates rise over time reflecting that the greatest social costs only appear far in the future and are thus heavily discounted in the initial periods. As time passes, however, and carbon levels rise, the benefits of discouraging additional carbon emissions becomes more pressing.

Under the uniform tax path shown in panel (a), the carbon tax rate starts at \$41/ton and climbs gradually over time to a long run value of \$78/ton. When the carbon tax can be differentiated by labor productivity (effectively a household's hourly wage), rates vary widely. In the first period, the tax rate on the lowest productivity households is \$10/ton and only rises to \$28/ton in the long run. In contrast, a household with the highest non-superstar productivity, the carbon tax begins at \$190/ton and tops out at \$445/ton. The enormous difference in tax rates results from low productivity households having lower average consumption (higher marginal utility of consumption). For similar reasons, the superstar carbon tax (not shown) is extremely high. It starts at \$8,400/ton and rises to almost \$20,000/ton.

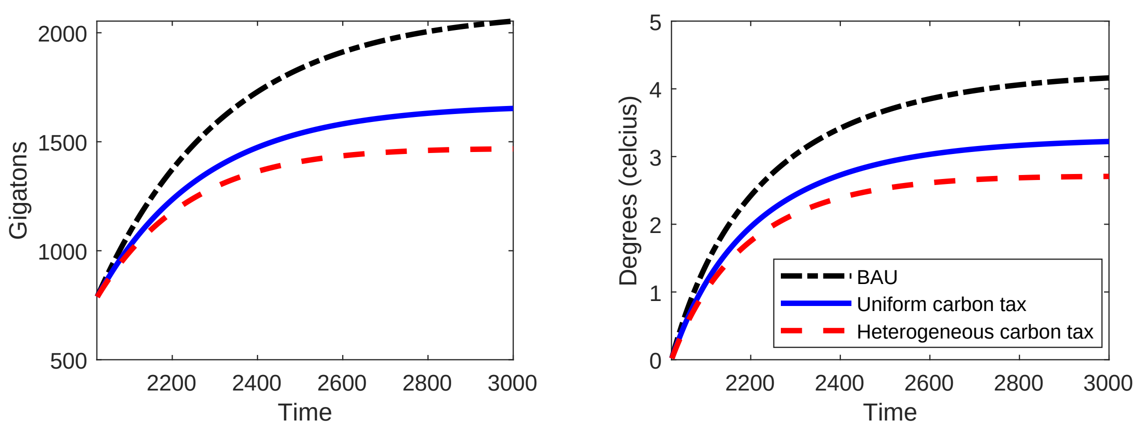
Because each household's carbon tax payment is rebated back as a lumpsum transfer, the wealth effect is shut off. As a result, the aggregate levels of labor, capital, consumption and output are virtually unchanged under either policy. However, the composition of these aggregates between dirty and clean goods does change, since the tax distorts each household's optimal consumption bundle toward a higher share of clean consumption.

While global temperatures still rise under both carbon tax policies, these fiscal interventions have a substantial effect on the evolution of the carbon stock and global temperatures over time, relative to the BAU scenario (Figure 3). The productivity-indexed carbon tax, which produces the greatest moderation in temperature, subtracts 0.5 degrees from the BAU path over 100 years and 1.1 degrees over 300 years. Under either carbon tax, the most sizeable gap in temperature emerges only after centuries have passed, and long after the economic transition has fully played out.

Figure 3: Carbon and Temperature

(a) Carbon Stock

(b) Temperature increase



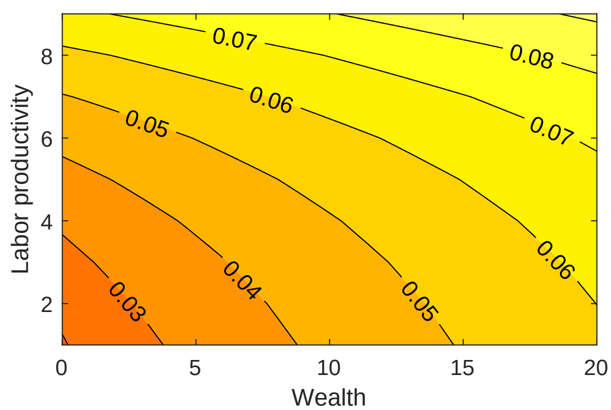
6.1 Welfare

Next, we compute the change in welfare from undergoing the policy-induced transitions relative to the BAU baseline and highlight the differential effects of the carbon tax across the wealth and income distribution and on average over time.

