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Xtpb: The Pooled Bewley Estimator of Long Run Relationships in Dynamic Heterogeneous Panels

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Xtpb: The Pooled Bewley Estimator of Long Run Relationships in Dynamic Heterogeneous Panels*

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Abstract

This paper introduces a new Stata command, xtpb, that implements the Chudik, Pesaran, and Smith (2023a) Pooled Bewley (PB) estimator of long-run relationships in dynamic heterogeneous panel-data models. The PB estimator is based on the Bewley (1979) transform of the autoregressive-distributed lag model and it is applicable under a similar setting as the widely used pooled mean group (PMG) estimator of Pesaran, Shin, and Smith (1999). Two bias-correction methods and a bootstrapping algorithm for more accurate small-sample inference robust to arbitrary cross-sectional dependence of errors are also implemented. An empirical illustration reproduces the PB estimates of the consumption function as in Chudik, Pesaran, and Smith (2023a).

Keywords: xtpb, pooled Bewley (PB) estimator, pooled mean group (PMG) estimator, heterogeneous dynamic panels, I(1) regressors, autoregressive-distributed lag model (ARDL), cross-sectional dependence, bias-correction, bootstrapping

JEL Codes: C12, C13, C23, C33

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1 Introduction

The estimation of long-run relationships is important for many applications in economics, including panel data settings with heterogeneous (or group-specific) short-run dynamics, and possibly non-stationary variables (outlined in Section 2 below). There are three widely-used estimators of cointegrating panels in the literature: the group-mean Fully Modified OLS (FMOLS) by Pedroni (2001a, 2001b), the Panel Dynamic OLS (PDOLS) by Mark and Sul (2003) and the Pooled Mean Group (PMG) estimator by Pesaran, Shin, and Smith (1999). Recently, Chudik, Pesaran, and Smith (2023a, hereafter CKP) proposed an alternative, Pooled Bewley (PB) estimator of long-run relationships.

This paper introduces the `xtpb` Stata command for PB estimation of long-run relationships. Similarly to other existing estimators in the literature (namely FMOLS, PDOLS, and PMG), the PB estimator is designed for panels with a sufficiently large time dimension (T), whereas the cross-section dimension of the panel can be small compared with T, or of the same order in magnitude. In practice this translates to T of at least 20 periods (for all of these estimators), according to the extensive simulations in CKP. Our `xtpb` command also features the bias-correction and bootstrapping options considered by CKP for more accurate and robust inference in the presence of arbitrary cross-sectional correlation of errors.

The remainder of this paper is organized as follows. Section 2 introduces the dynamic panel data model of interest and discusses the generality and the main limitations of the PB estimator. Section 3 describes the `xtpb` command and its options. Section 4 illustrates the use of the `xtpb` command by replicating the empirical example of CKP. Section 5 offers concluding remarks.

2 Dynamic panel-data model

Let y_{it} be the dependent variable for group i ($i = 1, 2, ..., N$) in period t ($t = 1, 2, ..., T$), and $\mathbf{x}_{it}$ be a $k \times 1$ vector of observations on k regressors for group i in period t. Let $\mathbf{w}_{it} = (y_{it}, \mathbf{x}_{it})'$ be a $(k+1) \times 1$ vector of observations on all variables for group i.

To understand generality and limitations of the Pooled Bewley and PMG estimators, it is useful to suppose $\mathbf{w}_{it}$ is given by the following group-specific VAR(p) model,

$$\mathbf{\Phi}_i(L)\left(\mathbf{w}_{it} - \mathbf{a}_i\right) = \mathbf{u}_{it} \tag{1}$$

for $i = 1, 2, ..., n$, and $t = 1, 2, ..., T$, where

$$\mathbf{\Phi}_i(L) = \mathbf{I}_m - \mathbf{\Phi}_{i1}L - \mathbf{\Phi}_{i2}L^2 - .... - \mathbf{\Phi}_{ip}L^p$$

