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Working Paper 2403

Research Department

<https://doi.org/10.24149/wp2403>

May 2024

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Geopolitical Oil Price Risk and Economic Fluctuations*

Lutz Kilian[†], Michael D. Plante[‡] and Alexander W. Richter[§]

May 21, 2024

Abstract

This paper seeks to understand the general equilibrium effects of time-varying geopolitical risk in oil markets. Answering this question requires simultaneously modeling several features including macroeconomic disasters and geopolitically driven oil production disasters, oil storage and precautionary savings, and the endogenous determination of uncertainty about output and the price of oil. We find that oil price uncertainty tends to be driven by macroeconomic uncertainty. Shifts in the probability of a geopolitically driven major oil supply disruption have meaningful effects on the price of oil and the macro economy, but the resulting oil price uncertainty is not a major driver of fluctuations in macroeconomic aggregates.

Keywords: Geopolitical risk, macroeconomic risk, time-varying uncertainty, rare disasters, oil, endogeneity, shock propagation, economic fluctuations, precautionary savings, inventories

JEL Classifications: E13, E22, E32, Q43

*This work was supported by computational resources provided by the BigTex High Performance Computing Group at the Federal Reserve Bank of Dallas. The views in this paper do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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1 INTRODUCTION

There has been growing interest in the impact of shifts in geopolitical risk in global commodity markets, in general, and in the oil market in particular in recent years. Historically, increases in oil price risk have been associated, for example, with uncertainty about the implications of the Iranian Revolution in 1979 and the outcome of the invasion of Kuwait in 1990. More recently, there was a surge in uncertainty about the possibility of Russia refusing to sell oil to Europe after the invasion of Ukraine in 2022 and then about the effectiveness of a price cap on Russian oil exports. Other recent sources of oil price uncertainty have included doubts about the ability of U.S. shale oil producers to maintain their production increases and about changes in OPEC oil production quotas.

There is a deep-rooted belief in macroeconomics that higher oil price uncertainty driven by geopolitical risk undermines business and consumer confidence, lowering domestic investment and consumption and hence real GDP.¹ For example, higher uncertainty may cause a delay in investment decisions, thereby lowering real GDP. This belief is based on insights from partial equilibrium models with exogenous oil price uncertainty. The intuition developed from these models need not hold in general equilibrium, however. Moreover, it is widely understood that uncertainty about the price of oil is not exogenous because it depends on the demand side of the oil market, reinforcing the importance of analyzing the implications of oil price uncertainty shocks in a general equilibrium framework.

In this paper we present results from a calibrated dynamic stochastic general equilibrium (DSGE) model of the global economy that is designed to address the question of how geopolitical oil price risk is linked to economic fluctuations. The model includes risk averse economic agents, an oil production sector, oil storage, and limited substitutability between oil and capital. The price

¹For example, Bernanke (1983), Ferderer (1996), Lee et al. (1995), Edelstein and Kilian (2009), Elder and Serletis (2010), Baumeister and Kilian (2016a), Ready (2018), and Gao et al. (2022) discuss the impact of oil price uncertainty on U.S. real activity, while Kilian (2009), Jo (2014), and Cross et al. (2022) discuss its impact on global real activity. The perception that oil price volatility matters for the transmission of oil price shocks to the economy also helped spawn a large literature on the asymmetric transmission of oil price shocks (see, e.g., Bernanke et al., 1997; Davis and Haltiwanger, 2001; Hooker, 1996, 2002; Kilian and Vigfusson, 2011; Leduc and Sill, 2004; Lee and Ni, 2002; Mork, 1989; Ramey and Vine, 2010).

of oil is determined endogenously. Since the model is global, we abstract from oil imports and exports and international capital flows. One key difference from earlier studies is that our model allows for both macroeconomic uncertainty and global oil price uncertainty and that uncertainty is determined endogenously. Building on Gourio (2012), the model includes macroeconomic and oil production disasters of stochastic length that occur with time-varying probabilities.

We find that shocks to the probability of a disaster have persistent effects on the price of oil and oil inventories, whether the disaster involves a reduction in economic growth (which may be viewed as the result of an economic crisis such as the Great Recession of 2008 or the Covid-19 recession of 2020) or a major oil production shortfall (modeled after events such as the 1973 and 1979 oil supply disruptions or the invasion of Kuwait in 1990). Shocks to the probability of a major exogenous decline in global oil production in the model cause persistent increases in oil price uncertainty, whether this disaster is actually realized or not. While these shocks have effects on real GDP and other macroeconomic aggregates that dwarf those found with more traditional stochastic volatility shocks, they are not a dominant driver of fluctuations in macroeconomic aggregates.

The model provides a better understanding of how oil price uncertainty is created and propagated to the economy. A central premise of the conventional narrative that oil price uncertainty is the key to understanding both the severity of the recession in the early 1980s and the absence of an expansion after 1986 is that any change in the price of oil is associated with an increase in oil price uncertainty. In some cases this narrative is implicit, for instance when the model of Bernanke (1983) is invoked in discussing the effect of the 1986 oil price shock. In other cases this premise is built into empirical models, such as when estimating a GARCH-in-mean VAR model that assumes the level and volatility of the oil price are driven by the same shock (e.g., Elder and Serletis, 2010).

Our DSGE model shows that changes in oil price uncertainty need not be an indication of exogenous shifts in the uncertainty about future oil supplies. We find that oil price uncertainty tends to be driven by macroeconomic uncertainty, which helps explain why higher oil price uncertainty historically has been associated with lower real activity. Uncertainty about the oil price may reflect exogenous macroeconomic uncertainty shocks even in the absence of level shocks, mirroring the

widely accepted conclusion that real oil price fluctuations respond to shifts in the global demand for oil (e.g., Kilian, 2009; Kilian and Murphy, 2014). In addition, increased oil price uncertainty may be caused by level shocks in the oil market and in the macro economy. Thus, not only are level and uncertainty shocks not the same, as implicitly assumed in VAR-GARCH models, but the effects of a level shock in the data generating process are not separable from those of a volatility shock, as assumed in VAR models with stochastic volatility. This casts doubt on the ability of these models to correctly identify exogenous oil price uncertainty shocks. It also calls into question a large body of empirical work that has produced seemingly robust evidence of large recessionary effects of oil price uncertainty shocks (e.g., Elder and Serletis, 2010; Gao et al., 2022; Jo, 2014).

Our work relates to several strands of the literature. First, it contributes to the large literature on the effects of uncertainty shocks on the macro economy (e.g., Berger et al., 2020; Bloom, 2009; Gourio, 2012; Jurado et al., 2015; Leduc and Liu, 2016; Ludvigson et al., 2021) by focusing on the interaction between macroeconomic and oil price uncertainty. We show that modeling geopolitical oil price risk and macroeconomic risk jointly is necessary for understanding the evolution of oil price uncertainty.

Second, our analysis contributes to the literature making the case that oil price uncertainty shocks in particular are recessionary (e.g., Başkaya et al., 2013; Bernanke, 1983; Drakos and Konstantinou, 2013; Gao et al., 2022; Ready, 2018) and affect oil production and storage (e.g., Cross et al., 2022; Kellogg, 2014). Our contribution is to extend this literature to a general equilibrium setting with endogenous oil prices and endogenous oil price uncertainty. Unlike earlier DSGE studies that incorporated stochastic volatility shocks to oil production, we account for the fact that geopolitical oil price risk is inherently one-sided. This risk reflects the stochastic arrival of oil production disasters driven by geopolitical events. Our analysis highlights that both the ability to store oil and the risk aversion of economic agents play a central role in propagating oil disaster shocks driven by geopolitical risk. For example, without storage the responses of both oil market variables and macroeconomic aggregates to higher oil production risk tend to be muted, suggesting that models without storage fail to capture the full effects of shifts in oil production risk. Oil

storage also matters for the responses of the global economy to macroeconomic disaster shocks.

Third, we contribute to a large literature emphasizing the endogeneity of fluctuations in the price of oil with respect to macroeconomic aggregates (e.g., Kilian, 2009; Kilian and Murphy, 2014). Whereas this earlier literature focused on showing that the level of the real price of oil is endogenously determined by oil demand and oil supply, our analysis shows that oil price uncertainty responds to both level and uncertainty shocks to macroeconomic aggregates, complicating the identification of exogenous oil price uncertainty shocks. Furthermore, we find that stochastic volatility shocks to oil production do not have strong recessionary effects, unlike shocks to the oil production disaster probability. These findings have important implications for empirical work seeking to establish the macroeconomic effects of shocks to oil price uncertainty (e.g., Cross et al., 2022; Elder and Serletis, 2010; Ferderer, 1996; Jo, 2014).

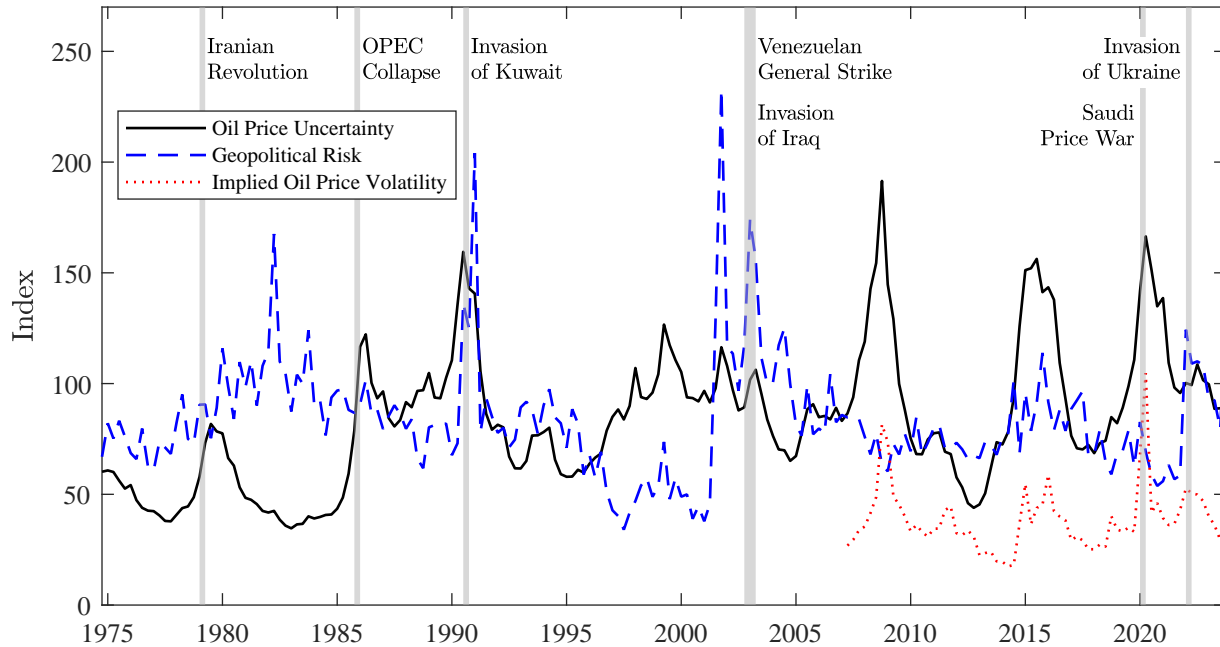
The remainder of the paper is organized as follows. In [Section 2](#), we highlight the importance of geopolitical risk for the economy. We propose an index of the uncertainty in the real price of oil building on Jurado et al. (2015), trace its evolution since the 1970s, and discuss the relationship between downside risk in oil production and oil price uncertainty. We also show that our index differs conceptually from the geopolitical risk index in Caldara and Iacoviello (2022) and other measures of oil price uncertainty used in the literature. [Section 3](#) reviews why many economists expect oil price uncertainty to slow economic activity. [Section 4](#) introduces a calibrated DSGE model of the global economy that elucidates the determination of oil price uncertainty and macroeconomic uncertainty. In [Section 5](#), we study the relationship between oil price uncertainty and macroeconomic uncertainty, the transmission of uncertainty shocks to the economy, and the ability of these shocks to explain fluctuations in economic growth and oil price fluctuations. Our analysis also sheds light on the key mechanisms in the model and the importance of downside risk. In [Section 6](#), we discuss the relationship between our work and earlier DSGE models of the transmission of oil price uncertainty shocks. [Section 7](#) discusses implications of our analysis for empirical models of the effects of oil price uncertainty shocks. The concluding remarks are in [Section 8](#).

2 THE IMPORTANCE OF GEOPOLITICAL RISK FOR THE ECONOMY

Time-variation in geopolitical risk is widely considered an important determinant of fluctuations in economic activity. The financial press, international organizations, rating agencies and the investment community all vie to assess these risks and their likely impact on the economy. Clearly, geopolitical events matter not only when they occur on rare occasions, but also when investors and consumers make decisions in anticipation of the possibility of such events. This fact is nowhere more apparent than when it comes to geopolitical risk in energy markets. For example, S&P Global and BlackRock list risks to energy security as one of the top geopolitical risks of 2024. This assessment is driven in no small part by concerns about OPEC quota decisions, global access to Russian oil amidst Ukrainian attacks on Russian oil infrastructure and efforts to tighten the G7 price cap, dwindling strategic oil reserves, disruptions of oil shipments in the Red Sea and possibly in the Persian Gulf, and concerns about a widening conflict between Israel and Iran. Many of these geopolitical events are low probability, but have potentially high impact on the economy.

The focus of our paper is to develop a better understanding of how time-variation in geopolitical risk in oil markets affects economic fluctuations. While we focus on the market for crude oil, our insights matter for other commodity markets as well. The literature has focused on oil price uncertainty arising from geopolitical events, rather than the downside risk to oil production caused by these events. Unlike the downside risks emphasized by market participants, which are inherently subjective because they relate to events that have not occurred, oil price uncertainty can be quantified using econometric methods. This does not make oil price uncertainty a good indicator of geopolitically driven downside risk, however, because downside risk and uncertainty need not go hand-in-hand. While downside geopolitical risk to oil production raises oil price uncertainty, not all surges in oil price uncertainty are driven by geopolitical events.

[Figure 1](#) quantifies the uncertainty about the price of oil in global oil markets since the modern oil market emerged in the early 1970s. We follow Jurado et al. (2015) in measuring oil price uncertainty (\mathcal{U}_{p^o}) as the one-quarter ahead conditional volatility of the unpredictable component

Figure 1: Oil price uncertainty, 1974Q4-2023Q4

Notes: The solid line shows the uncertainty about the percent change in the real price of oil obtained by deflating the U.S. refiners' acquisition cost for oil imports by the U.S. CPI for all urban consumers. The method used to quantify this uncertainty is based on Jurado et al. (2015). The dashed line is the quarterly average of the historical GPR series in Caldara and Iacoviello (2022). The dotted line is the option-implied crude oil price volatility index (OVX) published by the Chicago Board Options Exchange.

from a predictive model of the real price of oil.² This definition highlights the fact that what matters for economic decision making is not whether the price of oil has become more or less variable, but whether it has become more or less predictable.³ The real price of oil is defined as the U.S. refiners' acquisition cost for oil imports, as is standard in the literature, deflated by the implicit GDP deflator. The predictable component of the growth rate of the real price of oil is approximated using a diffusion index based on largely the same set of variables used by Jurado et al. (2015), augmented by the real price of oil, updated, and aggregated to quarterly frequency.