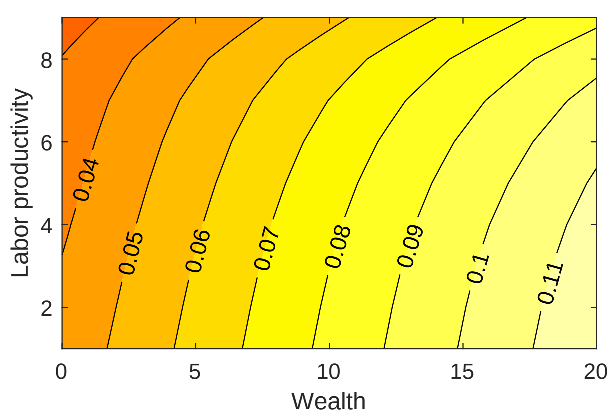
Figure 4 displays the change in welfare for all households according to their wealth and productivity in the initial distribution resulting from carbon taxation. The wealth levels shown cover 98 percent of households. In panel (a), where carbon taxes are uniform, all households gain, but the welfare gains are largest for the most productive households with high levels of wealth. While all households benefit from mitigated emissions, the costs of doing so, specifically distorting the composition of consumption, fall more heavily on the poor. This is evident in the heterogeneous tax case (panel b), which moves some of those distortions off of low-productivity households and onto high-productivity ones.

Figure 4: Welfare (consumption equivalents, percent)

(a) Uniform



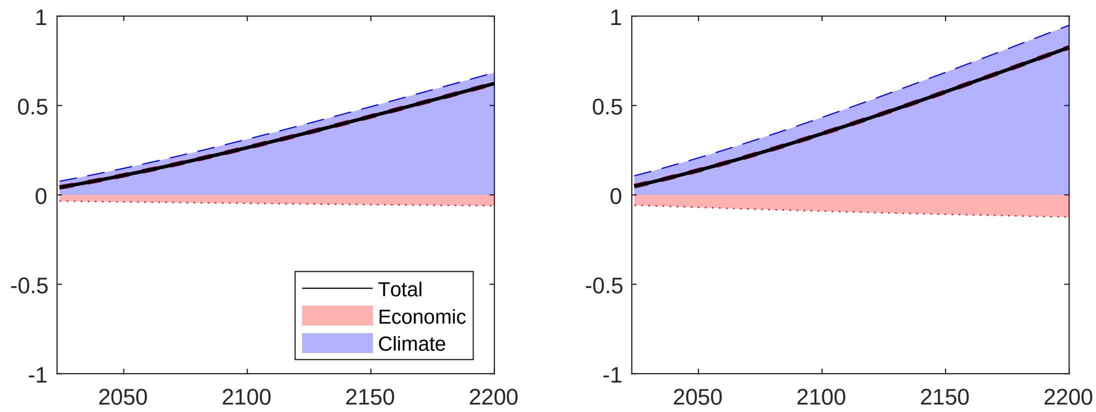
(b) Heterogeneous



There is a timing mismatch between the costs and benefits of taxing carbon. The consequences of unmitigated carbon build-up intensify over time so that the worst effects from business as usual are experienced well in the future. Meanwhile, any fiscal policy stringent enough to have a meaningful impact on the path of the carbon stock must impose immediate costs on households. The balance between these costs and benefits shifts over time. Figure 5 plots the evolution of average welfare, computed as consumption equivalents behind the veil of ignorance, and shows the decomposition in welfare between economic factors and climate improvement. As time moves forward, the benefit of a relative improvement in climate grows while the costs from consumption distortions remain roughly constant.