The group-specific VAR models in (1) can be equivalently re-written using the familiar error-correction representation,

$$\Delta\mathbf{w}_{it} = \mathbf{c}_i - \mathbf{\Pi}_i\mathbf{w}_{i,t-1} + \sum_{j=1}^{p-1}\mathbf{\Psi}_{ij}\Delta\mathbf{w}_{i,t-j} + \mathbf{u}_{it} \tag{2}$$

where $\mathbf{c}_i = \mathbf{\Phi}_i(1)\mathbf{a}_i$,

$$\begin{aligned} \mathbf{\Pi}_i &= -\mathbf{\Phi}_i(1) = -(\mathbf{I}_m - \mathbf{\Phi}_1 - .... - \mathbf{\Phi}_p) \\ \mathbf{\Psi}_{ij} &= -\sum_{\ell=j+1}^{p} \mathbf{\Phi}_\ell, \text{ for } j = 1, 2, ..., p-1 \end{aligned}$$

We assume variables given by (2) are integrated of order one, or $I(1)$ for short, with a single *homogeneous* cointegrating vector, in which case $\mathbf{\Pi}_i$ can be written as

$$\mathbf{\Pi}_i = \alpha_i \beta'$$

where α_i for $i = 1, 2, ..., n$ are $(k+1) \times 1$ group-specific vectors of error-correcting coefficients, and β is a $(k+1) \times 1$ common cointegrating vector (identified up to a constant of proportionality). Using the exact identifying restriction $\beta_1 = 1$, we write the normalized cointegrating vector as $\beta = \left(1, -\mathbf{b}'\right)'$, where $\mathbf{b}$ is a $k \times 1$ vector of long-run coefficients of interest.

We next derive the conditional representation for y_{it} in (2). Conformably with $\mathbf{w}_{it} = (y_{it}, \mathbf{x}_{it})'$, partition $\mathbf{u}_{it} = (u_{yit}, \mathbf{u}'_{xit})'$, $\alpha_i = (\alpha_{yi}, \alpha'_{xi})'$, $\mathbf{c}_i = (c_{yi}, \mathbf{c}'_{xi})'$,

$$\mathbf{\Psi}_{ij} = \begin{pmatrix} \psi_{yy,ij} & \psi'_{yx,ij} \\ \psi_{xy,ij} & \mathbf{\Psi}_{xx,ij} \end{pmatrix}, \text{ for } j = 1, 2, ..., p-1$$

and let

$$u_{yit} = E\left(u_{yit} \left| \mathbf{u}_{xit} \right.\right) + v_{it} = \delta'_{i0}\mathbf{u}_{xit} + v_{it} \tag{3}$$

Note that v_{it} is by construction uncorrelated with $\mathbf{u}_{xt}$. Using these notations, individual equations for y_{it} and $\mathbf{x}_{it}$ in (2) are

$$\Delta y_{it} = c_{yi} - \alpha_{yi}\xi_{i,t-1} + \sum_{j=1}^{p-1} \psi_{yy,ij}\Delta y_{i,t-j} + \sum_{j=1}^{p-1} \psi'_{yx,ij}\Delta \mathbf{x}_{i,t-j} + u_{yit} \tag{4}$$

and

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{xi} - \alpha_{xi}\xi_{i,t-1} + \sum_{j=1}^{p-1} \psi_{xy,ij}\Delta y_{i,t-j} + \sum_{j=1}^{p-1} \mathbf{\Psi}_{xx,ij}\Delta \mathbf{x}_{i,t-j} + \mathbf{u}_{xit} \tag{5}$$

where

$$\xi_{it} = y_{it} - \mathbf{b}'\mathbf{x}_{it}$$

is the error correcting term. Substituting (3) in (4), we have

$$\Delta y_{it} = c_{yi} - \alpha_{yi}\xi_{i,t-1} + \sum_{j=1}^{p-1} \psi_{yy,ij}\Delta y_{i,t-j} + \sum_{j=1}^{p-1} \psi'_{yx,ij}\Delta \mathbf{x}_{i,t-j} + \delta'_{i0}\mathbf{u}_{xit} + v_{it}$$

and substituting (5) for $\mathbf{u}_{xit}$ in the above expression, we obtain the following familiar conditional representation for Δy_{it},

$$\Delta y_{it} = d_i - \alpha_i\xi_{i,t-1} + \sum_{j=1}^{p-1} \varphi_{ij}\Delta y_{i,t-j} + \delta'_{i0}\Delta \mathbf{x}_{it} + \sum_{j=1}^{p-1} \delta'_{ij}\Delta \mathbf{x}_{i,t-j} + v_{it} \tag{6}$$

where $d_i = c_{yi} - \delta_{i0}' \mathbf{c}_{xi}$, $\alpha_i = \alpha_{yi} - \delta_{i0}' \alpha_{xi}$, $\varphi_{ij} = \psi_{yy,ij} - \delta_{i0}' \psi_{xy,ij}$, for $j = 1, 2, ..., p$, and $\delta_{ij}' = \psi_{yx,ij}' - \delta_{i0}' \mathbf{\Psi}_{xx,ij}$ for $j = 1, 2, ..., p$. It is useful to emphasize that the specification (6) is obtained from VAR representation (1), and, by construction, v_{it} is uncorrelated with $\Delta \mathbf{x}_{it}$. In addition, assuming lag order p is suitably chosen, the VAR innovations $\mathbf{u}_{it}$ as well as the error term v_{it} are serially uncorrelated.