We estimate the uncertainty about the price of oil from 1974Q4 through 2023Q4. [Figure 1](#) shows large spikes in 1979, 1986, and 1990 at the time of the Iranian Revolution, the collapse of

²Details of the construction of the uncertainty measure can be found in Appendix A.

³The definition of uncertainty in Jurado et al. (2015) is closely related to the formal measure of predictability in Diebold and Kilian (2001), since lack of predictability implies uncertainty.

OPEC, and the invasion of Kuwait. Not all geopolitical events are associated with surges in oil price uncertainty, however. For example, neither the outbreak of the Iran-Iraq War in late 1980 nor the outbreak of the Israel-Hamas War in the last quarter of 2023 had a discernible impact on the index.

The largest spike in oil price uncertainty in 2008 was not driven by geopolitical risk, but by macroeconomic risk created by the Great Recession. Similarly, the surge in oil price uncertainty in 2015 appears driven by market forces rather than geopolitics (see Baumeister and Kilian, 2016b), as was a smaller spike in uncertainty during the Asian Financial Crisis of the late 1990s. Sometimes, geopolitical events coincide with surges in macroeconomic risk, as was the case in early 2020 when the Covid-19 recession occurred at the same time as the Saudi price war in the oil market.

Figure 1 also shows that our oil price uncertainty index differs systematically from the geopolitical risk (GPR) index of Caldara and Iacoviello (2022), which quantifies the newspaper coverage of geopolitical events not limited to oil markets. The GPR index does not capture oil price uncertainty associated with macroeconomic risk, nor does it capture variation in oil price uncertainty clearly driven by geopolitical risk in oil markets. It is also very different from the oil price uncertainty measure. For example, the direction of these indices differs in the early 1980s. In addition, the relative magnitude of changes in these indices is quite different. The correlation of the indices is essentially zero. This highlights the importance of formally measuring oil price uncertainty.

Our oil price uncertainty index also differs conceptually from the implied volatility index (OVX) published by the Chicago Board Options Exchange, which is only available since 2007. The correlation between our index and the OVX (0.71) is much higher than its correlation with the GPR index. There is no indication that the implied volatility measure has an informational advantage, allowing it to respond to shifts in geopolitical uncertainty more quickly. While the timing of the peaks and troughs is broadly similar, the oil price uncertainty index is more persistent.⁴

⁴In related work, Gao et al. (2022) derive an index similar to the OVX series using oil options back to 1990Q1. The relationship between these indices is similar over the extended sample.

3 WHY ECONOMISTS THINK OIL PRICE UNCERTAINTY MATTERS

Interest in fluctuations in oil price uncertainty dates to the mid-1980s. Economists at the time observed that the economy entered a steep recession after the 1979/80 oil price surge, but a similarly large drop in the price of oil in 1986 did not cause a large economic expansion. This fact is consistent with two mutually exclusive narratives. One is that the relationship between oil prices and the U.S. economy is linear, which implies that the effect of oil price shocks on the economy is modest at best and that the recession in the early 1980s is explained in substantial part by other shocks (e.g., Barsky and Kilian, 2002). This explanation is consistent with DSGE models of the transmission of oil price shocks that predict that rising oil prices will modestly slow growth in oil-importing economies, as consumers' income is reduced and firms face higher production costs, and, conversely, falling oil prices will modestly stimulate growth in oil-importing economies (e.g., Backus and Crucini, 2000).⁵

The other narrative is that this relationship is nonlinear with positive oil price shocks having a disproportionately larger effects on the economy. Macroeconomists for many years have been partial to this interpretation. A leading explanation of the nonlinearity required to explain the disproportionately large effect of positive oil price shocks and the negligible effects of negative oil price shocks, is that the rise in oil price uncertainty associated with the 1979/80 oil price surge caused consumer spending and business fixed investment to drop, amplifying the effects of rising oil prices, whereas in 1986 an increase in oil price uncertainty associated with the fall in oil prices largely offset the stimulus from lower oil prices.⁶

The theoretical justification for this explanation relies on the real options theory of investment in Bernanke (1983) as well as the effect of rising uncertainty on precautionary savings and con-

⁵There have been many attempts to design macroeconomic models that amplify the transmission of oil price shocks (e.g., Aguiar-Conraria and Wen, 2007; Atkeson and Kehoe, 1999; Finn, 2000; Rotemberg and Woodford, 1996). Not only are these models not empirically supported in many cases, but being able to generate a larger recession after 1979/80 invariably makes it more difficult to explain the absence of an economic expansion in 1986.

⁶Whether oil price uncertainty increased or declined in 1986 has been a matter of debate. Whereas backward-looking GARCH estimates imply a rise in oil price uncertainty after the oil price dropped, but not leading up to the oil price decline, a strong case can be made that the collapse of OPEC that triggered the price decline reduced oil price uncertainty, since it showed that OPEC was unable to prop up oil prices.

sumer spending (e.g., Başkaya et al., 2013; Edelman and Kilian, 2009; Plante and Traum, 2012). The latter behavior arises from risk aversion embedded in the utility function. A closely related third argument is that firms build oil inventories when oil price volatility rises, which raises the real price of oil, lowers oil consumption, and hence depresses economic activity (e.g., Cross et al., 2022; Gao et al., 2022; Kilian, 2009; Kilian and Murphy, 2014).

3.1 REAL OPTIONS THEORY The most commonly cited reason why oil price uncertainty shocks matter for economic activity was articulated by Bernanke (1983) in a partial equilibrium setting.⁷ Bernanke's point is that—to the extent that the cash flow from an irreversible investment project depends on the price of oil or its derivatives—all else equal, increased uncertainty about the price of oil prompts firms to delay investments. As a result, investment expenditures drop and real output declines. Uncertainty for this purpose may be measured by the expected conditional volatility of the real price of oil over the relevant investment horizon. Exactly the same reasoning applies to purchases of energy-intensive consumer durables such as cars (see Edelman and Kilian, 2009).

There are several caveats to this application of real options theory. First, the quantitative importance of this channel depends on how important the real price of oil is for investment and durable consumption decisions and on the share of such expenditures in aggregate spending. For example, it seems intuitive that uncertainty about the price of oil would be important for decisions about oil drilling in Texas (see Kellogg, 2014). It is less obvious that it would be as important for investment in other sectors of the economy such as textile production or information technology, the expected profitability of which does not depend as much on oil prices.

Second, there is reason to believe that for longer-term investment projects, the variation over time in the uncertainty about the real price of oil is small. Consider an airline purchasing new planes that are expected to fly for 20 years. The cash flow from this investment clearly depends on fuel prices and an increase in expected fuel price uncertainty, all else equal, should cause the airline to delay the investment. However, the predictable component of the variance of the real price of oil quickly reverts to the unconditional variance at longer horizons, so one would not

⁷For related discussion of the effects of uncertainty shocks more generally see Pindyck (1991) and Bloom (2014).

expect variation in the conditional variance at monthly or quarterly frequency to have a large effect on the investment decision. This makes it implausible that the value of the real option is large.

Third, Bernanke (1983) takes the real oil price as exogenously given. This simplifying assumption does not hold in practice, complicating the analysis. The concern is that we may attribute to oil price uncertainty the effects of macroeconomic or financial uncertainty that are much more likely to affect the cash flow from the investment. Finally, Bernanke's analysis of the real options model is conducted in partial equilibrium. As discussed in [Section 4.4](#), it is unclear whether this result survives in general equilibrium. Thus, the overall importance of this channel for the aggregate economy is open to question.

3.2 PRECAUTIONARY SAVINGS A complementary reason first articulated in Edelstein and Kilian (2009) is that households' increased uncertainty about their future income in the wake of unexpected changes in the real price of oil will cause an increase in precautionary savings (or, equivalently, a reduction in consumer expenditures). In this interpretation, uncertainty may affect not merely spending on energy-intensive consumer durables such as cars, but other consumer expenditures as well. This argument has subsequently been formalized in Plante and Traum (2012) and Başkaya et al. (2013) within a small open economy DSGE model. Precautionary savings are driven by risk aversion embodied in the utility function. Thus, precautionary savings may also be caused by other uncertainty shocks, as discussed in Carroll and Kimball (2008).

While a response of precautionary savings to oil price uncertainty shocks may seem persuasive in a partial equilibrium setting, its quantitative importance becomes less obvious when moving to general equilibrium models. For example, higher uncertainty may cause households to work more in general equilibrium, allowing them to spend more, all else equal, and offsetting the precautionary savings motive for reducing consumption, as shown in Plante and Traum (2012).

3.3 PRECAUTIONARY INVENTORY DEMAND A third channel by which increased oil price uncertainty can reduce economic activity operates through precautionary demand for oil inventories. The notion of precautionary demand for oil was introduced in Kilian (2009) and expanded on in

Kilian and Murphy (2014) and Cross et al. (2022). These studies emphasized that higher precautionary demand driven by increases in oil price uncertainty, all else equal, will raise the real price of oil and reduce global economic activity by discouraging oil consumption. This analysis in turn builds on the theoretical insights in Alquist and Kilian (2010) of how mean-preserving shifts in the uncertainty about future oil supply shortfalls affect the real price of oil through inventory accumulation. DSGE models incorporating oil storage have been presented in Olovsson (2019) and Gao et al. (2022) who stress that firms hold precautionary oil inventories in response to oil price uncertainty.

4 A MODEL OF THE PROPAGATION OF UNCERTAINTY SHOCKS

In this section, we introduce a DSGE model of the global economy designed to elucidate the determinants of oil price uncertainty. Focusing on the global economy allows us to sidestep the complications involved in modeling oil importing or exporting economies and in aggregating the data at the global level, while focusing on the essence of the endogeneity problem. The model is not designed to accommodate all possible channels of the determination and transmission of oil price uncertainty shocks, but it includes the main mechanisms emphasized in the literature.

4.1 MODEL The model is a nonlinear stochastic growth model augmented to include oil production. Oil is used as an intermediate input by a representative firm that produces a final good. The model allows for precautionary savings as well as oil storage, which has been shown to play an important role in driving fluctuations in oil prices (see Kilian and Murphy, 2014). The distinguishing feature of the model is that it includes downside risk to both oil production and output growth. While downside risk to oil production can be thought of as arising from geopolitical events, downside risk to the macro economy represents rare, sharp economic downturns, such as the Great Recession or the COVID-19 Recession that are not otherwise captured by the model.

We follow Gourio (2012) in modeling such events as disasters that arrive with a small, but time-

varying probability.⁸ Time-variation in the probability of oil production and output growth disasters induces exogenous variation in oil price uncertainty and macroeconomic uncertainty. While the idea of modeling downside risk in the economy as growth disasters with time-varying probability is not new, we are the first to apply this approach to modelling geopolitical risk in the oil market.⁹ One advantage of this approach compared to the more traditional approach of subjecting oil production growth to an exogenous volatility shock is that it accounts for the fact that the risk agents are typically concerned with in practice is not two-sided. Rather these risks involve a sharp reduction in oil production. Such rare disasters matter not only because of their impact when they occur, but, more importantly, because agents' behavior reflects the anticipation of these disasters even when they are not realized in the data.

Productivity and Growth Disasters The growth rate of productivity, $g_t = a_t/a_{t-1}$, follows

$$\ln g_t = \ln \bar{g} + \sigma_g \varepsilon_{g,t} - \zeta_g (v_t^g - \bar{\pi}_1^g), \quad \varepsilon_{g,t} \sim \mathbb{N}(0, 1),$$

where \bar{g} is the steady state growth rate. The indicator variable v_t^g equals 1 if a growth disaster occurs and 0 otherwise. The transition matrix for v_t^g is summarized by

$$\Pr(v_{t+1}^g = 1 | v_t^g = 1) = \bar{q}^g, \quad \Pr(v_{t+1}^g = 1 | v_t^g = 0) = p_t^g,$$

where the probability of a growth disaster follows

$$\ln p_t^g = (1 - \rho_p^g) \ln \bar{p}^g + \rho_p^g \ln p_{t-1}^g + \sigma_p^g \varepsilon_{p,t}^g, \quad \varepsilon_{p,t}^g \sim \mathbb{N}(0, 1).$$

The size of the disaster is determined by ζ_g , and $\bar{\pi}_1^g = \frac{\bar{p}^g}{1 + \bar{p}^g - \bar{q}^g}$ is the unconditional probability of the disaster. Following Gourio (2012), capital is destroyed when the disaster occurs. Let k_t denote the inherited stock of capital and i_t denote investment. The capital stock evolves according to

$$k_{t+1} = e^{-\zeta_g v_{g,t+1}} ((1 - \delta)k_t + i_t - \phi(i_t/k_t)k_t),$$

⁸Related applications of disaster risk include Barro and Ursúa (2012), Gourio (2013), Gourio et al. (2013), Wachter (2013), Shen (2015), Farhi and Gabaix (2016), Olovsson (2019), Kim (2022), Kilian et al. (2024), among others.

⁹Olovsson (2019) uses a related approach, except that his model treats the probability of an oil disaster as constant.

where the functional form of the adjustment cost follows Jermann (1998) and is given by

$$\phi(i_t/k_t) = i_t/k_t - (a_1 + \frac{a_2}{1-1/\nu})(i_t/k_t)^{1-1/\nu}.$$

Final Goods Firm A representative firm maximizes profits by choosing investment (i_t), capital (k_t), labor (n_t), and oil (o_t) inputs. The firm produces a final good y_t using a Cobb-Douglas technology that aggregates labor and capital services, which are produced using a normalized CES production function that aggregates capital and oil with elasticity of substitution σ .

The firm's profit maximization problem is given by

$$V_t = \max_{n_t, k_{t+1}, o_t, i_t} y_t - i_t - p_t^o o_t - w_t n_t + E_t[x_{t+1} V_{t+1}]$$

subject to

$$\begin{aligned} k_{t+1} &= e^{-\zeta_g v_{g,t+1}} ((1 - \delta)k_t + i_t - \phi(i_t/k_t)k_t), \\ y_t &= y_0 (a_t n_t)^{1-\xi} \left((1 - \alpha)(k_t/k_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma} \right)^{\xi/(1-1/\sigma)}, \end{aligned}$$

where δ is the depreciation rate of capital, $1 - \xi$ is the share of labor in gross output, and α controls the share of oil in the capital services aggregate. The terms y_0 , k_0 , o_0 are scale factors that are set so that α is equal to the cost-share of oil in the capital services aggregator. These normalizations do not affect the substance of the results, but dramatically simplify the model calibration.¹⁰

The first-order conditions for the firm's problem are given by

$$\begin{aligned} w_t &= (1 - \xi)y_t/n_t, \\ p_t^o &= \xi \alpha \frac{(o_t/o_0)^{1-1/\sigma}}{(1-\alpha)(k_t/k_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma}} \frac{y_t}{o_t}, \\ E_t[x_{t+1} r_{t+1}^i] &= 1, \end{aligned}$$

where

$$\begin{aligned} r_{t+1}^i &\equiv e^{-\zeta_g v_{g,t+1}} (r_{t+1}^k + (1 - \delta + a_1 + \frac{a_2}{\nu-1})(i_{t+1}/k_{t+1})^{1-1/\nu}) p_{t+1}^k / p_t^k, \\ r_t^k &\equiv \xi (1 - \alpha) \frac{(k_t/k_0)^{1-1/\sigma}}{(1-\alpha)(k_t/k_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma}} \frac{y_t}{k_t}, \\ p_t^k &\equiv \frac{1}{1 - \phi'(i_t/k_t)} = \frac{1}{a_2} \left(\frac{i_t}{k_t} \right)^{1/\nu}. \end{aligned}$$

¹⁰A more detailed discussion of normalized CES production functions can be found in Klump et al. (2012).

