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QUALITATIVE SPECIFICATIONS OF THE BOND MARKET

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Research Paper

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*The views expressed in this paper are solely those of the authors, and should not be attributed to Southern Methodist University or to either the Federal Reserve Bank of Dallas or the Federal Reserve System.

Theoretical Macroeconomic Modelling and Qualitative Specifications of the Bond Market

by

William R. Russell
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1. Introduction

It is generally accepted that the common practice of specifying a commodity, a money, and a labor market will denote a complete theoretical macroeconomic model. By Walras' law, satisfaction of equilibrium conditions for all explicitly given markets is sufficient for equilibrium in an implicit bond market. So it is presumed that comparative statics analysis does not require the "redundant" introduction of a bond market.^{1,2} Alternatively, the implication is that no additional information can be derived from the bond market because any such information is fully implied by Walras' law and the explicit specification of other markets.³

If one were to take issue with the propriety of this procedure for specifying a macroeconomic model the battleground for resolution of the opposing contentions would not be in the science of economics. We are not concerned here with behavioral characteristics of a model nor will we be questioning the veracity of a model or its specifications. The concern is about the way economists define and present any general economic model, so the issue is a methodological one. Specifically, does the procedure generally followed by economists completely and properly define a theoretical macroeconomic model?

The meta-model question involved may amuse the philosopher of science but as economists we are more interested in the substantive ramifications this issue raises for macroeconomics. Consequently, we shall argue that

1) the modelling procedure as typically applied in theoretical macroeconomics analysis is usually incomplete, leading to ambiguous specification of a qualitative macroeconomic model. Specifically, we shall see that the state of affairs in the bond market can be ambiguous despite complete qualitative specifications of all the other markets.

2) Even with the acceptance of Walras' law, information vital to a model will be neglected if the bond market is omitted. In some cases, additional comparative statics results will become possible when qualitative specifications for the bond market are directly utilized in the analysis. To illustrate the issue, we use a simple theoretical macroeconomic model to investigate two comparative statics problems.

2. Specifications of a Model

If we follow the usual convention for presenting a theoretical macroeconomic model it is sufficient to include the aggregate behavior of economic agents in the commodity market, money market and labor market.

The relevant equilibrium conditions are:

$$(1) L^h(y, r, M^h/p) + L^f(y, r, M^f/p) = M/p$$

$$(2) C(y, r, M^h/p) + I(y, r, M^f/p) = y$$

$$(3) N^s(W/p) = N^d(W/p) = N ;$$

and for reference, we include as a bond market,

$$(4) B^d(y, r, M^h/p) = B^s(y, r, M^f/p),$$

where income supply, y , is determined by the equilibrium level of employment, N , and the fixed capital stock, K . Labor demand is a decreasing function of the real wage and its supply is an increasing function. Nominal wages are flexible and this market clears instantaneously.

M is the nominal money supply which is distributed between households, denoted M^h , and firms, denoted M^f ; r is the nominal rate of interest and p , the price level. B stands for real bonds with superscripts to indicate demand or supply. It is assumed that money demands of both firms (L^f) and households (L^h) are positively related to income and real money balances and inversely related to the interest rate. Furthermore, we assume

$$L_{M/p}^h + L_{M/p}^f < 1.$$

Consumption and investment demand for commodities are assumed to be increasing functions of income and real money balances, but are inversely related to the interest rate. Furthermore, it is assumed that

$$C_y + I_y < 1.$$

A given change in income induces a less than proportional change in income demanded. Or, alternatively, a state of excess supply of goods results from a given increase in income. Additionally, by consumers' budget constraint

$$C_y + S_y = 1.$$

Consider the effect of changes in each endogenous variable on the three markets. Since the model is a very familiar one, it requires very little elaboration. From a position of equilibrium in the commodity market an increase in either the rate of interest, the level of income, or the price level induces excess supply. From an equilibrium position in the money market, an increase in the interest rate induces excess supply but an increase in either income or the price level induces excess demand.