In addition to the assumption of a single cointegrating relationship, both PB and PMG estimators assume long-run causality runs from $\mathbf{x}_{it}$ to y_{it}, namely $\alpha_i \neq 0$ and $\alpha_{xi} = \mathbf{0}$ for all i. This is a limitation of these estimators. In the presence of two-way long-run causality, namely $\alpha_i \neq 0$, and $\alpha_{xi} \neq \mathbf{0}$, both PB and PMG estimators will be subject to a small bias. When $\alpha_{xi} = \mathbf{0}$, the representation for $\mathbf{x}_{it}$ simplifies to

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{xi} + \sum_{j=1}^{p-1} \psi_{xy,ij} \Delta y_{i,t-j} + \sum_{j=1}^{p-1} \mathbf{\Psi}_{xx,ij} \Delta \mathbf{x}_{i,t-j} + \mathbf{u}_{xit} \tag{7}$$

Our model, given by (6)-(7), allows for short-run feedbacks from y to $\mathbf{x}$ and vice versa, allows for heterogeneous contemporaneous correlation of u_{yit} and $\mathbf{u}_{xit}$, and allows for heterogeneity of all short-run parameters (including the coefficients α_i that determine the speed of convergence toward the long-run). In their exposition, CKP assume, purely for notational simplicity, that $p = k = 1$ and $\mathbf{c}_{xi} = 0$, but their estimator is valid for any value of p and for $\mathbf{c}_{xi} \neq 0$.

2.1 PB estimator

In contrast to PMG, which is obtained by maximizing a complex likelihood function (with no guarantee of finding the global maximum), the PB estimator is given by analytical formula.

The pooled Bewley estimator takes advantage of the Bewley (1979) transformation of the autoregressive distributed lag representation (6). Subtracting $(1 - \alpha_i) y_{it}$ from both sides of (6) and re-arranging, we have

$$\alpha_i y_{it} = d_i - (1 - \alpha_i) \Delta y_{it} + \alpha_i \mathbf{b}' \mathbf{x}_{it} + \sum_{j=1}^{p-1} \varphi_{ij} \Delta y_{i,t-j} + \delta_{i0}' \Delta \mathbf{x}_{it} + \sum_{j=1}^{p-1} \delta_{ij}' \Delta \mathbf{x}_{i,t-j} + v_{it}$$

Multiplying the equation above by α_i^{-1}, we obtain

$$y_{it} = \alpha_i^{-1} d_i + \mathbf{b}' \mathbf{x}_{it} + \psi_i' \Delta \mathbf{z}_{it} + \alpha_i^{-1} v_{it} \tag{8}$$

where $\Delta \mathbf{z}_{it} = \left(\Delta y_{it}, \Delta y_{i,t-1}, ..., \Delta y_{i,t-p+1}, \Delta \mathbf{x}_{it}', \Delta \mathbf{x}_{i,t-1}', ..., \Delta \mathbf{x}_{i,t-p+1}' \right)'$. Assuming observations are available for $t = 1, 2, ..., T$, we stack (8) for $t = p + 1, p + 2, ..., T$,

$$\mathbf{y}_i = \alpha_i^{-1} d_i \tau_{T-p} + \mathbf{X}_i \mathbf{b} + \Delta \mathbf{Z}_i \psi_i + \alpha_i^{-1} \mathbf{v}_i \tag{9}$$