Figure 5: Average welfare over time

(a) Uniform (b) Heterogeneous



7 Concluding Remarks

In this paper, we have studied the link between inequality and optimal carbon policies.

Empirically, we document that emissions embodied in household expenditures are higher per dollar for low-income and low-wealth households compared with high-income and high-wealth households. This suggests that a flat carbon tax would be regressive. We use these facts to motivate the use of non-homothetic preferences in our theoretical and quantitative analysis.

Theoretically, we study constrained-optimal policies in an environment in which the planner is not permitted to redistribute resources across agents. The constrained-optimal carbon tax is household-specific, featuring tax rates that increase with income. When carbon tax rates are further restricted to be uniform across households, the constrained-optimal carbon tax should optimally be set lower than the unconstrained optimal carbon tax.

Quantitatively, we measure the distributional effects of implementing either a uniform carbon tax or one that differentiates by household wages. Both cases are solved with individual rebates to remove wealth effects and keep the distribution of resources across households fixed. In this way, we quantify the climate and welfare effects of implementing the carbon tax policies prescribed by our theoretical findings. We find that both policies lead to Pareto improvements, benefiting all households regardless of their income or wealth.

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A Mathematical Appendix

Proof of Proposition 1. The proof consists of showing that all conditions for the constrained optimal allocation satisfy the conditions of a competitive equilibrium with taxes and transfers.

The constrained-optimal allocation is characterized by equations (15), (25) and (29). We want to show that the equations characterizing the constrained-optimal allocation solve the competitive equilibrium with taxes and transfers, defined in Definition 1 and characterized by equations (6)-(9). First, comparing (29) and (9), we get from simple observation that both coincide when τ_t^i is replaced by the optimal tax, $\tau_t^i = \frac{\upsilon\sigma_t}{u_{ct}^i}$.

Second, combine (6)-(8) by plugging (7) and (8) into (6) to get:

$$\sum_i \mu_i c_{c,t}^i + \sum_i \mu_i c_{d,t}^i = \sum \mu_i \varepsilon_t^i \tag{54}$$

where $g_t = 0$. To see that the constrained-optimal allocation satisfies this marketing clearing condition, multiply both sides of (25) by μ_i and sum over i to obtain:

$$\sum_i \mu_i c_{c,t}^i + \sum_i \mu_i c_{d,t}^i = \sum_i \mu_i \varepsilon_t^i \tag{55}$$

With $g_t = 0$, the budget constraint of the government is satisfied with transfers equal to $t_t^i = \tau_t^i c_{dt}^i$. QED.

Proof of Proposition 2. The proof follows from showing that the equations characterizing the constrained social planner's problem satisfy equations characterizing the competitive equilibrium, with taxes $\tau = \dfrac{\upsilon\sigma_t}{\sum_i \frac{\mu_i c_t^i}{\sum_i \mu_i c_t^i} u_{ct}^i}$ and transfers $t_t^i = \tau_t c_{dt}^i$.

The first order conditions for constrained social planner's problem are:

$$(c_{dt}^i) : \mu_i u_{dt}^i - \upsilon\mu_i\sigma_t - \lambda_t^i + \sum_{j\neq i} \eta_t^{ij}\left(c_{ct}^j + \bar{c}\right) - \sum_{j\neq i} \eta_t^{ji}\left(c_{ct}^j + \bar{c}\right) = 0 \tag{56}$$

$$(c_{ct}^i) : \mu_i u_{ct}^i - \lambda_t^i - \sum_{j\neq i} \eta_t^{ij} c_{dt}^j + \sum_{j\neq i} \eta_t^{ji} c_{dt}^j = 0 \tag{57}$$

$$(S_{t+1}) : -\beta^t x'(S_{t+1}) + \sigma_t\beta^t - \sigma_{t+1}\beta^{t+1}(1-\delta) = 0 \tag{58}$$

$$(\beta^t\lambda_t^i) : c_{ct}^i + c_{dt}^i = \varepsilon_i \tag{59}$$

$$(\beta^t\eta_t^{ij}) : (c_{ct}^i + \bar{c})c_{dt}^j = (c_{ct}^j + \bar{c})c_{dt}^i \tag{60}$$

where η_t^{ij} is the Lagrange multiplier on the constraint on allocations.