By budget constraints and Walras' law, the behavioral specifications enumerated for (1), (2), and (3) imply certain conditions in the bond market. We presume that bonds in the model are issued by firms and provide holders with interest income. Bond supply must be inversely related to the interest rate and real money balances, but positively related to income. Bond demand is positively related to the interest rate. However, budget constraints do not specifically imply particular direction of responses in bond demand for changes in real balances and income. Consequently, Walras' law does imply that an increase in the rate of interest induces excess demand but it does not provide a direction of response in the bond market to an increase in income or real balances.

It will prove useful at this time to review some conditions which must hold for bond demand and supply functions because of the constraints on firms and consumers. Any increase in income implies a higher activity level for firms and usually requires higher working capital and real capital. The issuance of bonds is the mode of financing such expansion. Thus, we must have

$$B_y^s = I_y + L_y^f > 0,$$

and for consumers, the budget constraint requires

$$1 - C_y - L_y^h = B_y^d, \text{ or, } B_y^d > 0 \text{ if and only if } C_y + L_y^h < 1.$$

The financing constraints imply B^s increases unambiguously as income increases but B^d may increase or decrease. Within their constraints, consumers may decrease bond demand to finance current consumption, additional money balances, or both. Should future consumption be considered

a normal good, then increases in income or wealth would reasonably lead to an increase in bond demand rather than a decrease. Although we are willing to accept this commonly made assumption notice that normality of future consumption is not implied by Walras' law.

Given a positive change in income, the specifications imply that a state of excess supply is induced in the commodity market and excess demand in the money market. The model does not dictate whether disequilibrium is larger in the money market or in the commodity market. Walras' law only requires that the sum of market excess demands equal zero. Of course, equilibrium in the bond market is clearly undisturbed if both demand and supply change in the same direction with equal magnitude as income changes. This is one of the assumptions Patinkin (1965) makes for his model.⁴ It should be clear that the behavioral specifications made for (1), (2), and (3) do not sufficiently constrain the bond market functions to compel this conclusion. Therefore, the bond market may be either in excess demand, excess supply, or in equilibrium in response to a change in income. We consider each of the three possible cases in greater detail.

Case 1. Bond market equilibrium is unaffected.

In this case,

$$(5) \quad B_y^s - I_y + L_y^f = B_y^d = S_y - L_y^h.$$

Note that the final expression on the right hand side of the equation (5) is determined by consumers while the left side is under the purview of firms. In this instance, individuals plan to add a value of bonds to their portfolios in an amount exactly equal to firms' increased needs for funds.

When income increases, individuals furnish exactly the additional net amount of money holdings and investment funds firms desire. Under the assumption of this first case, the matching of these plans must occur at the original price level and rate of interest. It is hard to imagine a theoretical justification for such a happy coincidence.

If $I_y = 0$, as is assumed in some specifications of the investment function, the above equation can be reduced to the condition $B_y^d = L_y^f$ and we can write

$$S_y = L_y^f + L_y^h.$$

In this version increased income induces an expansion of saving demand which will be exactly equal to the total increase in the demand for money; and the value of bonds individuals wish to add to their portfolios will equal the amount of money balances firms (independently determined) wish to add. Note that these equalities are not the result of market adjustments which may have brought about a resulting equilibrium in the bond market. In any event, disequilibria are induced in both the money and commodity markets.

Case 2. Excess demand is induced in the bond market.⁵

An implication of $B_y^d > B_y^s$ is that

$$L_y^h + L_y^f < S_y - I_y,$$

or simply that the excess supply of commodities is larger in magnitude than the excess demand for money balances. Substituting for B_y^s , this excess demand for bonds implies $B_y^d > I_y + L_y^f$. The demand for net additions to bond portfolios is greater than firms' desire for additional funds at the

prevailing prices and interest rate, i. e., before endogenous adjustments. If investment is not a function of income, the inequality is modified; consumers are willing to add to their bond portfolios a value which exceeds firms' additional needs for working capital in the form of money balances.