where $\mathbf{y}_i = (y_{i,p+1}, y_{i,p+2}, ..., y_{iT})'$, $\mathbf{X}_i = \left(\mathbf{x}_{i,p+1}', \mathbf{x}_{i,p+2}', ..., \mathbf{x}_{iT} \right)'$, $\Delta \mathbf{Z}_i = \left(\Delta \mathbf{z}_{i,p+1}', \Delta \mathbf{z}_{i,p+2}', ..., \Delta \mathbf{z}_{i,T}' \right)'$, $\mathbf{v}_i = (v_{i,p+1}, v_{i,p+2}, ..., v_{i,T})'$, and τ_{T-p} is a $(T - p) \times 1$ vector of ones. Define projection matrix $\mathbf{M}_\tau = \mathbf{I}_{T-p} - \tau_{T-p}\tau_{T-p}'/(T-p)$, and let $\tilde{\mathbf{y}}_i = (\tilde{y}_{i,p+1}, \tilde{y}_{i,p+2}, ..., \tilde{y}_{iT})' = \mathbf{M}_\tau \mathbf{y}_i$, $\tilde{\mathbf{X}}_i = \mathbf{M}_\tau \mathbf{X}_i$, $\Delta\tilde{\mathbf{Z}}_i = \mathbf{M}_\tau \Delta\mathbf{Z}_i$, and $\tilde{\mathbf{v}}_i = \mathbf{M}_\tau \mathbf{v}_i$. Multiplying (9) by $\mathbf{M}_\tau$, we have

$$\tilde{\mathbf{y}}_i = \tilde{\mathbf{X}}_i \mathbf{b} + \Delta\tilde{\mathbf{Z}}_i \psi_i + \alpha_i^{-1} \tilde{\mathbf{v}}_i$$

The PB estimator of $\mathbf{b}$ is given by

$$\hat{\mathbf{b}} = \left(\sum_{i=1}^{n} \tilde{\mathbf{X}}_i' \mathbf{M}_i \tilde{\mathbf{X}}_i \right)^{-1} \left(\sum_{i=1}^{n} \tilde{\mathbf{X}}_i' \mathbf{M}_i \tilde{\mathbf{y}}_i \right) \tag{10}$$

where

$$\mathbf{M}_i = \mathbf{P}_i - \mathbf{P}_i \Delta\tilde{\mathbf{Z}}_i \left(\Delta\tilde{\mathbf{Z}}_i' \mathbf{P}_i \Delta\tilde{\mathbf{Z}}_i \right)^{-1} \Delta\tilde{\mathbf{Z}}_i' \mathbf{P}_i$$

$$\mathbf{P}_i = \tilde{\mathbf{H}}_i \left(\tilde{\mathbf{H}}_i' \tilde{\mathbf{H}}_i \right)^{-1} \tilde{\mathbf{H}}_i'$$

$\tilde{\mathbf{H}}_i = \mathbf{M}_\tau \mathbf{H}_i$, and $\mathbf{H}_i = (\mathbf{y}_{i,-1}, \mathbf{y}_{i,-2}, ..., \mathbf{y}_{i,-p}, \mathbf{X}_i, \mathbf{X}_{i,-1}, ..., \mathbf{X}_{i,-p})$, in which $\mathbf{y}_{i,-\ell} = (y_{i,p+1-\ell}, y_{i,p+2-\ell}, ..., y_{i,T-\ell})'$ and $\mathbf{X}_{i,-\ell} = \left(\mathbf{x}_{i,p+1-\ell}', \mathbf{x}_{i,p+2-\ell}', ..., \mathbf{x}_{i,T-\ell} \right)'$ for $\ell = 1, 2, ..., p$.

2.2 Bias mitigation and robust inference

All of the existing dynamic panel data estimators of cointegrating relationships in the literature suffer from small sample bias that diminishes in T. The xtpb command implements two bias-mitigation procedures outlined in CKP, namely the half-panel jackknife and bootstrap methods, briefly outlined below. The xtpb command also implements the same options considered by CKP and Chudik et al. (2023b) for conducting inference: based on asymptotic critical values, and based on different bootstrapping options outlined in the next section. The reader is referred for full technical implementation details to Section S-2 of the online supplement of CKP.

Jackknife bias reduction

The jackknife bias correction method is given by

$$\tilde{\mathbf{b}}_{jk} = \tilde{\mathbf{b}}_{jk}(\kappa) = \hat{\mathbf{b}} - \kappa \left(\frac{\hat{\mathbf{b}}_a + \hat{\mathbf{b}}_b}{2} - \hat{\mathbf{b}} \right), \tag{11}$$

where $\hat{\mathbf{b}}$ is the PB estimator using the full sample, given by (10), and $\hat{\mathbf{b}}_a$ and $\hat{\mathbf{b}}_b$ are the PB estimators computed using the first and the second half sub-samples. Following CKP κ is set to $1/3$. This choice of κ is derived asymptotically to mitigate the $O\left(T^{-2}\right)$ bias of PB estimator when variables are integrated of order one.