Oil Production and Oil Disasters The production of oil is exogenous and given by

$$o_t^s = a_t^o e_t.$$

The assumption of exogenous oil production is commonly used in DSGE models of the oil market, given the paucity of data for the oil sector. The permanent component, a_t^o , reflects factors that influence the productive potential of the oil sector, including the evolution of oil reserves and technological progress that increases the ability of the sector to extract oil from current reserves. The transitory component reflects temporary changes in the production of oil not directly connected with those factors, such as weather-driven or geopolitical supply disruptions. The effect of oil production disasters on global oil production is modeled as transitory, given evidence that geopolitical supply disruptions have not had long-lasting effects on global oil production.

The permanent component is cointegrated with productivity in the rest of the economy,

$$\begin{aligned} a_t^o &= \kappa_0 g_t^{\kappa_1} \epsilon_{t-1}^{\kappa_2} a_{t-1}^o, \\ \epsilon_t &= a_t / a_t^o, \end{aligned}$$

where κ_1 determines the impact response of a growth shock on a_t^o , and κ_2 affects the speed at which a_t^o converges to a_t . This setup allows for a slow response of oil production to productivity growth shocks in the rest of the economy, which is a key feature of the data.¹¹

We allow for geopolitically driven oil production shortfalls in the transitory component,

$$\ln e_t = (1 - \rho_e) \ln \bar{e} + \rho_e \ln e_{t-1} + \sigma_e \varepsilon_{e,t} - \zeta_e (v_t^e - \bar{\pi}_1^e), \quad \varepsilon_{e,t} \sim \mathbb{N}(0, 1).$$

The indicator variable v_t^e equals 1 if an oil production disaster occurs and 0 otherwise. The transition matrix for v_t^e is summarized by

$$\Pr(v_{t+1}^e = 1 | v_t^e = 1) = \bar{q}^e, \quad \Pr(v_{t+1}^e = 1 | v_t^e = 0) = p_t^e,$$

¹¹When $\kappa_1 = 1$ and $\kappa_2 = 0$, $a_t^o = a_t$, so the production of oil responds immediately to changes in productivity elsewhere in the economy. This special case corresponds to the assumption made in Gao et al. (2022). We are the first to account for cointegration in a DSGE model with oil, but cointegrated TFP processes have been used in two-country international real business cycle models (e.g., Rabanal et al., 2011).

where the probability of an oil disaster follows

$$\ln p_t^e = (1 - \rho_p^e) \ln \bar{p}^e + \rho_p^e \ln p_{t-1}^e + \sigma_p^e \varepsilon_{p,t}^e, \quad \varepsilon_{p,t}^e \sim \mathbb{N}(0, 1).$$

The size of the disaster is determined by ζ_e , and $\bar{\pi}_1^e = \frac{\bar{p}^e}{1 + \bar{p}^e - \bar{q}^e}$ is the unconditional probability of the disaster. This process ensures that oil disasters have large but temporary effects on oil production.

Oil Storage A representative oil storage firm maximizes profits by choosing inventories, s_{t+1} , and how much oil to supply to the final goods firm, o_t . The firm's maximization problem is given by

$$V_t^o = \max_{o_t, s_{t+1}} p_t^o o_t + E_t[x_{t+1} V_{t+1}^o]$$

subject to

$$s_{t+1} = (1 - \omega)s_t + a_t^o e_t - o_t - \frac{\pi}{2} (s_t/a_t)^{-2} a_t,$$

where ω is the cost of storage. Following Gao et al. (2022), there is an adjustment cost, π , that prevents stockouts ($s = 0$) from occurring, as they are not observed in the global oil market.

The first-order condition for the storage firm is given by

$$1 = E_t[x_{t+1} r_{t+1}^s],$$

where

$$r_{t+1}^s \equiv ((1 - \omega + \pi(s_{t+1}/a_{t+1})^{-3})p_{t+1}^o)/p_t^o.$$

Household A representative household maximizes the present discounted value of utility by choosing consumption, c_t , hours worked, n_t , bond holdings, b_{t+1} , and equity shares, s_{t+1}^e , which have unit net supply. The household has Epstein-Zin recursive preferences to distinguish between risk aversion, γ , and the intertemporal elasticity of substitution, ψ (see Epstein and Zin, 1989).

The household's maximization problem is given by

$$J_t = \max_{c_t, n_t, s_{t+1}^e, b_{t+1}} \left((1 - \beta) u_t^{1-1/\psi} + \beta (E_t[J_{t+1}^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}} \right)^{\frac{1}{1-1/\psi}}$$

subject to

$$u_t = c_t^\chi (a_t(1 - n_t))^{1-\chi},$$

$$c_t + p_t^e s_{t+1}^e + b_{t+1}/r_t = w_t n_t + (p_t^e + d_t^e) s_t^e + b_t,$$

where β is the discount factor, χ is a preference parameter, p_t^e is the equity price, r_t is the risk-free rate, w_t is the wage rate, and d_t^e are dividends from firm ownership.

The first-order conditions for the household are given by

$$\chi w_t (1 - n_t) = (1 - \chi) c_t,$$

$$1 = E_t[x_{t+1} r_t],$$

$$1 = E_t[x_{t+1} r_{t+1}^e],$$

where

$$r_{t+1}^e \equiv (p_{t+1}^e + d_{t+1}^e)/p_t^e,$$

$$x_{t+1} \equiv \beta (u_{t+1}/u_t)^{1-1/\psi} (c_t/c_{t+1}) (J_{t+1}/z_t)^{1/\psi-\gamma},$$

$$z_t \equiv (E_t[J_{t+1}^{1-\gamma}])^{1/(1-\gamma)}.$$

The equity risk premium is defined as

$$r_t^{ex} \equiv r_t^e - r_{t-1}.$$

Market Clearing Aggregate firm dividends are given by

$$d_t^e = d_t^f + d_t^s - \vartheta (E_{t-1} k_t - \frac{1}{r_t} E_t k_{t+1}) - \vartheta (E_{t-1} s_t - \frac{1}{r_t} E_t s_{t+1}),$$

where

$$d_t^f = y_t - i_t - p_t^o o_t - w_t n_t,$$

$$d_t^s = p_t^o o_t.$$

Following Jermann (1998) and Gourio (2012), both the final goods and storage firms issue bonds to finance their assets, where ϑ determines leverage.

Asset market clearing implies that $s_t^e = 1$ and total bond issuance is given by

$$b_t \equiv b_t^f + b_t^s = \vartheta(E_{t-1}k_t + E_{t-1}s_t).$$

Market clearing in the goods market implies

$$c_t + i_t = y_t.$$

Due to the stochastic trend in productivity, we detrend the model by defining $\tilde{x}_t = x_t/a_t$. The detrending process introduces the growth terms $g_t = a_t/a_{t-1}$ and $g_{o,t} = a_t^o/a_{t-1}^o$. Appendix D provides the detrended equilibrium system of equations.

Uncertainty We follow Plante et al. (2018) and Bernstein et al. (2024) and define output uncertainty as the conditional volatility of log output growth, which is given by

$$\mathcal{U}_t^y = \sqrt{E_t[(\ln(y_{t+1}/y_t) - E_t[\ln(y_{t+1}/y_t)])^2]}.$$

This definition is equivalent to the uncertainty surrounding the level of log output because y_t is known at time t and cancels from the definition of \mathcal{U}_t^y . We define oil price uncertainty, $\mathcal{U}_t^{p_o}$, equivalently as the uncertainty surrounding $\ln(p_{t+1}^o/p_t^o)$ at time t .

4.2 SOLUTION METHOD Modeling the oil sector considerably increases the computational cost of solving the model compared to a model only including macroeconomic risk. The model has 8 state variables $(\ln e_t, k_t, s_t, v_t^g, v_t^e, \ln p_t^g, \ln p_t^e, \epsilon_{t-1})$, 5 of which are related to the oil market. There are 4 continuous and 2 discrete shocks. The existence of time-varying disaster risk prevents using perturbation methods, which are common in the stochastic volatility literature. We therefore employ a global solution method. Specifically, the model is solved using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work in Coleman (1991). The algorithm minimizes the Euler equation errors on each node in the state space and computes the maximum change in the policy functions. It then iterates until the maximum change is below a specified tolerance. Appendix C describes the solution method in more detail.

Table 1: Model calibration

Parameter	Value	Target
Discount Factor (β)	0.9945	$E(r)$
Risk Aversion (γ)	10	Gao et al. (2022), Croce (2014)
Intertemporal Elasticity (ψ)	2	Gao et al. (2022), Croce (2014)
Capital-Oil Elasticity of Substitution (σ)	0.13	$SD(\Delta p^o)$
Capital Depreciation Rate (δ)	0.025	Depreciation on fixed assets and durables
Capital-Oil Share of Production (ξ)	0.4043	Total economy labor share
Investment Adjustment Cost (ν)	1.7	$SD(\Delta i)$
Oil Inventory Depreciation Rate (ω)	0.025	Casassus et al. (2018), Gao et al. (2022)
Mean Growth Rate (\bar{g})	1.0039	$E(\Delta y)$
Leverage (ϑ)	0.9	$SD(r^{ex})$
Utility Weight on Leisure (χ)	0.47	Frisch labor supply elasticity of 2
Elasticity of Oil Supply to TFP (κ_1)	0	Newell and Prest (2019)
Oil Supply Adjustment Speed to TFP (κ_2)	0.05	Half life of 3.5 years
Growth Shock SD (σ_g)	0.01	$SD(\Delta y)$
Oil Production Shock Persistence (ρ_e)	0.6	$AC(\Delta o^s)$
Oil Production Shock SD (σ_e)	0.012	$SD(\Delta o^s)$
Growth Disaster Size (ζ_g)	0.025	$E(r^{ex})$
Probability of Entering Growth Disaster (\bar{p}_g)	0.0025	Occurs in expectation every 100 years
Probability of Exiting Growth Disaster (\bar{q}_g)	0.9	Gourio (2012)
Growth Disaster Probability Persistence (ρ_{pg})	0.8	$SD(\mathcal{U}_y)$
Growth Disaster Probability SD (σ_{pg})	1.2	$AC(\mathcal{U}_y)$
Oil Production Disaster Size (ζ_e)	0.05	Kilian (2008)
Probability of Entering Oil Disaster (\bar{p}_e)	0.015	Occurs in expectation every 15 Years
Probability of Exiting Oil Disaster (\bar{q}_e)	0.67	Expected duration of 3 quarters
Oil Disaster Probability Persistence (ρ_{pe})	0.8	$SD(\mathcal{U}_{p^o})$
Oil Disaster Probability SD (σ_{pe})	1.2	$AC(\mathcal{U}_{p^o})$

4.3 CALIBRATION Given the paucity of global macroeconomic data, we calibrate the model under the assumption that the world economy resembles the U.S. economy. The parameters shown in Table 1 are informed by moments in the data and the related literature.¹² The moments are computed using data from 1975Q1 to 2019Q4. Appendix B documents our data sources.

The discount factor β is set to 0.9945 to match the mean real interest rate. The relative risk aversion coefficient, γ , and intertemporal elasticity of substitution are set to 10 and 2, respectively, consistent with Gourio (2013), Croce (2014), Gao et al. (2022), and several other recent studies.¹³

¹²Estimation using Bayesian methods or the simulated method of moments is not possible due to the high dimensionality of the model. Even when using a supercomputer with 10,000 cores, it takes multiple days to solve our model.

¹³Swanson (2018) shows how to compute risk aversion under recursive preferences with an endogenous labor supply. Under our utility kernel, γ corresponds to risk aversion over consumption and leisure.

The elasticity of substitution between capital and oil, σ , is set to 0.13 to match the volatility of the growth rate of oil prices. Backus and Crucini (2000) adopt the same functional form of the production function and use a similar value (0.09). The Cobb-Douglas weight on capital services (ξ) is set to match the average labor share for the total economy. The investment adjustment cost parameter, ν , is set to match the volatility of per capita investment growth. The capital depreciation rate, δ , matches the annual average rate of depreciation on private fixed assets and durable goods. The oil inventory depreciation rate, ω , is set to 0.025 following Casassus et al. (2018) and Gao et al. (2022). As in other studies including Basu and Bundick (2017), the leverage parameter, ϑ , is set to 0.9 to help match the volatility of the equity premium.

The mean growth rate of productivity, \bar{g} , is set to 1.0039 to match the average growth rate of per capita real GDP. The standard deviation for the growth shock, σ_g , is set to 0.01 to help match the volatility of GDP growth. The calibration of the growth disaster parameters is guided by several moments in the data as well as the parameter choices in Gourio (2012). We set the size of the disaster, ζ_g , to 0.025 to match the mean equity premium. The mean probability of entering the disaster state, \bar{p}_g , is set to 0.0025, which implies that these disasters happen once every 100 years in expectation. The persistence, ρ_{pg} , and standard deviation, σ_{pg} , of this probability are set to 0.8 and 1.2, respectively, to help match the autocorrelation and volatility of output uncertainty. The fixed probability of exiting a growth disaster, \bar{q}_g , is set to 0.9, in line with Gourio (2012). This value implies that growth disasters, on average, last 2.5 years. As shown in Appendix E, the responses to an average growth disaster are very similar to those reported in Gourio (2012), who documents that his responses resemble the empirical estimates in Barro et al. (2013).