Combine equations (56) and (57) to obtain:

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \frac{1}{\mu_i u^i_{ct}} \left[\upsilon \mu_i \sigma_t - \sum_{j \neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) + \sum_{j \neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \right] \tag{61}$$

Equation (61) coincides with (9) for τ^i_t equal to:

$$\tau^i_t = \frac{1}{\mu_i u^i_{ct}} \left[\upsilon \mu_i \sigma_t - \sum_{j \neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) + \sum_{j \neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}). \right] \tag{62}$$

If we multiply both sides of equation (62) by $c^i_{c,t} + \bar{c} + c^i_{d,t}$ and sum across all i, we obtain:

$$\begin{aligned} \sum_i \tau^i_t \mu_i u^i_{ct} \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) =& \upsilon \sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \\ & - \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \sum_{j \neq i} \eta^{ij}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ & + \sum_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \sum_{j \neq i} \eta^{ji}_t (c^j_{ct} + \bar{c} + c^j_{dt}) \\ =& \upsilon \sigma_t \sum_i \mu_i \left(c^i_{ct} + \bar{c} + c^i_{dt} \right) \end{aligned} \tag{63}$$

Reorganizing terms, we get:

$$\tau_t = \frac{\upsilon \sigma_t}{\sum_i \frac{\mu_i c^i_t}{\sum_j \mu_j c^j_t} u^i_{ct}} \tag{64}$$

where $c^i_t \equiv c^i_{ct} + \bar{c} + c^i_{dt}$. Second, combine (6)-(8) by plugging (7) and (8) into (6) to get:

$$\sum_i \mu_i c^i_{c,t} + \sum_i \mu_i c^i_{d,t} = \sum \mu_i \varepsilon^i_t \tag{65}$$

where $G_t = 0$. To see that the constrained-optimal allocation satisfies this marketing clearing condition, multiply both sides of (25) by μ_i and sum over i to obtain:

$$\sum_i \mu_i c^i_{c,t} + \sum_i \mu_i c^i_{d,t} = \sum_i \mu_i \varepsilon^i_t \tag{66}$$

With $G_t = 0$, the budget constraint of the government is satisfied with transfers equal to $t^i_t = \tau_t c^i_{dt}$. QED.

Proof of Corollary 1. The proof consists in showing that the competitive equilibrium conditions with taxes coincide with the optimality condition (108) and (109). Evaluate the intratemporal consumption decision at the optimal taxes to get:

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{1 + \gamma \mu^\star_t}{1 - (1 - \gamma) \frac{\mu^\star_t}{1 + \mu^\star_t}} \tag{67}$$

After simple algebra, we get

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \mu^\star_t \tag{68}$$

which coincides with (109). To see that the allocation satisfies the budget constraint and market clearing conditions, plug the taxes into (33) to get

$$(1 + \gamma\mu^\star_t)c_{dt} + [1 - (1-\gamma)\frac{\mu^\star_t}{1+\mu^\star_t}]c_{ct} = \varepsilon^i_t + \tau_{ct}\bar{c} \tag{69}$$

$$c_{dt} + \gamma\mu^\star_t c_{dt} + c_{ct} - (1-\gamma)\frac{\mu^\star_t}{1+\mu^\star_t}c_{ct} = \varepsilon^i_t + (1-\gamma)\frac{\mu^\star_t}{1+\mu^\star_t}\bar{c} \tag{70}$$

$$c_{dt} + c_{ct} - \varepsilon^i_t = (1-\gamma)\frac{\mu^\star_t}{1+\mu^\star_t}(c_{ct} + \bar{c}) - \gamma\mu^\star_t c_{dt} \tag{71}$$

$$c_{dt} + c_{ct} - \varepsilon^i_t = \frac{\mu^\star_t}{1+\mu^\star_t}\{(1-\gamma)(c_{ct} + \bar{c}) - \gamma c_{dt}(1+\mu^\star_t)\} \tag{72}$$