Case 3. Excess supply is induced in the bond market.⁶

Under this specification, we have

$$B_y^d - S_y - L_y^h < I_y + L_y^f - B_y^s.$$

Hence, we observe that the desired net additions to household bond portfolios are less than the desired uses of funds by firms at the as yet unchanged values of the endogenous variables, price level and interest rate. If investment is not a function of income, the inequality reduces to

$$B_y^d < L_y^f$$

which implies that consumers' planned increments to bonds are insufficient to finance firms' planned additions to working capital in the form of money balances at pre-shock market prices and interest rates.

Every one of these cases is feasible and consistent with specifications made in the other markets. Indeed, it is because the general specifications of the other markets are not more stringent that Walras' law is not sufficient to denote exactly which case is obtained. The point to be made here is that a complete specification of a macroeconomic model as is usually presented by economists, i.e., making explicit specifications for the commodity, money, and labor markets, is just not sufficient (even upon

invoking Walras' law) to particularize the specification of the bond market. The bond market specification may be any one of the three cases we have discussed. Consequently, the specification of (1), (2), and (3) does not define a macroeconomic model with an unambiguous bond market. Each case listed above constitutes an optional specification for the bond market and when combined with (1), (2), and (3) will define a distinct qualitative macroeconomic model. The system (1), (2), and (3) specifies not one qualitative macroeconomic model, but three, when we consider the alternative specifications of the bond market with respect to income changes. Of course, if the implicit bond market specification is irrelevant and an explicit specification of the bond market is truly redundant, then comparative statics results should be invariant whether we use (1), (2), and (3) or the bond market specifications (4) along with (2) and (3). Exactly the same determination should be made of the direction of change in the endogenous variables regardless of which of three bond market cases we elect to use. To illustrate that the bond market is not "redundant" and does in fact matter we provide two examples.

3. Comparative Statics Analysis: Two Examples

For our first example, consider a decrease in risk aversion on the part of entrepreneurs because of an exogenous shock, β . As a result, they require a smaller risk premium and find additional investment optimal at the prevailing rate of interest. The expansion of expenditures is to be financed by a decreased demand for money and an increase in the supply of bonds. For purposes of our analysis we shall follow specifications consistent with our above discussion. Using the money and commodity markets⁷

the induced excess demand or supply for increases in some endogenous variables are shown in Table 1:

Table 1

Increase in	Explicit Model Specifications		Implicit Market Specifications*
	Commodity Mkt	Money Mkt	Bond Mkt
r	Exc. S	Exc. S	Exc. D
p	Exc. S	Exc. D	Exc.D, Eq., Exc.S
y	Exc. S	Exc. D	Exc.D, Eq., Exc.S .

*Implied by Walras' law.

The well-known ambiguity in qualitative specifications for the bond market⁸ when they are derived from Walras' law can be clearly seen in Table 1.

Using the specifications for the commodity and money markets and differentiating totally, we obtain the set of equations given below in matrix form:

$$(6) \begin{bmatrix} C_r + I_r & -\frac{1}{p^2} (M^h \cdot C_{M/p} + M^f \cdot I_{M/p}) \\ L_r^h + L_r^f & \frac{1}{p^2} (M \cdot -M^h \cdot L_{M/p}^h - M^f \cdot L_{M/p}^f) \end{bmatrix} \times \begin{bmatrix} dr/d\beta \\ dp/d\beta \end{bmatrix} = \begin{bmatrix} -I_\beta \\ -L_\beta^f \end{bmatrix}$$

We will use the term a_{ij} to refer to the element in the i th row and the j th column of the comparative static Jacobian matrix; $|D|$ refers to the determinant of the Jacobian and $|D_i|$, $i = 1, 2$ is the determinant with the i th column replaced by the column of constant terms. More specifically, $|D_1|$ and $|D_2|$ are used to solve for the sign of $dr/d\beta$ and $dp/d\beta$, respectively.