The value of κ_1 is set to 0, implying that productivity in the oil sector is unresponsive to changes in productivity in the rest of the economy within the first quarter. This is consistent with the view that oil production in the short run is determined entirely by geological constraints (see Newell and Prest, 2019). We set κ_2 to 0.05, so the half-life of the deviation between a_t^o and a_t is 3.5 years. The persistence and standard deviation of the oil production process, ρ_e and σ_e , are set to 0.6 and 0.012, respectively, to match the autocorrelation and volatility of global oil production. We set the

Table 2: Data and simulated moments

Moment	Data	Model	Moment	Data	Model
$E(\Delta y)$	0.391	0.309	$SD(\Delta p^o)$	14.39	14.73
$E(s/o)$	0.967	0.953	$SD(r^{ex})$	8.29	4.85
$E(p^o/y)$	0.045	0.049	$SD(\mathcal{U}_y)$	14.51	17.86
$E(r^{ex})$	2.178	2.206	$SD(\mathcal{U}_{p^o})$	29.95	34.87
$E(r)$	0.215	0.177	$AC(\Delta y)$	0.32	0.40
$SD(\Delta y)$	0.743	1.000	$AC(\Delta o^s)$	-0.11	0.08
$SD(\Delta i)$	1.947	2.026	$AC(\mathcal{U}_y)$	0.87	0.79
$SD(\Delta o^s)$	2.010	2.029	$AC(\mathcal{U}_{p^o})$	0.93	0.80

Notes: The model is calibrated to data from 1975Q1-2019Q4. $SD(\mathcal{U}_y)$ and $SD(\mathcal{U}_{p^o})$ have been normalized by $SD(\Delta y)$ and $SD(\Delta p^o)$, respectively, to be consistent with the normalization in Jurado et al. (2015).

size of the oil production disaster, ζ_e , to 0.05, in line with evidence in Kilian (2008). The mean probability of entering the oil disaster state, \bar{p}_e , is set to 0.015 so that disasters occur every 15 years in expectation, motivated by historical data on major OPEC supply disruptions. The persistence, ρ_{pg} , and standard deviation, σ_{pg} , of this probability are set to 0.8 and 1.2, respectively, to help match the autocorrelation and volatility of oil price uncertainty. The fixed probability of exiting an oil production disaster, \bar{q}_g , is set to 0.67 so that it lasts, on average, for 3 quarters, which implies a reduction in global oil production for about 3 years with the shortfall diminishing over time.¹⁴

To compute the model-implied moments, we simulate the model 10,000 times, each with 180 periods to match the length of the quarterly data we used to calibrate the model. We calculate moments of interest in each simulated data set and then compute the average moments across all simulations. Table 2 compares the data and model-implied moments. The model closely matches most of the targeted moments. This includes moments related to oil market dynamics (e.g., the standard deviation oil price growth, the standard deviation and autocorrelation of oil production growth, and the oil expenditure share), real activity (e.g., the standard deviation and autocorrelation

¹⁴It may seem that options data could be used to help with the calibration. This is not the case. One challenge is that tail probabilities estimated from equity options as in Barro and Liao (2021) do not help quantify macroeconomic tail risk, but only equity risk. The distinction between financial risk and macroeconomic risk has been emphasized in Gao et al. (2022) and Ludvigson et al. (2021). Likewise, oil disaster probabilities are not recoverable from oil options because these prices reflect both oil and macroeconomic disaster risk in unknown combinations.

of output growth and the standard deviation of investment), asset prices (e.g., the mean risk-free rate and equity risk premium), and uncertainty (e.g., the standard deviations of output and oil price uncertainty). Jointly matching all four of these key aspects of the economy gives us confidence that the model provides a good description of oil market, real activity, and uncertainty dynamics.

4.4 DISCUSSION Our model incorporates precautionary savings by households in response to higher oil price uncertainty as well as storage, the two main economic mechanisms that are thought to propagate oil price uncertainty shocks. While we do not model real options arising from irreversible investment as in Bernanke (1983), our model features limited substitutability between capital and oil. This feature causes the expected return on investment to decline when the probability of an oil production disaster increases, generating recessionary effects in the model.

Although Bernanke's theoretical analysis is often cited in support of models of oil price volatility shocks, it is not well appreciated that Bernanke was not modeling a monthly or quarterly oil price volatility shock, holding constant the conditional mean of the price of oil. Rather, he envisioned agents being uncertain about whether the price of oil would permanently move to a higher level or not, which is a different thought experiment. In his model there are two types of capital that differ by their oil efficiency. The irreversibility of the investment decision causes risk averse agents to postpone the acquisition of either type of capital. The difficulty in generalizing this model to general equilibrium is that it requires aggregating different types of capital across many firms.

A closely related model that deals with the aggregation of different types of capital in general equilibrium was proposed by Atkeson and Kehoe (1999). In their putty-clay model there is a continuum of capital goods index by their oil efficiency. Existing capital goods use oil in fixed proportions, so, in the short run, there is no substitutability between capital and oil. However, firms may invest in new capital with different oil efficiency in response to changes in the price of oil. Although this point is not the focus of Atkeson and Kehoe (1999), their model implies that higher oil price uncertainty would reduce investment, as discussed in Plante and Traum (2012).

The reason we do not incorporate the putty-clay framework within our model is that two key assumptions made by Atkeson and Kehoe (1999) do not hold in our model. One is that the price

of oil is exogenous; the other is that under their assumptions oil consumption does not respond to the price of oil on impact. These assumptions not only allow Atkeson and Kehoe to abstract from storage, but they allow them to aggregate across different types of capital without the need to track the distribution of capital types. The fact that the infinite dimensional state space of capital stocks in the model can be reduced to a one-dimensional space facilitates the solution of the model.

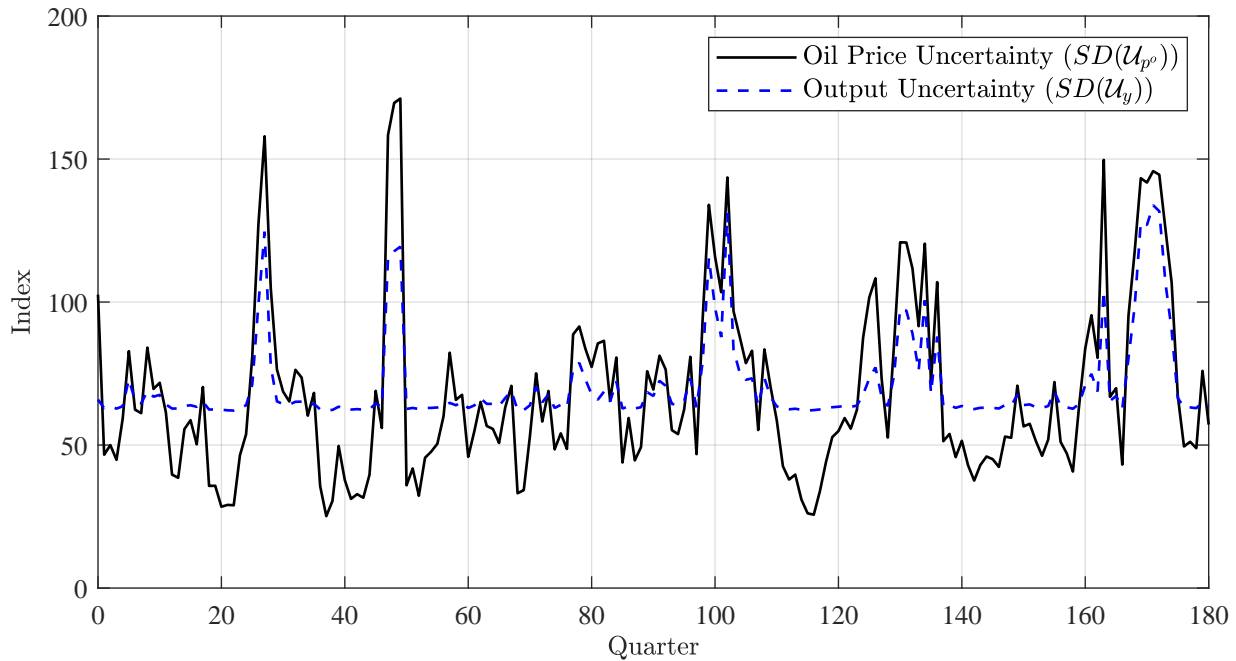
In contrast, in our model the price of oil is endogenously determined. Suppose, for example, that there is an oil supply shock. In that case, we must add storage to the model because otherwise equilibrium in the oil market is unattainable. If oil consumption is predetermined and hence unresponsive to the oil price fluctuations caused by the oil supply shock, oil inventories must absorb any imbalances in the oil market each period. It can be shown that, as a result, the oil inventory moments of the simulated model data differ substantially from the oil inventory moments in the actual data. It may seem that this problem could be addressed by dropping the assumption that oil consumption is unresponsive to the price of oil, but this would render the capital stock intractable, which is why we do not consider the putty-clay framework in our model. However, our model with disaster risk generates investment and real GDP responses that are qualitatively consistent with those in models of irreversible investment. The reason is that in our model risk averse agents are reluctant to invest given the limited substitutability between capital and oil.

5 WHAT IS THE ROLE OF UNCERTAINTY SHOCKS?

As discussed in [Section 2](#), economists' intuition about the impact of oil price uncertainty shocks continues to be based on insights from partial equilibrium models or from general equilibrium models with exogenous oil price volatility shocks but no fluctuations in macroeconomic uncertainty. This section reexamines this question within the context of a general equilibrium model with endogenous time-varying uncertainty about the price of oil as well as real GDP growth.

5.1 THE RELATIONSHIP BETWEEN OIL PRICE UNCERTAINTY AND REAL GDP UNCERTAINTY

The DSGE model provides a useful benchmark for what we would expect the relationship between

Figure 2: Simulation draw of uncertainty series

Notes: $SD(U_y)$ and $SD(U_{p^o})$ have been normalized by $SD(\Delta y)$ and $SD(\Delta p^o)$, respectively, to be consistent with Jurado et al. (2015).

measured oil price uncertainty and measured uncertainty in real GDP growth to be in the data. Figure 2, which depicts simulated data from this model, serves three purposes. First, it shows that the DSGE model is capable of generating fluctuations in oil price uncertainty that are qualitatively similar to those shown in Figure 1. Second, it illustrates that output uncertainty and oil price uncertainty are not independent. In particular, major increases in oil price uncertainty tend to be associated with major increases in output uncertainty. The model helps us understand to what extent this relationship is driven by exogenous shifts in macroeconomic risk, exogenous shifts in geopolitical risk in oil markets, or by other shocks. Third, it shows that oil price uncertainty tends to be more volatile than output uncertainty.

5.2 THE TRANSMISSION OF UNCERTAINTY SHOCKS In practice, economic agents will rarely witness disasters. Typically, disasters matter only because there is a small probability that they may be realized in the future. As we highlight in this section, changes in the probability of a disaster, which generate fluctuations in uncertainty, are sufficient to create large responses in the oil market

and in the macro economy even when no disasters actually occur.

Oil Disaster Probability Figure 3 shows the responses of key model variables when the exogenous oil disaster probability is increased by 5, 10, 20, and 40 percentage points (pp), respectively. Higher odds of an oil production disaster generate stronger precautionary storage demand, reflected in a persistent build-up of oil inventories. This raises the price of oil, with the initial increase ranging from about 4% for a 5pp shock to the disaster probability to 16% for the 40pp shock.

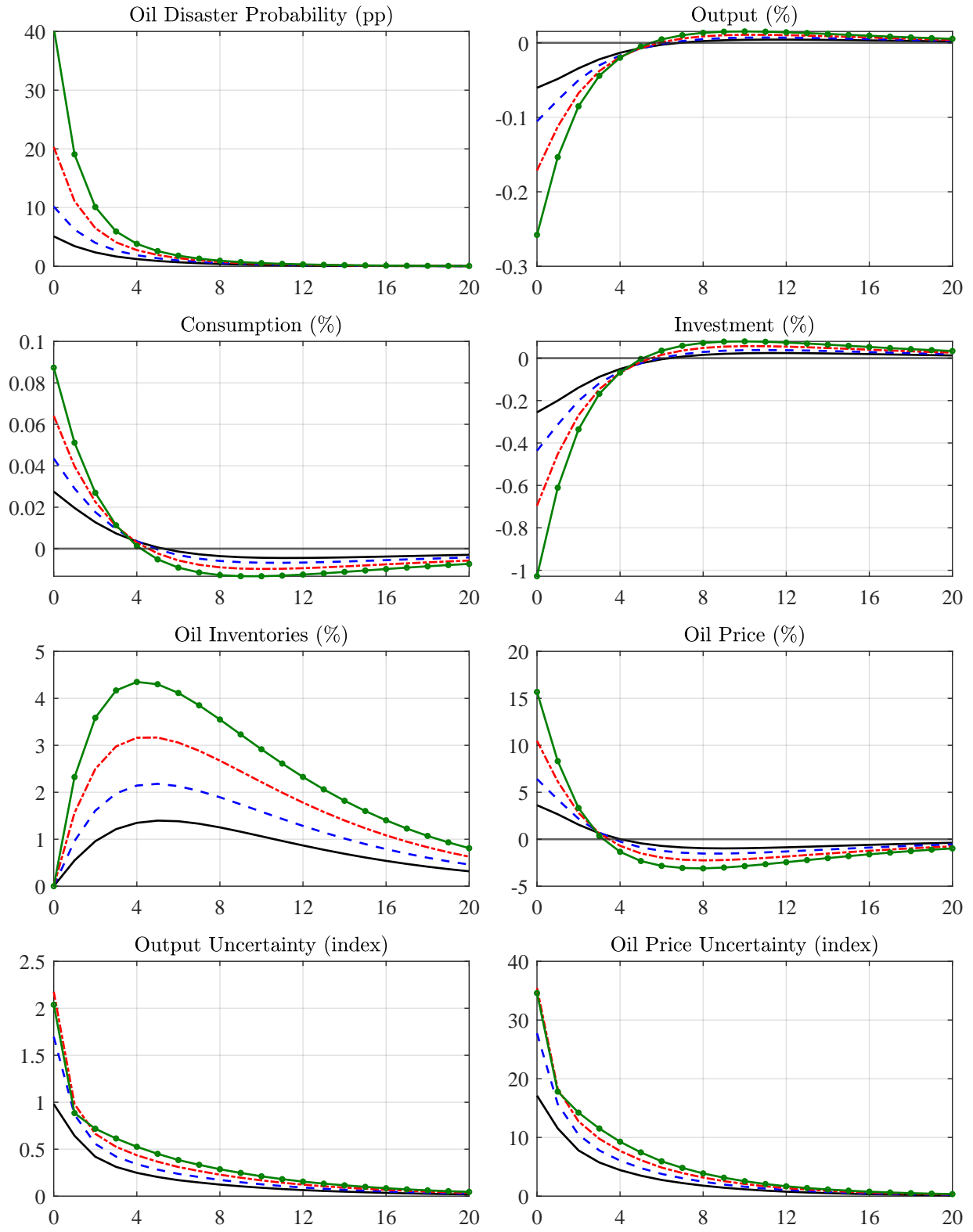
The negative effect on investment spending arises for two distinct reasons. First, an oil production disaster, if it were to occur, would reduce the return to capital, since oil and capital are complements in production of the final good. Thus, the higher probability of such a disaster lowers the expected return from investing in capital. Second, the return to capital today declines because higher precautionary demand for oil inventories raises the price of oil. Together, these two effects push down output, but the overall magnitude is modest since the investment share in output is small and there is a slight increase in consumption.

Like the consumption response to rising macroeconomic disaster risk in Gourio (2012), the consumption response is marginally positive, indicating that the precautionary savings motive is not the primary driver of the consumption response. Mechanically, this occurs because households in the model lack alternative investment opportunities. With a temporarily lower incentive to invest, households have no choice but to consume more today and less in the future.¹⁵

Higher odds of an oil production disaster raise both output uncertainty and oil price uncertainty, but the effects on oil price uncertainty are an order of magnitude larger. For example, a 5pp increase in the probability of an oil disaster raises the output uncertainty index only by 1, but the oil price uncertainty index by 17. Thus, the oil disaster probability shock effectively is an exogenous oil price uncertainty shock. The model shows that the recessionary effects of this shock are reflected in real GDP immediately, but are short-lived.