Bootstrap bias reduction

The bootstrap bias-corrected PB estimator, using R bootstrap replications, is given by

$$\tilde{\mathbf{b}_R} = \hat{\mathbf{b}} - \hat{bias}_R, \tag{12}$$

where $\hat{\mathbf{b}}$ is the original (uncorrected) PB estimator, given by (10), and, following CKP, $\hat{bias}_R$ is its bias estimate computed by the sieve wild bootstrap procedure outlined below.

1. Given $\hat{\mathbf{b}}$, estimate the remaining unknown coefficients in (6)-(7) by least squares, and compute residuals denoted by $\hat{v}_{it}, \hat{\mathbf{u}}_{x,it}$.

2. For each $r = 1, 2, ..., R$, generate new draws for $\hat{v}_{it}^{(r)} = e_{it}^{(r)} \hat{v}_{it}$, and $\hat{\mathbf{u}}_{x,it}^{(r)} = e_{it}^{(r)} \hat{\mathbf{u}}_{x,it}$, where $e_{it}^{(r)}$ is randomly drawn from Rademacher distribution,

$$e_{it}^{(r)} = \begin{cases} -1, & \text{with probability } 1/2 \\ 1, & \text{with probability } 1/2 \end{cases} .$$

   Given the estimated parameters of (6)-(7) from Step 1 and the initial values, generate the simulated series $y_{it}^{(r)}, \mathbf{x}_{it}^{(r)}$, and compute the corresponding bootstrap estimates $\hat{\mathbf{b}}^{(r)}$ for $r = 1, 2, ..., R$.

3. The estimate of the bias is computed as $\hat{bias}_R = \left[R^{-1} \sum_{r=1}^{R} \hat{\mathbf{b}}^{(r)} - \hat{\mathbf{b}} \right]$.

In addition to generating bootstrap samples based on (6) and (7), we also consider the option of setting $\psi_{xy,ij} = \mathbf{0}$ in (7), and the option of setting $\mathbf{x}_{it}^{(r)} = \mathbf{x}_{it}$. The choice of different lag orders in (6) and (7) are also allowed. In addition, we also allow for the option to set $e_{it}^{(r)} = e_t^{(r)}$, which results in bootstrapping the columns of cross-sectionally stacked residuals. This is an important option, since it allows for arbitrary cross-section correlation of residuals. All of these options are described in the next section.

3 The Xtpb Stata Command

The `xtpb` command uses mata functions from the `moremata` package by Jann (2005). Installation of the `moremata` package is required.

3.1 Syntax

`xtpb` *depvar* [*indepvars*] [*if*] [*in*]

[, `lagorder(#)` `biascorrect(`*string*`)` `fulldisplay` `errorcorrect(`*string*`)` `residuals(`*string*`)` `bootstrap(`*#reps [, bootstrap_options]*`)`]

The `xtpb` command supports balanced and unbalanced panel data, and requires the dataset to be `xtset`. Variable inputs are also compatible with time series operators. The post-estimation command `predict` cannot be used to obtain residuals of the error-correcting equations in (7). These residuals as well as de-meaned error-correction terms can be obtained using the options below. The post-estimation command `predict` for fitted values returns $\hat{\mathbf{b}}'\mathbf{x}_{it}$.

3.2 Options

`lagorder(#)` Number of lags, p, for the dependent variable and regressors in the level representation. The default is `lagorder(1)`, which sets $p = 1$. Recall, p is the lag order in the levels, which corresponds to $p - 1$ lags for the first-differences in the error-correction representation (6).

`biascorrect(`*string*`)` Choice of small-T bias-correction. `biascorrect(`*none*`)` implements no bias correction. `biascorrect(`*jackknife*`)` implements half-panel jackknife bias correction. `biascorrect(`*bootstrap*`)` implements bias correction based on stochastic simulation (bootstrapping). The default is `biascorrect(`*none*`)`.

`fulldisplay` Choice to display all regression outputs used to calculate the error correction coefficients.

`errorcorrect(`*string*`)` Choice to generate a variable of specified name that contains the error correction terms.

`residuals(`*string*`)` Choice to generate a variable of specified name that contains residuals derived from the error correction regressions.