From our specifications, a_{11} , a_{12} , $a_{21} < 0$ and $a_{22} > 0$. Consequently,

$$|D| < 0, \text{ and}$$

$$(7) \quad |D_1| = -I_\beta(a_{22}) - (-L_\beta^f)(a_{12}).$$

With $I_\beta > 0$, $L_\beta^f < 0$, the sign of the latter determinant is ambiguous and, thus, the direction of change in the equilibrium rate of interest is unknown.

With a decrease in risk aversion of entrepreneurs, the equilibrium rate of interest may increase or decrease.

To determine the response of equilibrium price level, we write:

$$(8) \quad |D_2| = a_{11}(-L_\beta^f) - a_{21}(-I_\beta).$$

This determinant is negative, and therefore, the equilibrium price level increases as entrepreneurs' risk aversion decreases.

The above analysis relies on the explicit specifications of the commodity and money markets. Consider another qualitative macroeconomic model based on explicit specifications of the commodity and bond markets. Suppose we make the following specifications:

Table 2

Increase in	Explicit Model Specifications		Implicit Market Specifications*
	Commodity Mkt.	Bond Mkt.	Money Mkt.
r	Exc. S	Exc. D	Exc.D, Eq., Exc.S
p	Exc. S	Exc. S	Exc.D
y	Exc. S	Exc. D	Exc.D, Eq., Exc.S .

* Implied by Walras' law.

While complete qualitative specifications are made for the commodity and bond markets, money market conditions cannot be made unambiguous by applying Walras' law (in the absence of additional specifications).

Nonetheless, suppose we proceed to analyze the given macroeconomic problem using the commodity and bond markets of a general model rather than the commodity and money markets.⁹

Totally differentiating the market conditions for the commodity and bond markets, we obtain in matrix form the following:

$$(9) \begin{bmatrix} C_r + I_r & -\frac{1}{p^2} (M^h \cdot C_{M/p} + M^f \cdot I_{M/p}) \\ (B_r^d - B_r^s) & -\frac{1}{p^2} (M^h \cdot B_{M/p}^d - M^f \cdot B_{M/p}^s) \end{bmatrix} \times \begin{bmatrix} dr/d\beta \\ dp/d\beta \end{bmatrix} = \begin{bmatrix} -I_\beta \\ B_\beta^s \end{bmatrix}$$

Denoting elements in the Jacobian as b_{ij} from our assumptions, b_{11} , b_{12} , and $b_{22} < 0$ and $b_{21} > 0$; and thus, $|D| > 0$. The sign of $dr/d\beta$ will, therefore, be of the same sign as $|D_1|$ which is:

$$(10) \quad |D_1| = -I_\beta (b_{22}) - B_\beta^s (b_{12}).$$

With $I_\beta, B_\beta^s > 0$, this determinant is positive. The equilibrium rate of interest increases as risk aversion decreases.

For the equilibrium price level change, we find

$$(11) \quad |D_2| = b_{11}(B_\beta^s) - b_{21}(-I_\beta).$$

But under our assumptions, the two terms on the right hand side of equation (11) are of different signs. The direction of change in equilibrium price is indeterminate.

We may observe from Tables 1 and 2 that the explicit specifications made for both analyses are not contradictory. If we accept both sets to be the qualitative specifications of all markets in a general macroeconomic model, then the solutions we obtain for the price level and interest rate are applicable to this model. Notice that both directions of change could not have been obtained if either of the previously omitted markets was not explicitly specified.