We need to show that the right hand side of (72) equals zero. Notice that for preferences given by (3), equation (34) simplifies to

$$\frac{1-\gamma}{\gamma}\frac{(c^i_{ct} + \bar{c})}{c^i_{dt}} = 1 + \mu^\star_t \tag{73}$$

$$(1-\gamma)\left(c^i_{ct} + \bar{c}\right) = \gamma c^i_{dt}(1 + \mu^\star_t) \tag{74}$$

Plug into (72) to obtain

$$c_{dt} + c_{ct} = \varepsilon^i_t \tag{75}$$

That feasibility constraints are satisfied, follow from adding over all i following the steps in the proof of Proposition 2. QED.

Proof of Proposition 3. The first part of the proof comes from comparing the intratemporal optimality condition on clean and dirty consumption in the quantitative model economy (9) with the constrained optimal on (91). It follows that the constrained optimal carbon tax equals the one characterized in Proposition 1 and equals:

$$\tau^i_t = \frac{\upsilon\sigma_t}{u^i_{ct}}$$

To prove that the uniform carbon tax takes the form in (32), notice that equation (106) coincides with (9) for τ_t^i equal to:

$$\tau_t^i = \frac{1}{\mu_i u_{ct}^i}\left[\upsilon\mu_i\sigma_t - \sum_{j\neq i}\eta_t^{ij}(c_{ct}^j + \bar{c} + c_{dt}^j) + \sum_{j\neq i}\eta_t^{ji}(c_{ct}^j + \bar{c} + c_{dt}^j)\right] \tag{76}$$

which coincides with (62) in the proof of Proposition 2. The rest of the proof follows the same steps from (62) onwards in the proof of Proposition 2.

B Constrained Optimum in the Quantitative Model

Let $\{\alpha_i\}_{\forall i}$ be an arbitrary set of Pareto weights with $\sum_i \alpha_i = 1$. The socially optimal allocation is the sequence $\{c_{jt}^i, n_t^i, k_{t+1}^i, K_{jt}, N_{jt}, S_t\}_{t=0,j=c,d}^{\infty}$ that solves the social planner's problem, which is to maximize

$$\mathbf{E}_0 \sum_i \alpha_i \left[\sum_{t=0}^{\infty}\beta^t\left(u(c_{ct}^i, c_{dt}^i) - v(n_t^i) - x(S_{t+1})\right)\right] \tag{77}$$

subject to the carbon cycle (1), pricing rules given by $w_t = F_{Njt},\ r_t = F_{Kjt}$ for $j = c, d$ and $p_{dt} = 1$ the consumer´s budget constraint (38) and (39), the feasibility constraints (41)-(44).

Plug in the labor and capital market clearing constraints into (43) and (44) to rewrite the budget set of the planner's problem as:

$$S_{t+1} = (1-\delta)S_t + \upsilon\sum_i \mu_i c_{dt}^i \tag{78}$$

$$c_{dt}^i + c_{ct}^i + k_{t+1}^i - k_t^i = F_N\left(\sum_i \mu_i k_t^i, \sum_i \mu_i \varepsilon_t^i n_t^i\right)\varepsilon_t^i n_t^i + [F_K\left(\sum_i \mu_i k_t^i, \sum_i \mu_i \varepsilon_t^i n_t^i\right) - \delta_k]k_t^i \tag{79}$$

$$k_{t+1}^i \geq 0 \tag{80}$$

$$\sum_i \mu_i[c_{ct}^i + k_{t+1}^i - (1-\delta_k)k_t^i] = F(K_{ct}, N_{ct}) \tag{81}$$

$$\sum_i \mu_i c_{dt}^i = F\left(\sum_i \mu_i k_t^i - K_{ct}, \sum_i \mu_i \varepsilon_t^i n_t^i - N_{ct}\right) \tag{82}$$