`bootstrap(`*#reps [, bootstrap_options]*`)` Choice to compute bootstrapped confidence intervals (CIs) in addition to the asymptotic CIs. The number of bootstrap replications must be specified, a large value is recommended (at least 2000). Bootstrapped confidence intervals are automatically computed when `biascorrect(`*bootstrap*`)` is specified.

Bootstrap Options

`csrobust` Choice of re-sampling of residuals in the bootstrapping algorithm for computation of bootstrapped confidence intervals. If not specified, it is assumed there is no cross-sectional dependence of errors. `csrobust` allows for arbitrary cross-sectional dependence of errors by resampling column vectors of cross-sectionally stacked residuals. The default is to not specify `csrobust`.

`btx(`*string*`)` Choice of bootstrapping algorithm for re-sampling regressors in x_{it}. `btx(`*fixed*`)` or no `btx` specification conducts bootstrapping conditional on x_{it}, namely regressors are fixed across the bootstrap replications. `btx(`*varx*`)` resamples regressors in bootstrap replications according to the VAR model in first differences of regressors. `btx(`*varxy*`)` re-samples regressors in bootstrap replications according to the VAR model in first differences of regressors augmented with lags of the first differences of the dependent variable. The default is `btx(`*fixed*`)`.

`btx_lagorder(#)` choice of lag order for the marginal model for regressors used for resampling in bootstrapping. Lag order is specified in the level representation, similarly to `lagorder`. When `btx_lagorder` is not specified, it is set equal to `lagorder`.

`bcialpha(#)` Choice of nominal level for bootstrapped CIs. The default is `bcialpha(0.95)`, which computes 95 percent CIs. Any value between 0 and 1 can be chosen.

`seed(#)` Choice of a random seed for reproducibility. If not specified, the default is `seed(123456)`.

3.3 Stored Results

`xtpb` stores the following results to `e()`.

Scalars			
`e(N)`	Number of groups	`e(Tavg)`	Average obs. per group
`e(Tmin)`	Minimum obs. per group	`e(Tmax)`	Maximum obs. per group
`e(bcialpha)`	Bootstrap CI nominal level		
Macros			
`e(timevar)`	Time period identifier	`e(panelvar)`	Cross section identifier
`e(cmd)`	Name of estimation command	`e(depvar)`	Name of dependent variable
Functions			
`e(sample)`	Marks estimation sample		
Matrices			
`e(b)`	Coefficient vector	`e(table)`	Matrix containing coefficients with their asymptotic standard errors, test statistics, p-values, and confidence intervals
`e(BCI)`	Bootstrap confidence intervals	`e(V)`	Asymptotic variance-covariance matrix of the coefficient vector
`e(ec)`	Error correction coefficients	`e(ec_rsq)`	Group-specific error correction regression r-squared and adjusted r-squared
`e(ec_#)`	Group-specific error correction regression coefficients with their standard errors, test statistics, p-values, and confidence intervals		

4 Replicating Chudik, Pesaran, and Smith (2023a)

Using the `xtpb` command, we replicate the PB estimation results for the consumption function empirical example by CKP featuring OECD countries. This application was originally considered by Pesaran, Shin, and Smith (1999). The consumption function specification is given by

$$\Delta c_{it} = d_i - \alpha_i \left(c_{i,t-1} - b_1 y^d_{i,t-1} - b_2 \pi_{i,t-1} \right) + \delta_{i1} \Delta y^d_{it} + \delta_{i2} \Delta \pi_{it} + v_{it}$$

for country $i = 1, 2, ..., n$, where c_{it} is the logarithm of real consumption per capita, y^d_{it} is the logarithm of real per capita disposable income, and π_{it} is the rate of inflation. The data is taken from Pesaran, Shin, and Smith (1999). It consists of $n = 24$ countries and a slightly unbalanced time period covering 1960-1993, with $T_{\min} = 32$ and $T_{\max} = 33$. We replicate the PB estimation results reported in Table 3 of CKP. We start by computing original PB estimates of long run coefficients, b_1 and b_2, without any bias correction and using standard asymptotic critical values for inference:

```
. use pBewley_data_file.dta, clear

. xtset country time

Panel variable: country (strongly balanced)
 Time variable: time, 1960 to 1993
         Delta: 1 unit

. xtpb con infl inc, lagorder(1)

Pooled Bewley Estimation of Long-Run Relationship in Dynamic Heterogenous Pane
> l
------------------------------------------------------------------------------
> -
Group variable: country                     Total number of observations = 791
                                            Number of groups = 24
                                            Obs per group: min = 32
                                                           max = 33
                                                           avg = 32.958333
Original Pooled Bewley (PB) estimator without bias correction.
Inference below conducted based on asymptotic standard errors.
------------------------------------------------------------------------------
         con | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        infl |  -.1341816   .0642033    -2.09   0.037    -.2600178   -.0083453
         inc |   .9122914   .0343783    26.54   0.000     .8449112    .9796717
------------------------------------------------------------------------------

Bootstrap confidence intervals (CIs) were not computed.
To compute bootstrap CIs, use option ´bootstrap()´ and associated suboptions.
-----------------------------------------------------------------------------
```