The implication is clear. If we ignore qualitative information from the bond market and only use the explicitly specified information for the commodity and money markets within a general equilibrium model (which includes the specification of all markets), we will not necessarily be able to derive some of the comparative statics results implied by the full model. Another configuration of markets from the same general equilibrium system might be necessary to provide additional results. In our example, we are able to derive the result for the rate of interest when we use the bond and commodity markets. Neglecting the bond market in this case necessarily entails the abandonment of the means to secure the direction of change in the equilibrium rate of interest. By the same token, we would not recommend the restriction of macroeconomic analysis to the commodity and bond markets for we have also seen that this would imply foregoing some other results. In our example, it would have then been impossible to secure the comparative statics result for the price level. We may observe that for the problem investigated here, the limitation of information to specifications of the bond and money markets will yield neither the direction of change in the rate of interest nor that of the price level. We hasten to add that this does not imply that such a combination should never be explored. For different models or other problems, that configuration may prove useful.

A second, more familiar, example will provide us with the opportunity to explicate another aspect of our general theme. In the first example, simply introducing the bond market is sufficient to solve for both interest rate and price movements. This will now change. Suppose there is an exogenous increase in labor supply. We continue to assume flexible money wages and a labor market which quickly clears so this translates into an

increase in income. If we totally differentiate equations (1) and (2) as before, collect terms and write in matrix notation, we have the following:

$$(12) \begin{bmatrix} C_r + I_r & -\frac{1}{p^2} (M^h \cdot C_{M/p} + M^f \cdot I_{M/p}) \\ L_r^h + L_r^f & \frac{1}{p^2} (M \cdot M^h \cdot L_{M/p}^h - M^f \cdot L_{M/p}^f) \end{bmatrix} \times \begin{bmatrix} dr/dy \\ dp/dy \end{bmatrix} = \begin{bmatrix} 1 - C_y - I_y \\ -L_y^h - L_y^f \end{bmatrix}$$

Using the commodity and money markets, therefore, we obtain the following:

$$(13) |D| = a_{11} a_{22} - a_{21} a_{12} < 0,$$

where a_{11} , a_{12} , and $a_{21} < 0$, and $a_{22} > 0$. Furthermore, with $C_y + I_y < 1$ and $L_y^h + L_y^f > 0$, we have

$$(14) |D_1| = (1 - C_y - I_y) a_{22} - (-L_y^h - L_y^f) a_{12}.$$

The two terms on the right hand side of equation (14) are of different sign and therefore $|D_1|$ cannot be signed. Unless quantitative information is imposed, the direction of change in the rate of interest in response to an increase in income is indeterminate if we only use information specified for the money and commodity markets.

To determine dp/dy , we need

$$(15) |D_2| = a_{11}(-L_y^h - L_y^f) - a_{21}(1 - C_y - I_y).$$

Under our specifications, this sign is positive. Thus, the exogenous increase in income decreases the equilibrium price level but its effect on the rate of interest is indeterminate.

The analysis is now duplicated using the commodity and bond markets. Totally differentiating the relevant market equations, we obtain:

$$(16) \begin{bmatrix} C_r + I_r & -\frac{1}{p^2} (M^h \cdot C_{M/p} + M^f \cdot I_{M/p}) \\ (B_r^d - B_r^s) & -\frac{1}{p^2} (M^h \cdot B_{M/p}^d - M^f \cdot B_{M/p}^s) \end{bmatrix} \times \begin{bmatrix} dr/dy \\ dp/dy \end{bmatrix} = \begin{bmatrix} 1 - C_y - I_y \\ B_y^s - B_y^d \end{bmatrix}$$

From the assumptions, b_{11} , b_{12} , and $b_{22} < 0$ and $b_{21} > 0$, we see that

$$|D| > 0.$$

For dr/dy , we need the sign of $|D_1|$ which is

$$(17) |D_1| = b_{22}(1 - C_y - I_y) - b_{12}(B_y^s - B_y^d).$$

Earlier discussion of the three possible alternative impacts of an income change on the bond market makes it clear that we cannot sign the last term in the equation. Likewise, for the price level direction of change we need the sign of

$$(18) |D_2| = b_{11}(B_y^s - B_y^d) - b_{21}(1 - C_y - I_y).$$

The first term on the right hand side of equation (18) cannot be signed.