The first order conditions for this problem are:

$$(c^i_{dt}) : \alpha_i u^i_{dt} - \upsilon \mu_i \sigma_t - \mu_i \lambda_{dt} - \mu_i \lambda^i_t = 0 \tag{83}$$

$$(c^i_{ct}) : \alpha_i u^i_{ct} - \mu_i \lambda_{ct} - \mu_i \lambda^i_t = 0 \tag{84}$$

$$(n^i_t) : -\alpha_i v^i_{n_t} + \mu_i \lambda^i_t F_{Nt} \varepsilon^i_t + \sum_j \mu_j \lambda^j_t (F_{NNt} \mu_i \varepsilon^i_t \varepsilon^j_t n^j_t + F_{KNt} \mu_i \varepsilon^i_t k^j_t) + \lambda_{dt} F_{Nt} \mu_i \varepsilon^i_t = 0 \tag{85}$$

$$(k^i_{t+1}) : -\mu_i(\lambda_{ct} + \lambda^i_t) + \beta \mu_i \mathbf{E}_t \left[\lambda^i_{t+1}(1-\delta_k) + \lambda^i_{t+1} F_{Kt+1} + \phi^i_{t+1}\right] \tag{86}$$

$$+ \beta \mathbf{E}_t \left[\sum_j \mu_j \lambda^j_{t+1} (F_{NKt+1} \mu_i \varepsilon^j_{t+1} n^j_{t+1} + F_{KKt+1} \mu_i k^j_{t+1})\right] + \beta \mu_i \mathbf{E}_t \left[\lambda_{dt+1} F_{Kt+1} + \lambda_{ct+1}(1-\delta_k)\right] = 0$$

$$(S_{t+1}) : \sigma_t - \mathbf{E}_t \left\{x'(S_{t+1}) + \beta(1-\delta)\sigma_{t+1}\right\} = 0 \tag{87}$$

$$(N_{ct}) : \lambda_{ct} F_{Nt} - \lambda_{dt} F_{Nt} = 0 \tag{88}$$

$$(K_{ct}) : \lambda_{ct} F_{Kt} - \lambda_{dt} F_{Kt} = 0 \tag{89}$$

where $\beta^t \sigma_t$, $\beta^t \mu_i \lambda^i_t$, $\beta^t \phi^i_t$, $\beta^t \lambda_{dt}$, $\beta^t \lambda_{ct}$ are the Lagrange multipliers on the carbon cycle and the constraints (79)-(82), respectively.

At an interior solution, optimality for a utilitarian planner implies the following intratemporal wedge for clean and dirty consumption:

$$\frac{u^i_{dt}}{u^i_{ct}} = \frac{\nu \sigma_t + \lambda_{dt} + \lambda^i_t}{\lambda_{ct} + \lambda^i_t} \tag{90}$$

From (88)–(89), we have that $\lambda_{ct} = \lambda_{dt}$ so we can write (90) as:

$$\frac{u^i_{dt}}{u^i_{ct}} = 1 + \frac{\nu \sigma_t}{u^i_{c_t}} \tag{91}$$

which coincides with the one in the simple model, equalizing the consumption marginal rate of substitution between clean and dirty goods with the relative social price that includes the cost of carbon emissions. In this economy, the social cost of carbon is the expected present value of the future climate damages associated with an extra unit of emission. Iterating on (87), it equals:

$$\sigma_t = \mathbf{E}_t \sum_{j=1}^{\infty} \left[\beta(1-\delta)\right]^{j-1} x'(S_{t+j}) \tag{92}$$