¹⁵The sign of the consumption response could change in a model with additional frictions. We are unable to explore this possibility because modeling these frictions would increase the number of state variables in the model to the point of making the already unusually large DSGE model intractable. What matters for our purposes is that the overall effect on real GDP is clearly negative, reflecting the response of investment. Thus, as long as the consumption response is modest, its sign is largely immaterial for assessing the recessionary effect of oil disaster probability shocks.

Figure 3: Responses to an oil production disaster probability shock



Notes: Responses in deviations from the baseline. Simulations assume no disasters are realized.

The responses do not change proportionately with the shock size. For example, the responses of output and the price of oil to a 20pp increase in the oil disaster probability are only 2.8 times larger than when the probability rises by 5pp. This result highlights that exogenous variation in uncertainty transmits to the macro economy nonlinearly. Raising the disaster probability beyond a certain point lowers oil price uncertainty because the agents in the model perceive the disaster as increasingly likely. At the same time, the recessionary effect of the disaster probability shock strengthens.

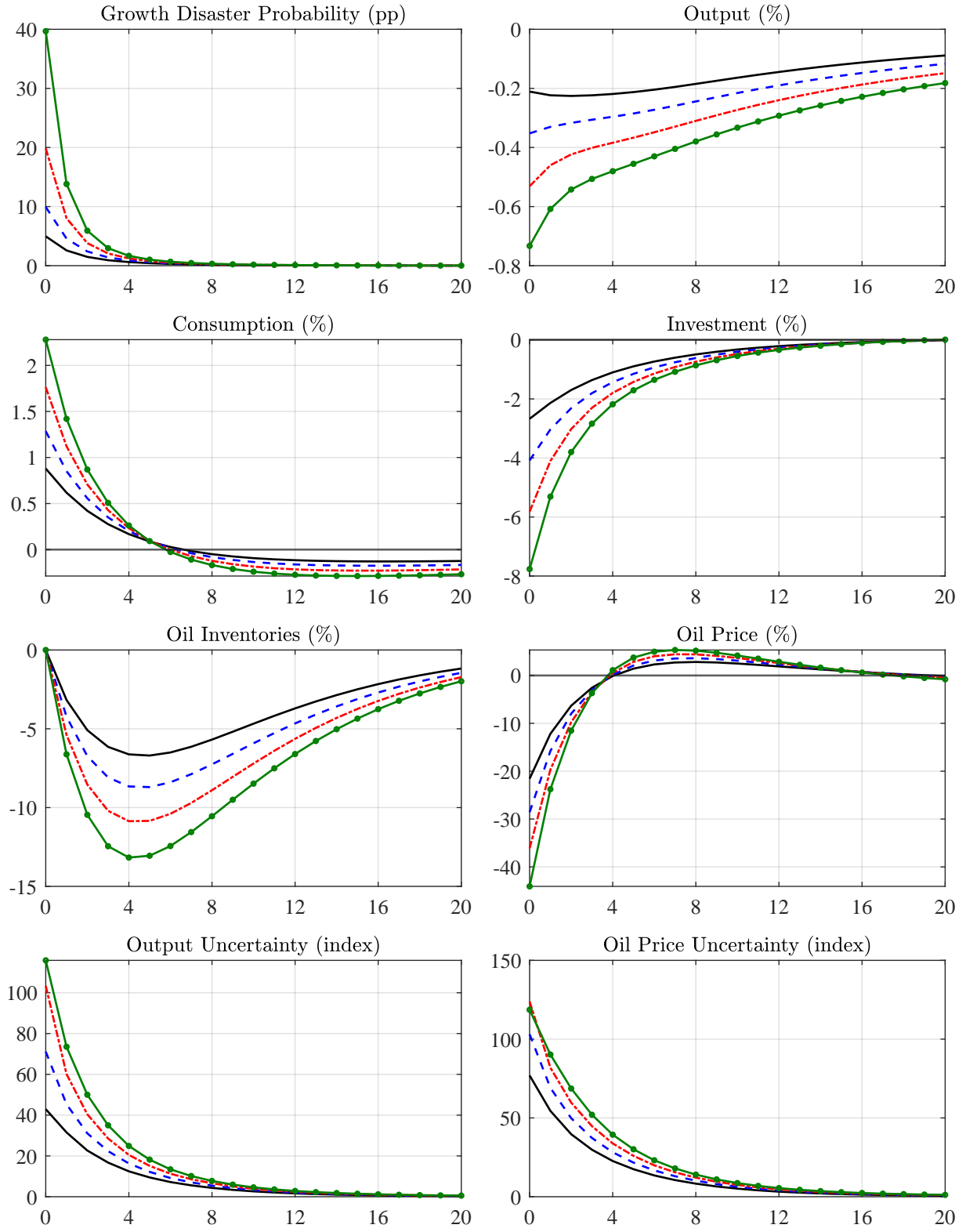
Growth Disaster Probability A growth disaster acts like a negative demand shock in the oil market by reducing real activity and lowering oil demand. This plays a key role in understanding how the oil market responds to an increased probability of an output growth disaster.

Figure 4 shows the responses when the exogenous disaster probability increases by 5, 10, 20, and 40pp, respectively. An increase in the probability of a growth disaster has substantial, albeit short-lived, effects on the price of oil. For example, a 5pp increase in the probability causes the price of oil to decline by 22% on impact. There are two related but somewhat distinct mechanisms at play. First, as in Gourio (2012), the higher probability directly reduces the expected return to capital, which lowers oil demand today since capital and oil are complements. Second, lower current and expected oil demand also reduces the expected return from holding oil inventories. As a result, oil currently held in storage is sold off, pushing down the oil price even further.

Although the reduction in the price of oil is beneficial for the economy, the net effect of this probability shock on output is negative. In fact, the decline in output is much larger than from the oil disaster probability shock of the same magnitude. This is because the growth probability shock transmits directly through TFP rather than through the share of oil in output, which is small.

In related work, Gourio (2012) showed that an increase in the probability of a growth disaster causes the uncertainty about equity prices to rise. Our results show that the same shock also has a major effect on uncertainty in output and in the price of oil. Thus, the growth disaster probability shock, like the oil disaster probability shock, can be viewed as an exogenous uncertainty shock. In response to this shock, there is a decline in real activity that persists for several years.

Figure 4: Responses to a growth disaster probability shock



Notes: Responses in deviations from the baseline. Simulations assume no disasters are realized.

Table 3: Decomposition of key volatilities

Moment	Data	Model		
		Baseline	No Output Disaster Risk	No Output Disaster Risk or Oil Production Disaster Risk
$SD(\Delta y)$	0.74	1.00	0.64	0.63
$SD(\Delta i)$	1.95	2.03	1.02	0.92
$SD(\Delta o^s)$	2.01	2.03	2.00	1.35
$SD(\Delta p^o)$	14.39	14.73	5.70	2.08
$SD(r^{ex})$	8.29	4.85	0.29	0.29
$SD(\mathcal{U}_y)$	14.51	17.86	0.70	0.13
$SD(\mathcal{U}_{p^o})$	29.95	34.87	15.13	1.02

Notes: $SD(\mathcal{U}_y)$ and $SD(\mathcal{U}_{p^o})$ have been normalized by $SD(\Delta y)$ and $SD(\Delta p^o)$ in the baseline model, respectively, to be consistent with Jurado et al. (2015).

As in Figure 3, the responses do not scale proportionately with the increase in the growth disaster probability. For example, a 5pp increase leads to a 22% decline in the price of oil, whereas the oil price declines by 36% when the probability rises by 20pp. This is true for the other variables shown in the figure, as well, once again highlighting the nonlinearity in the transmission of uncertainty. As the growth disaster probability increases further, the recessionary effect strengthens but the response of oil price uncertainty declines, reflecting greater certainty about a reduction in output.

The key difference between the two disaster probability shocks is that the growth disaster probability shock has substantial effects on both uncertainty variables, whereas the oil disaster probability shocks does not. Thus, comovement between oil price uncertainty and output uncertainty, as documented in Figure 2, tends to reflect shifts in macroeconomic risk rather than geopolitical risk.

5.3 HOW MUCH VOLATILITY IS DUE TO OUTPUT RISK AND GEOPOLITICAL RISK? Table 3

shows that the model generally does an excellent job at capturing the volatility of the data, except for slightly overstating the standard deviations of the two uncertainty measures. Dropping the output disaster risk from the DSGE model substantially lowers the ability of the model to explain the volatility of the data. The resulting model not only substantially understates the standard deviation

of the two uncertainty series, but also understates most other data moments in the table.

Dropping both output disaster risk and the geopolitical risk underlying oil production disasters from the model further lowers the volatilities. For example, it removes virtually all variability in the two uncertainty measures and much of the variability in the real price of oil. It also widens the gap between the volatility in macroeconomic aggregates in the model and the data. We conclude that both macroeconomic disaster risk and geopolitical oil price risk are essential features for understanding output uncertainty and oil price uncertainty and their relationship with the economy.

These results suggest that output disaster risk plays an important role in driving economic fluctuations. In contrast, oil price uncertainty does not. This follows from the fact that dropping oil production disaster risk substantially lowers the volatility of oil price uncertainty without much of an effect on the volatility of macroeconomic aggregates.¹⁶

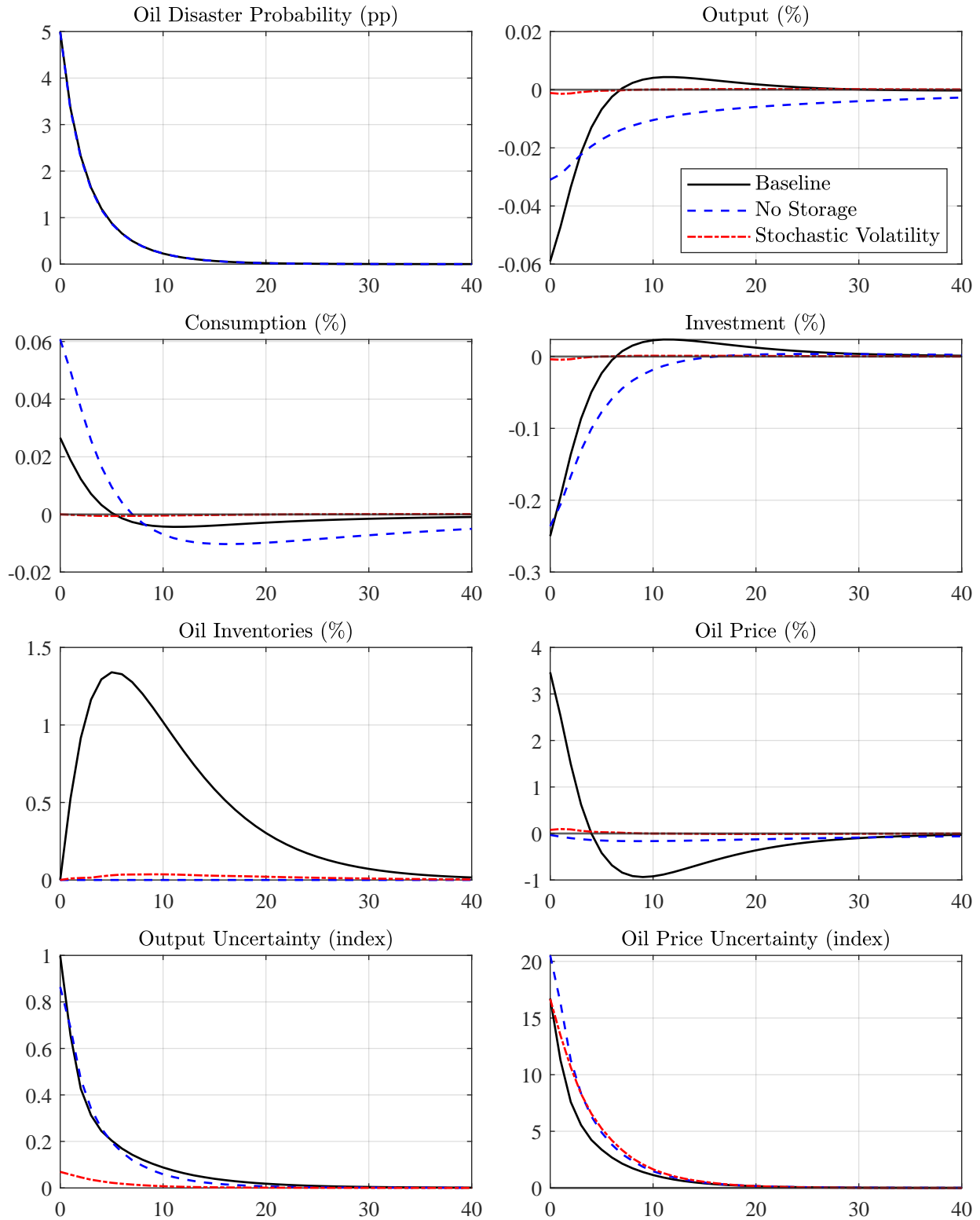
5.4 ALTERNATIVE MODEL SPECIFICATIONS In this section, we highlight key features of our model by considering alternative specifications. We first illustrate the central role of oil storage. We then contrast our model with earlier models incorporating stochastic volatility shocks.

The role of storage There are important movements in oil inventories whenever the probability of a disaster increases. These movements affect the price of oil and, therefore, the evolution of macroeconomic aggregates and uncertainty. In this section, we investigate how important storage is for those responses by comparing the baseline results to those from a model without storage.

Figure 5 shows the responses for the oil disaster probability shock. The key difference is that the price of oil declines slightly in the model without storage, whereas it increases substantially in the baseline model. In the absence of storage, the response of the oil price is driven entirely by the expectation of lower output, which reduces the demand for oil and modestly lowers its price. Given the muted response of the price of oil, the impact effect on output is also reduced. However, even in the absence of storage, the effect on oil price uncertainty remains substantial.