Neither change in r nor change in p can be determined by using the bond and commodity market combination without additional qualitative specifications for the bond market.

Under Patinkin's specification (our case 1 in section 2) the resulting signs are

$$dr/dy, dp/dy < 0.$$

The second case is one where excess demand is induced. If so, the signs are

$$dr/dy < 0 \text{ and } dp/dy = ?$$

While in the third and final case excess supply is created. Then

$$dr/dy = ? \text{ and } dp/dy < 0.$$

Using the three alternative specifications for the bond market we obtain three different sets of results from our analysis of the commodity and bond

markets. Observe that in the third case, the equilibrium rate of interest may increase; but if the specification were that of the first or second case, the equilibrium rate of interest must decrease. Importantly, each set is consistent with results from the analysis carried out using the commodity and money markets.

The comparative statics results of this example using (1), (2), (3) and (4) with Patinkin's specification (case 1) are the same as those obtained using the same system with case 2 specification of the bond market. There is a decrease in both price and interest rate. It should be noted that this does not mean that comparative statics results will invariably be identical for these two bond market specifications in every macroeconomic system.

If the macromodel consists of (1), (2), (3), and (4) with the bond market specification of case 3, then in this example the price level will decrease but the change in rate of interest may increase or decrease. It may be observed that these are also the conclusions which would be drawn from a system where models differing only by bond market specifications are commingled. In this case, no distinguishing qualitative information from the bond market can be admitted and this is equivalent to denying any qualitative information from the bond market.

4. The Bond Market and Walras' Law.

For comparative statics analysis to be fruitful, it is necessary that a model be defined to indicate the nature of disequilibria introduced by any exogenous change. Often, the impact due to a change in an exogenous variable is assumed to take place in only one explicit market. By Walras' law, in such instances there is an implicit effect in the bond market. Yet, an exogenous change may reasonably affect other explicitly specified

markets.¹⁰ Recall our example of a decrease in risk aversion of entrepreneurs. Since the investment expansion must be financed, we are required to specify portfolio adjustments. Our assumption of a decrease in the demand for money balances together with an increase in the supply of bonds is not the only one possible. But it does inspire the resulting indeterminacy of the rate of interest when analysis is restricted to the commodity and money markets; and we find that when we use the bond market and commodity market in the analysis, the change in rate of interest is determinate.

In the second example, we see that a change in income is explicitly specified to increase excess demand for money and excess supply of commodities, but the effect on the implicit bond market is ambiguous. Three cases are consistent with our explicit specification of the commodity and money markets. Depending on which case is chosen, we have seen that comparative statics results can vary when income is changed. In this example, the use of the commodity and bond markets for the analysis is not by itself sufficient for determining the direction of change in the equilibrium rate of interest. It is required that bond market specifications shed some light on whether an increase in income creates an excess demand or excess supply. This is an additional theoretical specification on the bond market because Walras' law is simply not sufficient for this determination.

In both our examples indeterminacy of the direction of change in the endogenous rate of interest is due to a lack of information about the bond market because analysis is restricted to information about the commodity and money markets. The remedy in the first example is simply to use the commodity and bond markets for the analysis. By so doing, relevant

qualitative information about the bond market could be utilized in the analysis. In the second example we observe an additional ambiguity. A set of three possible qualitative specifications for the bond market are consistent with Walras' law. The macroeconomic model remains ambiguous unless a choice is made. Suppose we augment the money and commodity market conditions with a specification that an increase in income creates an excess demand in the bond market. This resulting macroeconomic model is not equivalent to another case obtained by combining the same money and commodity market conditions with a bond market in which an increase in income creates an excess supply. Different specifications imply different theoretical models. In addition, they can produce different qualitative results as we have seen in our second example. Thus, a theoretical model specified without an explicit bond market will be ambiguous despite Walras' law.¹¹