For ease of notation we define the following objects:

$$\Delta^n_t \equiv \sum_j \mu_j \lambda^j_t (F_{NNt} \varepsilon^i_t \varepsilon^j_t n^j_t + F_{KNt} \varepsilon^i_t k^j_t) \tag{93}$$

$$\Delta^k_t \equiv \beta \mathbf{E}_t \sum_j \mu_j \lambda^j_{t+1} (F_{NKt+1} \varepsilon^j_{t+1} n^j_{t+1} + F_{KKt+1} k^j_{t+1}) \tag{94}$$

Using these definitions, the intratemporal wedge between consumption and leisure is given by

$$\frac{u^i_{nt}}{u^i_{ct}} = F_{Nt}\varepsilon^i_t + \frac{\Delta^n_t}{u^i_{ct}} \tag{95}$$

The intratemporal condition shows that the social planner internalizes how much people decide to work determines the equilibrium salary and interest rate, impacting the uninsurable income. Similarly, the intertemporal wedge incorporates the effect of the individual's saving decision over the equilibrium prices, and the Euler equation is given by:

$$u^i_{ct} = \beta \mathbf{E}_t \left[u^i_{c,t+1} \left(F_{Kt+1} - \delta_k + 1 \right) + \phi^i_{t+1} \right] + \frac{\Delta^k_t}{u^i_{ct+1}} \tag{96}$$

Constrained-Optimal Income taxes. Comparing the constrained-optimal Euler equation (96) with the market optimality condition (46), it follows that a capital income tax/subsidy is required if the constrained-optimal allocation is to be implemented in the market economy. The capital income tax must satisfy

$$\tau^i_{kt+1} = \frac{-\Delta_k}{u^i_{ct+1}}. \tag{97}$$

Similarly, a labor income tax is required to implement the constrained-optimal allocation. The optimal tax rate comes from comparing the constrained optimality condition (95) with (45) and equals

$$\tau^i_{nt} = \frac{-\Delta_n}{u^i_{ct} F_{Nt} \varepsilon^i_t} \tag{98}$$

B.1 Uniform Constrained-Optimal Allocation

As in the simple model economy, uniformity of the carbon tax must be added as an exogenous constraint. We also restrict attention to utility functions over consumption that take the form (3). The constrained optimal allocation comes from maximizing (77) subject to (78)-(82) together with (31).

The first order conditions for the constrained social planner are:

$$(c_{d,t}^i): \alpha_i u_{dt}^i - \upsilon\mu_i\sigma_t - \mu_i\lambda_{dt} - \mu_i\lambda_t^i + \sum_{j\neq i}\eta_t^{ij}\left(c_{c,t}^j + \bar{c}\right) - \sum_{j\neq i}\eta_t^{ji}\left(c_{c,t}^j + \bar{c}\right) = 0 \tag{99}$$

$$(c_{c,t}^i): \alpha_i u_{ct}^i - \mu_i\lambda_{ct} - \mu_i\lambda_t^i - \sum_{j\neq i}\eta_t^{ij}c_{d,t}^j + \sum_{j\neq i}\eta_t^{ji}c_{d,t}^j = 0 \tag{100}$$

$$(n_t^i): -\alpha_i v_{n_t}^i + \mu_i\lambda_t^i F_{Nt}\varepsilon_t^i + \sum_j \mu_j\lambda_t^j(F_{NNt}\mu_i\varepsilon_t^i\varepsilon_t^j n_t^j + F_{KNt}\mu_i\varepsilon_t^i k_t^j) + \lambda_{dt}F_{Nt}\mu_i\varepsilon_t^i = 0 \tag{101}$$