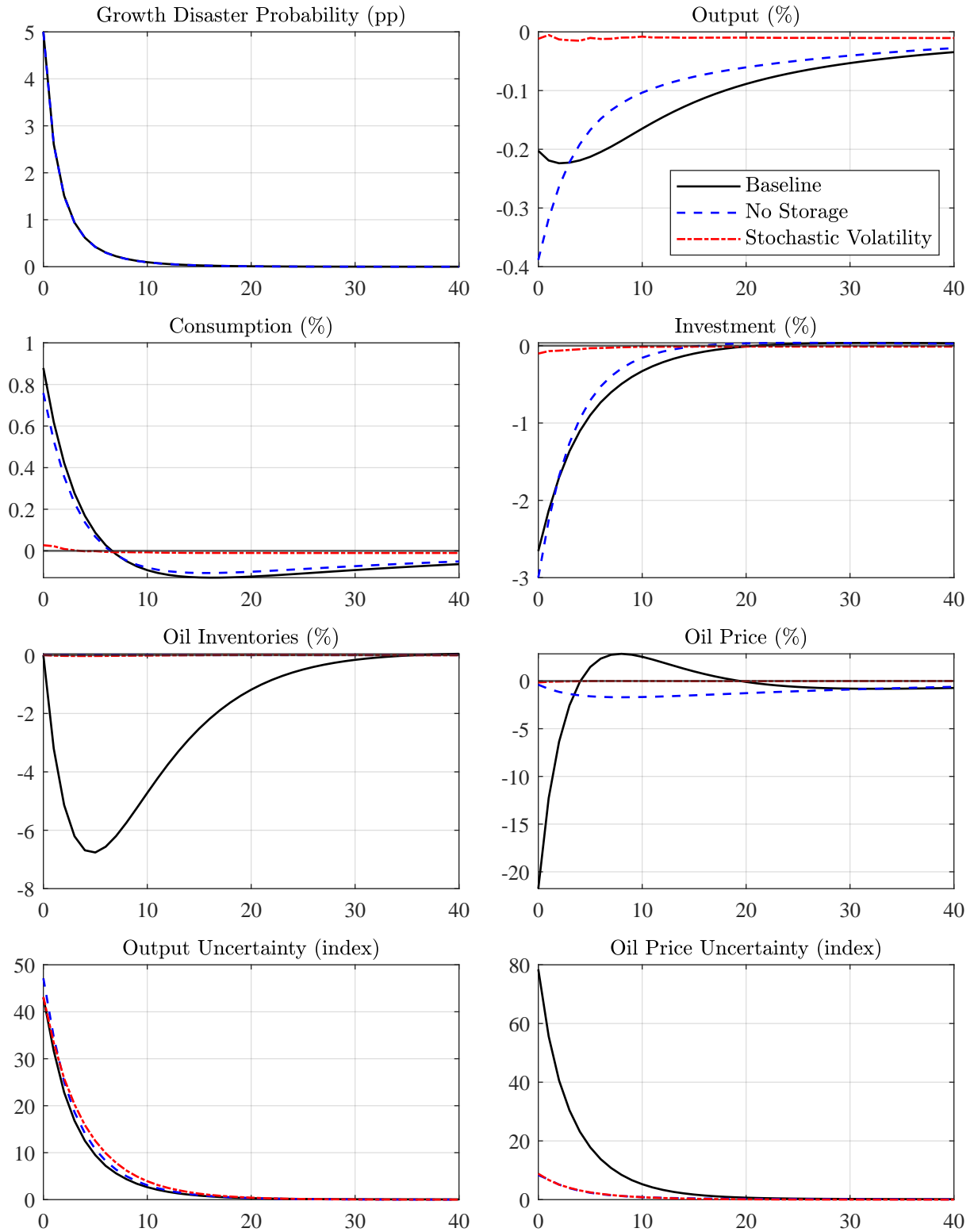
¹⁶The analysis in Table 3 conditions on the same calibration as the baseline model. Recalibrating the model in the last column results in a systematic mismatch with the data moments. While raising the volatility of the shocks to oil production and productivity growth allows the model to match $SD(\Delta o^s)$ and $SD(\Delta y)$, the recalibrated model is unable to match $SD(\Delta p^o)$ and $SD(\Delta i)$ or, for that matter, the volatilities of uncertainty.

Figure 5: Responses to an oil production disaster probability shock



Notes: Responses in deviations from the baseline. Simulations assume no disasters are realized.

Figure 6: Responses to a growth disaster probability shock



Notes: Responses in deviations from the baseline. Simulations assume no disasters are realized.

Figure 6 shows that storage also plays a key role in the propagation of a growth disaster probability shock. In the model with storage, a higher probability of a disaster leads to a reduction in oil inventories due to the greater likelihood of a recession. This causes a substantial decline in the price of oil, which does not occur in the no-storage model. Since lower oil prices offset some of the negative effects of this shock on the macro economy, the impact effect on output is larger when the model does not contain storage. Oil price uncertainty is much less responsive to a change in the growth disaster probability. Overall, our results demonstrate that storage is a key ingredient for understanding the effects of uncertainty in both the oil market and the macro economy.

Stochastic Volatility Stochastic volatility (SV) is an alternative way of generating time-varying oil price uncertainty that has been used in previous studies (e.g., Başkaya et al., 2013; Gao et al., 2022; Plante and Traum, 2012).¹⁷ In this section, we compare the results from a model where oil production uncertainty is generated by SV rather than a time-varying probability of disaster.

Specifically, we introduce an exogenous volatility shock into productivity and oil production,

$$\begin{aligned}\ln g_t &= \ln \bar{g} + \sigma_{g,t-1} \varepsilon_{g,t}, \\ \ln e_t &= (1 - \rho_e) \ln \bar{e} + \rho_e \ln e_{t-1} + \sigma_{e,t-1} \varepsilon_{e,t}, \\ \ln \sigma_{g,t} &= (1 - \rho_{sv}^g) \ln \bar{\sigma}_g + \rho_{sv}^g \ln \sigma_{g,t-1} + \sigma_{sv}^g \varepsilon_{sv,t}^g, \\ \ln \sigma_{e,t} &= (1 - \rho_{sv}^e) \ln \bar{\sigma}_e + \rho_{sv}^e \ln \sigma_{e,t-1} + \sigma_{sv}^e \varepsilon_{sv,t}^e,\end{aligned}$$

where all shocks are standard normally distributed. The parameters of the level processes are unchanged. The persistence of both SV processes, ρ_{sv}^g and ρ_{sv}^e , is set to 0.8 to match the persistence of the disaster probability processes. The standard deviation of the growth SV shock, $\varepsilon_{sv,t}^g$ is set to 0.09 to match the volatility of output growth uncertainty. Analogously, the standard deviation of the oil production SV shock, $\varepsilon_{sv,t}^e$ is set to 0.22 to match the volatility of oil price uncertainty.

Figure 5 compares the responses to an SV shock in oil production, $\varepsilon_{sv,t}^e$, to the responses to our baseline oil disaster probability shock, $\varepsilon_{p,t}^e$. The $\varepsilon_{sv,t}^g$ shock is set so the SV specification generates

¹⁷Stochastic volatility has also been used to model exogenous uncertainty shocks in a number of other settings including fiscal policy (Fernández-Villaverde et al., 2015), monetary policy (Mumtaz and Zanetti, 2013), household preferences (Basu and Bundick, 2017), and the global interest rate (Fernández-Villaverde et al., 2011).

the same impact effect on oil price uncertainty as the oil disaster probability shock. Qualitatively, these shocks move the model variables in the same direction, but there are quantitatively significant differences. While the SV shock generates sizable fluctuations in both output uncertainty and oil price uncertainty, it otherwise has little effect on the macro economy and the oil market.

The key difference between the two modeling choices is that an oil disaster introduces a source of downside risk into the economy because it makes a sharp drop in oil production more likely. As a result, when the probability of a disaster increases, it not only increases uncertainty but also shifts the conditional mean of economic outcomes. This generates a stronger precautionary demand motive, which pushes up the price of oil. The SV shock, on the other hand, is akin to a mean-preserving spread. It generates a sizeable increase in uncertainty but has little effect on the conditional mean. Hence, the response of the price of oil and output is muted. A similar result holds when replacing the growth probability shock with a SV shock on productivity, as shown in [Figure 6](#). We conclude that SV shocks are unable to capture the effects of uncertainty shocks associated with major geopolitical events that affect the oil market.¹⁸

6 RELATIONSHIP TO THE EXISTING LITERATURE

We are not the first to study the transmission of oil price uncertainty shocks within a DSGE model. For example, Başkaya et al. (2013) analyze the business cycle implications of oil price uncertainty for a stylized oil-importing small open economy. One important difference is that oil price uncertainty is exogenous in their model. In contrast, our analysis is specifically designed to make explicit that oil price uncertainty in general depends on all level shocks as well as macroeconomic uncertainty shocks. Allowing for both macroeconomic and oil price uncertainty is important when assessing their respective role in explaining economic fluctuations, as illustrated in [Section 5.3](#).

¹⁸Similar to our responses for the SV specification, Gao et al. (2022) find small impacts of oil production volatility shocks in their baseline model. They show that the responses are amplified when markups are assumed to be time-varying such that the markup falls with oil consumption. The responses are even larger when level and volatility shocks to oil production are also assumed to be negatively correlated. The empirical support for these assumptions is not clear. For example, time-varying markups are a standard feature of micro-founded New Keynesian models. Plante and Traum (2014) examine such a model where oil is an intermediate input and find that SV shocks to the price of oil have a negligible effect on the macro economy, casting doubt on the reduced-form setup used in Gao et al. (2022).

Another important difference is their use of SV shocks. Our results in [Section 5.4](#) show that models with SV are unable to match important features of geopolitically driven oil production events.

Our DSGE model shares some features with the theoretical model in Gao et al. (2022) in that both models allow the price of oil to be endogenously determined and are concerned with uncertainty shocks. One key difference is that their model features SV shocks to the production of oil and to productivity growth rather than disaster risk. Another difference is that Gao et al. do not examine the response of oil price uncertainty to other shocks in their model, nor does their work address the implications of the endogeneity of oil price uncertainty for empirical work.

The study related most closely to ours is perhaps Olovsson (2019) with the important difference that while Olovsson allows for oil storage and oil production disasters, his model does not allow the disaster probability to vary over time, nor does it include macroeconomic disasters. In addition, the focus of his paper is not on uncertainty. Our DSGE model also differs from the theoretical model in Ready (2018) who does not study responses to oil price uncertainty shocks, but reports comparative statics with respect to a regime shift in the slope of the oil production process.

Finally, what sets our work apart from all these earlier studies is that we address the implications of the endogeneity of oil price uncertainty for the identification of oil price uncertainty shocks in empirical work. This question has not been addressed in prior work.

7 IMPLICATIONS FOR EMPIRICAL WORK

The model in [Section 4](#) highlights that oil price uncertainty endogenously responds not only to exogenous uncertainty about future oil production driven by geopolitical events, but also to exogenous uncertainty about the future path of the economy. Thus, geopolitically driven oil price uncertainty shocks differ in general from shocks to observed oil price uncertainty, as measured by the method of Jurado et al. (2015) or the OVX crude oil volatility index. This result is consistent with practitioners' understanding that uncertainty about the oil price reflects not only uncertainty about future oil production, but also uncertainty about future oil consumption driven by macroeconomic uncertainty, financial uncertainty, and possibly policy uncertainty. For example, market

commentators in recent years have routinely highlighted the role of uncertainty about the prospects of the Chinese economy, the resolution of the Covid-19 pandemic, and whether the U.S. economy is about to enter a recession in assessing the uncertainty about the price of oil. In addition, the model shows that oil price uncertainty responds to level shocks in the macro economy, invalidating the premise of exogenous oil price uncertainty shocks even when there are no exogenous shocks to macroeconomic uncertainty. Notably, a disaster shifting real activity has substantial effects on oil price uncertainty.

Our analysis highlights the difficulty of separating shocks to the level of the price of oil from exogenous shocks to oil price volatility, given that oil price volatility naturally arises even in the absence of exogenous uncertainty shocks. More generally, the premise that both positive and negative oil price shocks are associated with increases in oil price uncertainty is not supported by the DSGE model. For example, a negative growth disaster probability shock raises the real price of oil but lowers oil price uncertainty. Likewise, a negative shock to oil production raises the real price of oil, but lowers oil price uncertainty.

Defining and measuring an exogenous oil price uncertainty shock in a structural vector autoregressive (VAR) model setting is not straightforward. Our model shows that VAR models with GARCH errors that postulate that every oil price volatility shock is also a level shock to the price of oil are inherently misspecified. Empirical models need to allow for the possibility that level and uncertainty shocks may evolve differently. For example, a shock to the level of oil production moves the price of oil without having a material effect on oil price uncertainty.

Models that break the link between level and volatility shocks such as VAR models with SV have their own limitations. Our analysis implies that empirical measures of oil price uncertainty shocks and shocks to the level of the price of oil are not independent, as assumed in VAR models with SV. The reason is that oil price uncertainty is endogenous and driven by the same shocks as the price of oil. For example, a growth disaster probability shock not only raises oil price uncertainty, but higher uncertainty also causes storage demand to increase, raising the real price of oil.¹⁹

¹⁹Moreover, as shown in [Section 5](#), while an increase in oil price uncertainty may alternatively be generated by an SV shock to oil production or an exogenous shock to the probability of a major decline in oil production, only the

The insight that oil price uncertainty is simultaneously determined with macroeconomic aggregates applies not only to GARCH and SV models but also to recursively identified linear VAR models that order oil price uncertainty first in the spirit of Bloom (2009), such as Gao et al. (2022).²⁰ Thus, the seemingly robust empirical evidence from VAR models that oil price uncertainty shocks substantially lower real activity must be viewed with caution.

8 CONCLUDING REMARKS

It is widely believed that exogenous shifts in oil price uncertainty help explain the disproportionate impact of positive oil price shocks on the economy and the muted response to negative oil price shocks. This belief is based on economic intuition derived from partial equilibrium models that treat oil price uncertainty as exogenous with respect to the economy. We showed that this intuition does not generalize to general equilibrium settings with endogenous oil price uncertainty. Oil price uncertainty responds to level shocks in the macro economy and exogenous macroeconomic uncertainty shocks, in addition to exogenous geopolitically driven oil price uncertainty shocks. This fact not only invalidates standard empirical models of the transmission of oil price uncertainty shocks to the economy that provide seemingly robust evidence supporting the conventional wisdom, but it calls into question the premise of these partial equilibrium models.

The use of a DSGE model helps define unambiguously the sources of fluctuations in oil price uncertainty. We show that shocks to the probability of disasters have major effects on global oil market variables, whether the exogenous disaster involves a reduction in economic growth or a major oil production shortfall driven by a geopolitical event. Our analysis highlights the importance of jointly modeling macroeconomic risk and geopolitical risk. We show that oil price uncertainty tends to be driven by macroeconomic risk more than geopolitical risk, which helps explain its seemingly recessionary effects.

Geopolitical risk matters mainly for understanding oil price volatility. Shocks to the probability

latter generates large recessionary effects. Thus, one would not expect large recessionary effects of shocks to oil price uncertainty when estimating VAR models with SV.

²⁰Exploring alternative recursive orderings does not address this concern, as shown in Kilian et al. (2024).

of a major decline in global oil production driven by geopolitical events cause persistent changes in oil price uncertainty, whether the disaster is actually realized or not. While these shocks have effects on real GDP and other macroeconomic aggregates that dwarf those found with more traditional SV shocks, we find that geopolitically driven oil price uncertainty shocks are not a major driver of fluctuations in macroeconomic aggregates.

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Online Appendix: Geopolitical Oil Price Risk and Economic Fluctuations*

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May 21, 2024

ABSTRACT

This appendix describes the methodology for constructing a time series of oil price uncertainty, the data sources and transformations, and the global solution method for the DSGE model, and then presents the responses to a macroeconomic disaster and oil production disaster.

*The views expressed in this paper are our own and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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A MEASURING UNCERTAINTY

Our method of constructing quarterly measures of uncertainty builds on Jurado et al. (2015). We first summarize the key steps of the estimation process before discussing the data used in the estimation.

A.1 METHODOLOGY Let $\mathbf{Y}_t = (y_{1,t}, \dots, y_{N_y,t})'$ be a vector of data containing N_y variables. Our objective is to estimate the 1-quarter ahead uncertainty about select elements of \mathbf{Y}_t , defined as

$$\mathcal{U}_t^j \equiv \sqrt{E[(y_{j,t+1} - E[y_{j,t+1}|I_t])^2|I_t]},$$

where the expectation is taken with respect to the information set I_t and j refers to the variable of interest. There are four steps:

1. Generate forecast errors for $y_{j,t+1}$ using a forecasting model that includes lags of the variable y_j , estimated factors extracted from a panel of predictor variables, $\hat{\mathbf{F}}_t$, and a set of additional predictors contained in a vector \mathbf{W}_t .
2. Fit autoregressive models for the factors in $\hat{\mathbf{F}}_t$ and the variables in \mathbf{W}_t and generate residuals for each variable.
3. Estimate a stochastic volatility model for each residual.
4. Calculate \mathcal{U}_t^j .