Whether or not the direction of interest rate change can be determined may depend on how the bond market is specified in the model as we see in our second example. More troublesome is the fact that a change in rate of interest can appear to be indeterminate in a macroeconomic model (when it is not) if the bond market is not used in the solution procedure. This is illustrated by our first example. Even with explicit qualitative specifications of the bond market, analysis restricted to the commodity and money markets cannot detect this variation. To uncover the additional result, qualitative bond market specifications must be used in the analysis. In both examples, the direction of change in the equilibrium rate of interest is determined by admitting bond market information explicitly into our analysis.¹²

The problem is a methodological one, as we point out in our introduction. In the absence of information about the nature of bond market disequilibrium initiated by some change in a system variable or parameter, a qualitative macroeconomic model which specifies only the commodity and money markets is not well-defined.¹³

The same argument can be applied generally. Assume that $n-1$ markets are "fully" specified, as in an econometric model where functional forms and parameter values are given. It is surely the case that by Walras' law the n th market is completely defined. Or, if the n th market is now specified, it cannot provide any information not already contained in the "fully" specified $n-1$ markets. While these statements are true, they do not contradict the thrust of the argument we make in this paper. Any doubt should be resolved by recognizing that no theoretical macroeconomic model is specified in full detail. Some information is necessarily omitted when a theoretical macroeconomic model is presented. This omission is what makes the model a general one and, therefore, a family of particular fully-described models. The specification of a theoretical model is not (and should not be) sufficiently particularized to define a model with equations of given functional forms and values of all parameters. Rather, a family of models is specified wherein every element, (i.e., every particular model) shares a common set of desired characteristics. In this sense, the $n-1$ markets of a theoretical model are not "fully" specified and, accordingly, the n th market is not. In particular, for the same qualitative specifications the algebraic sum of excess demands over the $n-1$ markets may be positive or negative, implying that the excess demand for the n th market may be negative or positive, respectively. An explicit qualitative specification made to choose one or the other for the n th market will

introduce information beyond that implied by Walras' law. Depending on the problem under investigation this designation of excess demand or supply may provide the link for additional comparative statics results. Clearly this can result only because we deal with theoretical models which by their nature must be general and are not "fully" specified in the sense we use above.

Any theoretical macroeconomic model defines a family of particular, full-information models, not a specific one. Therefore, Walras' law does not always indicate whether excess supply or excess demand exists in the bond market. If there is excess supply in the commodity market and excess demand in the money market, the bond market may have either excess demand or excess supply. This is the situation in our examples. When a sign condition is indicated for the bond market, this provides additional information not contained in the rest of the model. Thus, Walras' law is not a sufficient basis for presuming that all the information we need from the bond market is automatically being utilized merely by analyzing the explicit commodity and money markets.

5. Conclusion

Economists appear to believe that Walras' law permits a complete specification of a theoretical model without the explicit specification of a bond market. However, when a bond market is not specified the disequilibrium condition induced in the bond market by any exogenous (or endogenous) change must be deduced by Walras' law. This entails the signing of the algebraic sum of excess demands over the explicitly specified markets. Since we refrain from specifying magnitudes of excess demands in the markets of qualitative general equilibrium models, this determination

cannot be made. Macroeconomic specifications do not ordinarily include sufficient information to define the state of affairs in the bond market for all circumstances. As a result, it is often unclear whether a change in some system variable causes no change, excess demand, or excess supply in the bond market. An economist should be no more satisfied with this than if, for any other market, the model specifications did not indicate whether excess supply or demand is created as a system variable changes. More emphatically, would we be willing to present theoretical specifications which leave uncertain whether an increase in rate of interest creates an excess demand or excess supply in the money market?¹⁴ This is the state of affairs we are passively admitting in the bond market. Without information to define whether an excess supply or demand results from a change in a parameter, or system variable, some comparative statics results may be lost.