$$\begin{aligned}(k_{t+1}^i): & -\mu_i(\lambda_{ct} + \lambda_t^i) + \beta\mu_i\mathbf{E}_t\left[\lambda_{t+1}^i(1-\delta_k) + \lambda_{t+1}^i F_{Kt+1} + \phi_{t+1}^i\right] \\ & + \beta\mathbf{E}_t\left[\sum_j \mu_j\lambda_{t+1}^j(F_{NKt+1}\mu_i\varepsilon_{t+1}^j n_{t+1}^j + F_{KKt+1}\mu_i k_{t+1}^j)\right] + \beta\mu_i\mathbf{E}_t\left[\lambda_{dt+1}F_{Kt+1} + \lambda_{ct+1}(1-\delta_k)\right] = 0\end{aligned} \tag{102}$$

$$(S_{t+1}): \sigma_t - \mathbf{E}_t\left\{x'(S_{t+1}) + \beta(1-\delta)\sigma_{t+1}\right\} = 0 \tag{103}$$

$$(N_{ct}): \lambda_{ct}F_{Nt} - \lambda_{dt}F_{Nt} = 0 \tag{104}$$

$$(K_{ct}): \lambda_{ct}F_{Kt} - \lambda_{dt}F_{Kt} = 0 \tag{105}$$

where $\beta^t\eta_t^{ij}$ is the Lagrange multiplier on the constraint (31).

From equations (104) and (105), we obtain $\lambda_{ct} = \lambda_{dt}$. Then combine equations (99) and (100) to obtain:

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \frac{1}{\alpha_i u_{ct}^i}\left[\upsilon\mu_i\sigma_t - \sum_{j\neq i}\eta_t^{ij}(c_{ct}^j + \bar{c} + c_{dt}^j) + \sum_{j\neq i}\eta_t^{ji}(c_{ct}^j + \bar{c} + c_{dt}^j)\right] \tag{106}$$

where the social cost of carbon, σ_t, is given by (92).

If we multiply both sides of equation (62) by $c_{c,t}^i + \bar{c} + c_{d,t}^i$ and sum across all i, we obtain:

$$\begin{aligned}\sum_i \tau_t^i\mu_i u_{ct}^i\left(c_{ct}^i + \bar{c} + c_{dt}^i\right) =& \upsilon\sigma_t\sum_i \mu_i\left(c_{ct}^i + \bar{c} + c_{dt}^i\right) \\ & - \sum_i\left(c_{ct}^i + \bar{c} + c_{dt}^i\right)\sum_{j\neq i}\eta_t^{ij}(c_{ct}^j + \bar{c} + c_{dt}^j) \\ & + \sum_i\left(c_{ct}^i + \bar{c} + c_{dt}^i\right)\sum_{j\neq i}\eta_t^{ji}(c_{ct}^j + \bar{c} + c_{dt}^j) \\ =& \upsilon\sigma_t\sum_i \mu_i\left(c_{ct}^i + \bar{c} + c_{dt}^i\right)\end{aligned} \tag{107}$$

Reorganizing terms, we can define the social cost of carbon in units of average consumption at the uniform constrained allocation as:

$$\mu_t^\star \equiv \frac{\upsilon\sigma_t}{\sum_i \frac{\mu_i c_t^i}{\sum_j \mu_j c_t^j}u_{ct}^i} \tag{108}$$

where $c_t^i \equiv c_{ct}^i + \bar{c} + c_{dt}^i$. Thus we can write (106) as

$$\frac{u_{dt}^i}{u_{ct}^i} = 1 + \mu_t^\star \tag{109}$$

The intra and intertemporal wedges coincide with the ones in the economy without the constraint (31) and are given by (95) and (96).

C Data Appendix

We redo the empirical analysis using household data from the Panel Study of Income Dynamics (PSID). Compared to the CEX, the PSID has the advantage that it contains a more complete representation of household wealth, including financial and nonfinancial assets and debt. On the other hand, the PSID expenditure data is more aggregated, compared with the CEX. Thus, the PSID analysis presented here complements our CEX analysis and provides a useful robustness exercise.

We combine the PSID expenditure data with the EPA emissions data, in a similar way as described in Section 2. As shown in Figure 6, the embodied emissions intensity is decreasing in both income and in wealth.

Figure 6: Embodied emissions

(a) Income

(b) Wealth