Factors Let $\mathbf{X}_t = (X_{1,t}, \dots, X_{N_x,t})'$ be a vector of predictors that are available for forecasting. These data are transformed to be stationary. It is assumed that the transformed variables have an approximate factor structure,

$$X_{i,t} = \Lambda_i^{F'} \mathbf{F}_t + e_{i,t}^X,$$

where \mathbf{F}_t is a $r_F \times 1$ vector of latent factors, $\Lambda_i^{F'}$ is a $1 \times r_F$ vector of loadings for variable i and the idiosyncratic errors are given by $e_{i,t}^X$. The estimated factors, denoted as $\hat{\mathbf{F}}_t$, are estimated using principal components and the number of factors is selected using the criterion of Bai and Ng (2002). Each of the factors is assumed to follow an autoregressive process with two lags,

$$\begin{aligned} F_t &= \Phi^F(L)F_{t-1} + v_t^F, \\ v_t^F &= \sigma_t^F \epsilon_t^F, \quad \epsilon_t^F \sim \mathbb{N}(0, 1), \\ \ln(\sigma_t^F)^2 &= \alpha^F + \beta^F \ln(\sigma_{t-1}^F)^2 + \tau^F \eta_t^F, \quad \eta_t^F \sim \mathbb{N}(0, 1), \end{aligned}$$

where $\Phi^F(L)$ is a lag polynomial.

Additional predictors The $r_W \times 1$ vector \mathbf{W}_t includes the squared values of the first factor in \hat{F}_t and a set of N_G factors estimated using principal components on the squared values of the variables in \mathbf{X}_t . Each variable in \mathbf{W}_t is assumed to follow an autoregressive process with two lags,

$$\begin{aligned} W_t &= \Phi^W(L)W_{t-1} + v_t^W, \\ v_t^W &= \sigma_t^W \epsilon_t^W, \quad \epsilon_t^W \sim \mathbb{N}(0, 1), \\ \ln(\sigma_t^W)^2 &= \alpha^W + \beta^W \ln(\sigma_{t-1}^W)^2 + \tau^W \eta_t^W, \quad \eta_t^W \sim \mathbb{N}(0, 1), \end{aligned}$$

where $\Phi^F(W)$ is a lag polynomial.

Forecasting Model A forecast for $y_{j,t+1}$ is produced with the factor-augmented forecasting model,

$$\begin{aligned} y_{j,t+1} &= \phi_j^Y(L)y_{j,t} + \gamma_j^F(L)\hat{\mathbf{F}}_t + \gamma_j^W(L)\mathbf{W}_t + \nu_{j,t+1}^Y, \\ \nu_t^y &= \sigma_t^y \epsilon_t^y, \quad \epsilon_t^y \sim \mathbb{N}(0, 1) \\ \ln(\sigma_t^y)^2 &= \alpha^y + \beta^y \ln(\sigma_{t-1}^y)^2 + \tau^y \eta_t^y, \quad \eta_t^y \sim \mathbb{N}(0, 1), \end{aligned}$$

where $\phi_j^Y(L)$, $\gamma_j^F(L)$, and $\gamma_j^W(L)$ are lag polynomials of orders 2, 1, and 1, respectively. As in Jurado et al. (2015, footnote 10), a hard threshold is applied to remove any variables from the forecasting model that do not have incremental predictive power.

Uncertainty Define $\mathbf{Z}_t \equiv (\hat{\mathbf{F}}_t', \mathbf{W}_t')$ as a vector that collects the estimated factors and the additional predictors contained in \mathbf{W}_t . Then let $\mathcal{Z}_t \equiv (\mathbf{Z}_t', \dots, \mathbf{Z}_{t-q+1}')$ and $Y_{j,t} = (y_{j,t}, \dots, y_{j,t-q+1})'$, where $q = 2$. The FAVAR model can be written in companion form as

$$\begin{pmatrix} \mathcal{Z}_t \\ Y_{j,t} \end{pmatrix} = \begin{pmatrix} \Phi^{\mathcal{Z}} & 0 \\ \Lambda_j' & \Phi_j^Y \end{pmatrix} \begin{pmatrix} \mathcal{Z}_{t-1} \\ Y_{j,t-1} \end{pmatrix} + \begin{pmatrix} \mathcal{V}_t^{\mathcal{Z}} \\ \mathcal{V}_{j,t}^Y \end{pmatrix} \iff \mathcal{Y}_{j,t} = \Phi_j^{\mathcal{Y}} \mathcal{Y}_{j,t-1} + \mathcal{V}_{j,t}^{\mathcal{Y}}.$$

The forecast error variance is

$$\Omega_{j,t}^{\mathcal{Y}}(1) \equiv E_t[(\mathcal{Y}_{j,t+1} - E_t \mathcal{Y}_{j,t+1})(\mathcal{Y}_{j,t+1} - E_t \mathcal{Y}_{j,t+1})'],$$

where $E_t \mathcal{Y}_{j,t+1} = \Phi_j^{\mathcal{Y}} \mathcal{Y}_{j,t}$. The forecast error variances can be calculated as

$$\Omega_{j,t}^{\mathcal{Y}}(1) = E_t[\mathcal{V}_{j,t+1}^{\mathcal{Y}} \mathcal{V}_{j,t+1}^{\mathcal{Y}' }].$$

The uncertainty of $y_{j,t+1}$ is

$$\mathcal{U}_t^j = \sqrt{1_j' \Omega_{j,t}^{\mathcal{Y}}(1) 1_j},$$

where 1 is a selection vector and j refers to the growth rate of real GDP and inflation adjusted U.S. refiners' acquisition cost of imported crude oil, respectively.

A.2 DATA Our dataset includes most of the financial and macroeconomic variables listed in the data appendix of Ludvigson et al. (2021) plus real GDP and the inflation-adjusted U.S. refiners' acquisition cost of imported crude oil.

The macroeconomic variables are from the April 2024 vintage of the FRED-MD database with the following modifications.

- We linearly interpolate the missing values of $UMCSENT_x$ that occur through 1977.
- We set the missing value of $CP3M_x$ for 4/1/2020 to its value on 3/1/2020.
- We set the missing value of $COMPAPFF_x$ for 4/1/2020, to its value on 3/1/2020.

Monthly data are averaged by quarter and transformed to stationarity using the code in the FRED-MD database. Both real GDP and the real price of oil are log-differenced. The data set starts begins in 1974Q1. The sample begins in 1974Q2, because we lose one observation due to differencing.

The financial variables are obtained from FRED-MD, CRSP and the Fama-French database. Returns are aggregated by summing the three monthly values in each quarter.

B DATA SOURCES

We use the following time-series provided by Haver Analytics:

1. **Consumer Price Index for All Urban Consumers:** Not seasonally Adjusted, Monthly, Index (PCUN@USECON)
2. **World Production of Crude Oil Including Lease Condensate**
Not Seasonally Adjusted, Thousands of Barrels per Day
(Monthly, AWOACAU@ENERGY; Quarterly, BWOACAU@ENERGY)
3. **United States: Petroleum Products Expenditures**
Annual, Millions of Dollars (ZUSPATCV@USENERGY)
4. **US Crude Oil Imported Acquisition Cost by Refiners**
Not Seasonally Adjusted, Quarterly, Dollars per Barrel (CUSIQABF@USENERGY)
5. **Civilian Noninstitutional Population: 16 Years & Over**
Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)
6. **Gross Domestic Product: Implicit Price Deflator**
Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)
7. **Gross Domestic Product**
Seasonally Adjusted, Quarterly, Billions of Dollars (GDP@USECON)
8. **Gross Domestic Product**
Annual, Millions of Dollars (GDPY@USNA)

9. **Personal Consumption Expenditures: Nondurable Goods**
Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)
10. **Personal Consumption Expenditures: Services**
Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)
11. **Personal Consumption Expenditures: Durable Goods**
Seasonally Adjusted, Quarterly, Billions of Dollars (CD@USECON)
12. **Private Fixed Investment**
Seasonally Adjusted, Quarterly, Billions of Dollars (F@USECON)
13. **Total Economy: Labor share**
Seasonally Adjusted, Quarterly, Percent (LXEBL@USNA)
14. **Net Stock: Private Fixed Assets**, Annual, Billions of Dollars (EPT@CAPSTOCK)
15. **Net Stock: Durable Goods**, Annual, Billions of Dollars (EDT@CAPSTOCK)
16. **Depreciation: Private Fixed Assets**, Annual, Billions of Dollars (KPT@CAPSTOCK)
17. **Depreciation: Durable Goods**, Annual, Billions of Dollars (KDT@CAPSTOCK)
18. **CBOE Crude Oil Volatility Index (OVX)**, Daily, Index (SPOVX@DAILY)

We also used the following data sources:

1. **FRED-MD**, Monthly Databases for Macroeconomic Research. The data is available at <https://research.stlouisfed.org/econ/mccracken/fred-databases> (McCracken, 2024). Under Monthly Data, we use the April 2024 vintage.
2. **Fama-French**, Database. The data is available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html (Fama and French, 2024).
3. **WRDS**, Stock Market Indexes. The data is available at <https://wrds-www.wharton.upenn.edu> (Wharton Research Data Services, 2024).
4. **Geopolitical Risk Index**, Historical series (GPRH). The data is available at <https://www.matteoiacoviello.com/gpr.htm> (Caldara and Iacoviello, 2024)
5. **Global Oil Inventories Monthly**, Millions of Barrels per Day (Inv_t) from Kilian (2022).

We applied the following data transformations:

1. **Per Capita Real Output:** $Y_t = 10^9 \times GDP_t / ((DGDP_t/100)(1000 \times LN16N_t))$.
2. **Per Capita Real Consumption:** $C_t = 10^9(CN_t + CS_t) / ((DGDP_t/100)(1000 \times LN16N_t))$.
3. **Per Capita Real Investment:** $I_t = 10^9(F_t + CD_t) / ((DGDP_t/100)(1000 \times LN16N_t))$.

4. **Depreciation Rate:** $\delta = (1 + \frac{1}{T/4} \sum_{t=1}^{T/4} (KPT_t + KDT_t) / (EPT_{t-1} + EDT_{t-1}))^{1/4} - 1$.
5. **Capital Services Share:** $\xi = 1 - \frac{1}{T} \sum_{t=1}^T LXEBL/100$.
6. **Real Price of Oil:** $p_t^o = CUSIQABF_t / (DGDP_t/100)$.
7. **Expenditure Share of Oil:** $ZUSPATCV_t / GDPY_t$.
8. **Oil Consumption:** $o_t = \text{Days per Month} \times AWOACAUF_t / 1000 - (INV_t - INV_{t-1})$.
9. **Inventory-Oil Consumption Share:** $INV_t / \sum_{j=t-2}^t o_j$ for $t = 3, 6, \dots, 3T$.
10. **CPI Inflation Rate:** $\pi_t^{cpi} = 100 \times (PCUN_t / PCUN_{t-1} - 1)$.
11. **Asset Returns:** We use two time series from the Fama-French data library:
 - Net nominal risk-free rate, monthly, percent (RF)
 - Net nominal excess market return, monthly, percent ($MKTmRF$)

Define the market return as

$$RM_t = MKTmRF_t + RF_t.$$

The gross quarterly analogues of the Fama-French series and CPI inflation are given by

$$RF_t^Q = \prod_{j=t-2}^t (1 + RF_j/100), \quad RM_t^Q = \prod_{j=t-2}^t (1 + RM_j/100), \quad \pi_t^Q = \prod_{j=t-2}^t (1 + \pi_j^{cpi}/100)$$

for $t = 3, 6, \dots, 3T$, so the quarterly real risk-free rate and equity premium are given by

$$r_t = 100 \times (RF_t^Q / \pi_t^Q - 1), \quad r_t^{ex} = 100 \times (RM_t^Q / \pi_t^Q - 1) - r_t.$$

All empirical targets are computed using quarterly data, except the expenditure share of oil which is based on annual data.

C SOLUTION METHOD

The equilibrium system of the DSGE model is summarized by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1}) | \mathbf{z}_t, \vartheta] = 0$, where g is a vector-valued function, \mathbf{x}_t is the vector of model variables, ε_t is the vector of shocks, $\mathbf{z}_t = [\ln e_t, k_t, s_t, v_t^g, v_t^e, \ln p_t^g, \ln p_t^e, \varepsilon_{t-1}]$ is the vector of states, and ϑ is the vector of parameters.

We discretize the continuous shocks, $\{\varepsilon_g, \varepsilon_e, \varepsilon_p^g, \varepsilon_p^e\}$ using the Markov chain in Rouwenhorst (1995). The bounds of the six continuous state variables are chosen so there is minimal extrapolation over 99% of the ergodic distribution. Specifically, the bounds on capital, k_t , range from -20% to $+10\%$, the bounds on storage, s_t , range from -50% to $+70\%$, and the bounds on the error

correction term, ϵ_{t-1} , range from -35% to $+15\%$ of the deterministic steady state. The bounds on transitory component of oil production range from -15% to $+10\%$ of the deterministic steady state in levels, and are then converted to logs. The bounds on the probability of a growth disaster, p_t^g , are set to $[0.00001, 0.8]$, while the bounds on the probability of an oil disaster, p_t^e , are set to $[0.00001, 1]$. Both are converted to logs, consistent with the specifications of the processes. We discretize $\ln e_t$, k_t , and ϵ_{t-1} into 6 points, s_t into 7 points, and $\ln p_t^g$ and $\ln p_t^e$ into 13 points. All of the grids for the continuous states are evenly-spaced. There are also binary indicators for whether the economy is in a growth disaster or an oil disaster, forming 4 distinct outcomes. The product of the points in each dimension, D , is the total number of nodes in the state space ($D = 1,022,112$).

The realization of \mathbf{z}_t on node d is denoted $\mathbf{z}_t(d)$. The Rouwenhorst method provides integration nodes for the continuous shocks, $[\varepsilon_{g,t+1}(m), \varepsilon_{e,t+1}(m), \varepsilon_{p,t+1}^g(m), \varepsilon_{p,t+1}^e(m)]$. The transition matrices for the discrete states determine the integration weights for their future realizations, $[v_{t+1}^g(m), v_{t+1}^e(m)]$. The weight for a particular realization of the continuous and discrete shocks is $\phi(m)$, where $m \in \{1, \dots, M\}$ and M is the product of the number of realizations of each shock. The oil production shock and the two probability shocks have the same number of realizations as the corresponding state variable. We specify that the growth shock, ε_g , has 6 states. Each discrete state has two possible outcomes. In total, there are $M = 24,336$ possible shock realizations.