In practice, we may safely ignore the bond market when information from it is not relevant for our investigation. But the fact is, when only the commodity market and money markets enter the analysis we are, de facto, simply ignoring qualitative information particular to the bond market. The existence of Walras' law does not mean that bond market information is automatically being processed in our analysis. We are only presuming that whatever may be happening in the bond market will not be in violation of Walras' law. Unless both sign and magnitude conditions on the excess demands of the money and commodity markets are available, it will not in principle be possible to know whether excess supply or demand exists in the bond market. Our examples fully illustrate that in such cases the lack of information can be important.

A complete theoretical macroeconomic model should be one which includes qualitative information from all markets (including a bond

market) in order to avoid ambiguity in the model. The common deviation from this premise is probably due to our willingness to believe that Walras' law makes bond market specification redundant. But redundancy exists only if a model is "fully" specified. When we are dealing with theoretical models, this is not the case. Consequently, Walras' law should not be relied upon to provide us with sufficient information to avoid ambiguity in the bond sector of a theoretical macroeconomic model.

Footnotes

*Department of Economics, Southern Methodist University and Federal Reserve Bank of Dallas, respectively. We wish to thank Professors Gerald P. Dwyer, Thomas R. Saving, Nathan Balke, and Dan Slottje for helpful comments. Any remaining errors are solely our responsibility.

¹ Examples of specifying only the commodity and money markets begin with Hicks (1937). This treatment continues in many excellent macroeconomic texts including Dornbush and Fisher (1978) and Branson (1979).

² Yet more recently, the importance of the bond market is being emphasized, for example, in works of Friedman(1981) and Fackler(1985).

³ Consider the following examples:

"The answer ... seems to be ... that it makes no difference whether one works with money or securities, provided first, that one is concerned only with the determination of the equilibrium rate of interest."
Johnson (1961).

"After all, the money market could be dropped from the IS-LM model just as easily as the bond market."
Silber (1970).

"The conventional wisdom says that there is no difference between the two (money or bond markets). For the money market coupled with the commodities market is exactly the same system as the securities (loans) market coupled with the commodities market (by Walras Law)."

Akerlof (1973).

⁴ See Patinkin (1965) for a discussion of the bond market specification.

⁵ This is probably the most commonly made assumption. See Gapinski (1982) p.41, although his justification centers on change in bond demand because his model does not appear to permit an income induced change in bond supply.

⁶ Brunner and Meltzer (1972) assume an increase in y increases excess supply on the credit market in their model. See pp. 962-3.

⁷ Meyer (1980) asserts "Walras' Law permits us to treat any of the three markets as redundant, even the commodity market. To preserve similarity to the conventional IS-LM analysis, we will continue to drop the bond market" p.95.

⁸ See Witte (1966) and McCaleb and Sellon (1980) for a discussion of the ambiguous signs for excess demand functions consistent with Walras' Law.

⁹ See Barro (1984), "We shall obtain the same results regardless of which pair of aggregate-consistency conditions that we examine. Usually,

macroeconomists look at the condition for clearing the commodity market, --, and at the one for money to be willingly held,--. We shall find it convenient to follow this practice in our analysis. However, remember that the results do not change if we substitute for one of these conditions the condition that the credit market clear, --." p.126.

¹⁰ Witte (1966) and Friedman (1981) make this point very well.

¹¹ McCaleb and Sellon (1980) provide valid criticism of Meyer (1975) but overstate with "In summary, then, we have shown that the omitted bond market is, in fact, implicitly specified in the IS-LM model and that qualitative restrictions on the excess demand functions in the product and money markets place qualitative restrictions on the bond market. In particular, the excess demand functions for bonds is completely specified and depends upon the endogenous variables y and r and all the exogenous shift parameters in the explicit functions." p. 409.

¹² On p. 74, Witte (1966) does "--treat the money equation as the redundant one" but this is harmless in his investigation.

¹³ For a related but very different view, consider Pearsall (1980) wherein he argues that "-- the usual descriptions of a model's dynamic behavior make it possible to do comparative statics even when the model is incomplete" p.234. Certainly, the correspondence principle may be inspiration for additional model specification.

¹⁴ This case is illustrated in Table 2.

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