The vector of policy functions and the realization on node d are denoted by \mathbf{pf}_t and $\mathbf{pf}_t(d)$, where $\mathbf{pf}_t \equiv [n(\mathbf{z}_t), o(\mathbf{z}_t), J(\mathbf{z}_t), p^e(\mathbf{z}_t), r(\mathbf{z}_t)]$. The following steps outline our algorithm:

1. Use the Sims (2002) `gensys` algorithm to solve the log-linear model without any disasters or time-varying probabilities. Then map the solution for the policy functions to the discretized state space, copying the solution on the dimensions that were excluded from the linear model. This provides an initial conjecture, \mathbf{pf}_0 , for the nonlinear algorithm.
2. On iteration $j \in \{1, 2, \dots\}$ and each node $d \in \{1, \dots, D\}$, use Chris Sims' `csolve` to find the $\mathbf{pf}_t(d)$ that satisfies $E[g(\cdot)|\mathbf{z}_t(d), \vartheta] \approx 0$. Guess $\mathbf{pf}_t(d) = \mathbf{pf}_{j-1}(d)$. Then
 - (a) Solve for all variables dated at time t , given $\mathbf{pf}_t(d)$ and $\mathbf{z}_t(d)$.
 - (b) Linearly interpolate the policy functions, \mathbf{pf}_{j-1} , at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{pf}_{t+1}(m)$ on every integration node, $m \in \{1, \dots, M\}$.
 - (c) Given $\{\mathbf{pf}_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{s}_{t+1}(m)$ and compute

$$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1})|\mathbf{z}_t(d), \vartheta] \approx \sum_{m=1}^M \phi(m)g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$

When `csolve` has converged, set $\mathbf{pf}_j(d) = \mathbf{pf}_t(d)$.

3. Repeat step 2 until $\text{maxdist}_j < 10^{-4}$, where $\text{maxdist}_j \equiv \max\{|\mathbf{pf}_j - \mathbf{pf}_{j-1}|/\mathbf{pf}_{j-1}\}$. When that criterion is satisfied, the algorithm has converged to an approximate solution.

D DETRENDED EQUILIBRIUM

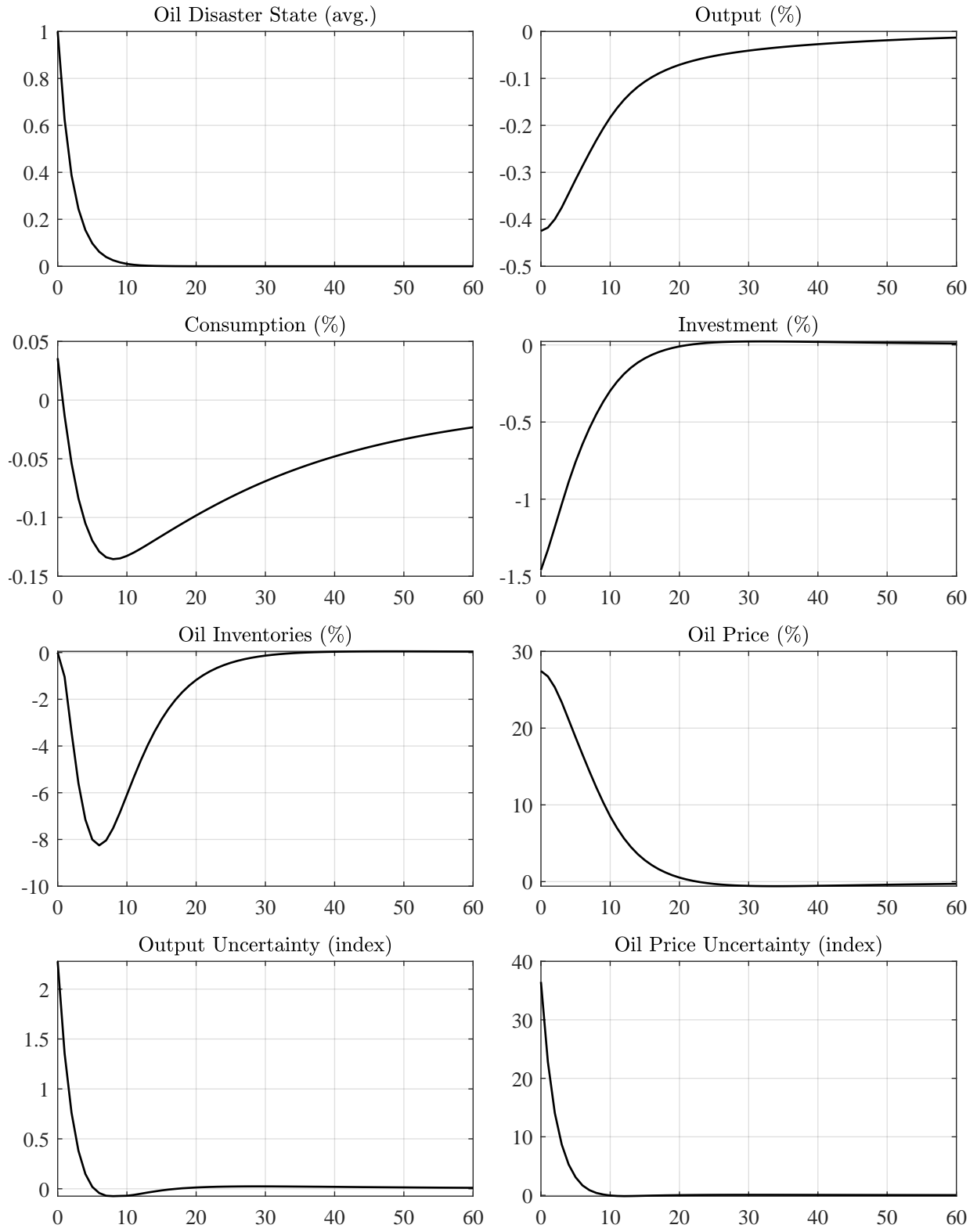
We detrend the model by defining $\tilde{x}_t = x_t/a_t$. The equilibrium system is given by

$$\begin{aligned}
 \tilde{w}_t &= (1 - \xi)\tilde{y}_t/n_t \\
 p_t^o &= \xi\alpha \frac{(\tilde{o}_t/o_0)^{1-1/\sigma}}{(1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma} + \alpha(\tilde{o}_t/o_0)^{1-1/\sigma}} \frac{\tilde{y}_t}{\tilde{o}_t} \\
 E_t[x_{t+1}r_{t+1}^i] &= 1 \\
 r_t^i &= e^{-\zeta v_t} \frac{1}{p_{t-1}^k} (r_t^k + (1 - \delta + a_1 + \frac{a_2}{\nu-1}(\tilde{i}_t/\tilde{k}_t)^{1-1/\nu})p_t^k) \\
 p_t^k &= \frac{1}{a_2}(\tilde{i}_t/\tilde{k}_t)^{1/\nu} \\
 r_t^k &= \xi(1 - \alpha) \frac{(\tilde{k}_t/k_0)^{1-1/\sigma}}{(1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma} + \alpha(\tilde{o}_t/o_0)^{1-1/\sigma}} \frac{\tilde{y}_t}{\tilde{k}_t} \\
 E_t[x_{t+1}r_{t+1}^s] &= 1 \\
 r_t^s &= \frac{1}{p_{t-1}^o} (1 - \omega + \pi \tilde{s}_t^{-3})p_t^o \\
 \chi \tilde{w}_t \ell_t &= (1 - \chi)\tilde{c}_t \\
 x_t &= (\beta/g_t^g)(\tilde{u}_t/\tilde{u}_{t-1})^{1-1/\psi} (\tilde{c}_{t-1}/\tilde{c}_t)(\tilde{J}_t/\tilde{z}_{t-1})^{1/\psi-\gamma} \\
 \tilde{u}_t &= \tilde{c}_t^X \ell_t^{1-X} \\
 \tilde{z}_t &= (E_t[(g_{t+1}J_{t+1})^{1-\gamma}])^{1/(1-\gamma)} \\
 \tilde{J}_t &= \left((1 - \beta)\tilde{u}_t^{1-1/\psi} + \beta\tilde{z}_t^{1-1/\psi} \right)^{\frac{1}{1-1/\psi}} \\
 \tilde{y}_t &= y_0 n_t^{1-\xi} \left((1 - \alpha)(\tilde{k}_t/k_0)^{1-1/\sigma} + \alpha(\tilde{o}_t/o_0)^{1-1/\sigma} \right)^{\xi/(1-1/\sigma)} \\
 g_{t+1}\tilde{k}_{t+1} &= e^{-\zeta g v_{t+1}} (1 - \delta + a_1 + \frac{a_2}{1-1/\nu}(\tilde{i}_t/\tilde{k}_t)^{1-1/\nu})\tilde{k}_t \\
 g_{t+1}\tilde{s}_{t+1} &= (1 - \omega)\tilde{s}_t + \tilde{o}_t^s - \tilde{o}_t - \frac{\pi}{2}\tilde{s}_t^{-2} \\
 \tilde{o}_t^s &= e_t/\epsilon_t \\
 \tilde{c}_t + \tilde{i}_t &= \tilde{y}_t \\
 n_t + \ell_t &= 1 \\
 g_{o,t} &= \kappa_0 g_t^{\kappa_1} \epsilon_{t-1}^{\kappa_2} \\
 \epsilon_t &= (g_t/g_{o,t})\epsilon_{t-1} \\
 \ln g_t &= \ln \bar{g} + \sigma_g \varepsilon_{g,t} - \zeta_g (v_t^g - \bar{\pi}_1^g) \\
 \ln e_t &= (1 - \rho_e) \ln \bar{e} + \rho_e \ln e_{t-1} + \sigma_e \varepsilon_{e,t} - \zeta_e (v_t^e - \bar{\pi}_1^e) \\
 Pr(v_{t+1}^g = 1 | v_t^g = 1) &= \bar{q}^g, \quad Pr(v_{t+1}^g = 1 | v_t^g = 0) = p_t^g \\
 Pr(v_{t+1}^e = 1 | v_t^e = 1) &= \bar{q}^e, \quad Pr(v_{t+1}^e = 1 | v_t^e = 0) = p_t^e \\
 \ln p_t^g &= (1 - \rho_p^g) \ln \bar{p}^g + \rho_p^g \ln p_{t-1}^g + \sigma_p^g \varepsilon_{p,t}^g \\
 \ln p_t^e &= (1 - \rho_p^e) \ln \bar{p}^e + \rho_p^e \ln p_{t-1}^e + \sigma_p^e \varepsilon_{p,t}^e \\
 E_t[x_{t+1}r_t] &= 1 \\
 1 &= E_t[x_{t+1}r_{t+1}^e] \\
 r_t^e &= g_t(\tilde{p}_t^e + \tilde{d}_t^e)/\tilde{p}_{t-1}^e \\
 \tilde{d}_t^e &= \xi \tilde{y}_t - \tilde{i}_t - \vartheta_f (E_{t-1}\tilde{k}_t - \frac{1}{r_t} E_t[g_{t+1}\tilde{k}_{t+1}]) - \vartheta_s (E_{t-1}\tilde{s}_t - \frac{1}{r_t} E_t[g_{t+1}\tilde{s}_{t+1}])
 \end{aligned}$$

E RESPONSES TO DISASTER REALIZATIONS

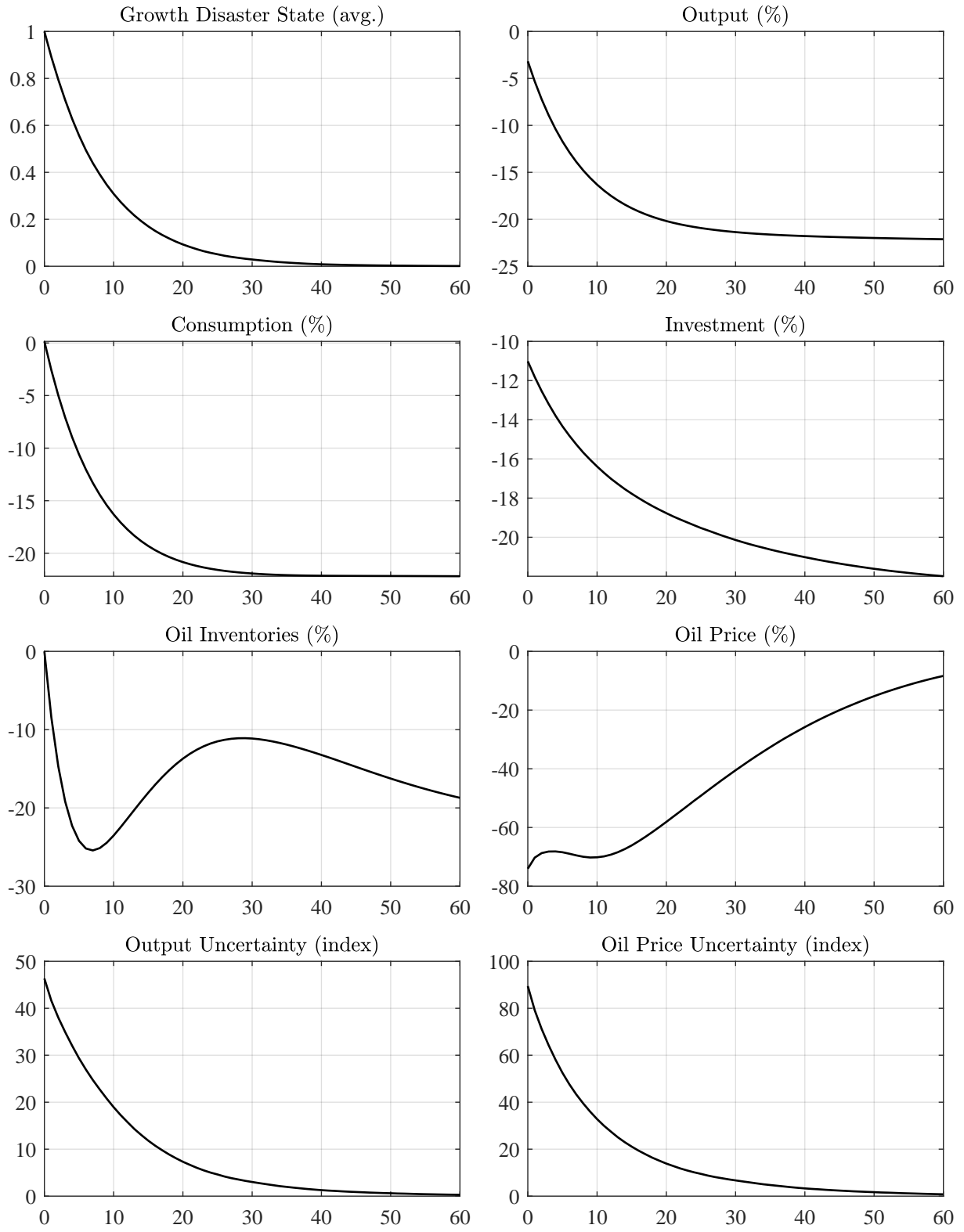
Figures 1 and 2 show how the economy responds to oil production and growth disaster realization, respectively. The disaster occurs in the initial period and follows its expected path in future periods.

Figure 1: Responses to an oil production disaster realization



Notes: Responses in deviations from the baseline.

Figure 2: Responses to a growth disaster realization



Notes: Responses in deviations from the baseline.

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