Next, we compute jackknife bias-corrected PB estimates of long-run coefficients by using the option `biascorrect(`*jackknife*`)`. In addition, we also compute bootstrapped confidence intervals by using the option `bootstrap`. Regarding the bootstrapping algorithm, we follow the same choices as in CKP. Specifically, the suboptions `btx(`*varx*`)` and `btx_lagorder(2)` specify that bootstrap replications for regressors are generated based on country-specific VAR models (7) with $\psi_{xy,ij} = \mathbf{0}$ and lag order $p = 2$. The suboption `csrobust` ensures that bootstrap confidence intervals are robust to arbitrary cross section dependence of errors. The number of bootstrap replications is set to 10,000 within the `bootstrap` option.

```
. xtpb con inc infl, lagorder(1) biascorrect(jackknife) bootstrap(10000, csrob
> ust btx(varx) btx_lagorder(2) seed(123456))

Pooled Bewley Estimation of Long-Run Relationship in Dynamic Heterogenous Pane
> l
------------------------------------------------------------------------------
> -
Group variable: country                     Total number of observations= 791
                                            Number of groups = 24
                                            Obs per group: min = 32
                                                           max = 33
                                                           avg = 32.958333

jackknife bias-corrected Pooled Bewley (PB) estimator.
Inference below conducted based on asymptotic standard errors.
```

```
-----------------------------------------------------------------------------
         con | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+---------------------------------------------------------------
         inc |   .9257776   .0405451    22.83   0.000     .8463106    1.005245
        infl |  -.1199915   .0861496    -1.39   0.164    -.2888415    .0488586
-----------------------------------------------------------------------------
Computing bootstrap CIs based on 10000 bootstrap replications...
Bootstrapping progress:
----------10%
----------20%
----------30%
----------40%
----------50%
----------60%
----------70%
----------80%
----------90%
----------Complete!

95 percent bootstrapped confidence intervals:
         inc |  .8348745   1.016681
        infl | -.3447268   .1047439
Bootstrapping allowed for arbitrary cross sectional dependence of errors.
To change confidence interval coverage, use suboption ´bcialpha()´ of option ´
> bootstrap()´
----------------------------------------------------------------------------
```

To replicate the PB estimates in the bottom part of Table 3 in CKP, we compute simulation-based bias-corrected PB estimates with 95 percent bootstrapped confidence intervals by specifying the option `biascorrect(`*bootstrap*`)`.

```
. xtpb con inc infl, lagorder(1) biascorrect(bootstrap) bootstrap(10000, csrob
> ust btx(varx) btx_lagorder(2) seed(123456))
Bootstrapping progress:
----------10%
----------20%
----------30%
----------40%
----------50%
----------60%
----------70%
----------80%
----------90%
----------Complete!

Pooled Bewley Estimation of Long-Run Relationship in Dynamic Heterogenous Pane
> l
-----------------------------------------------------------------------------
> -
Group variable: country                    Total number of observations= 791
                                           Number of groups = 24
                                           Obs per group: min = 32
                                                          max = 33
                                                          avg = 32.958333

Bias-corrected Pooled Bewley (PB) estimator using stochastic simulations
based on 10000 replications.
Inference below conducted based on asymptotic standard errors.
-----------------------------------------------------------------------------
             |
```

```
         con | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .9209627   .0349202    26.37   0.000     .8525203    .9894051
        infl |  -.1245002   .0616862    -2.02   0.044    -.2454029   -.0035974
------------------------------------------------------------------------------

95 percent bootstrapped confidence intervals:
         inc |   .8296397   1.012286
        infl |  -.3137327   .0647324
Bootstrap CIs are based on 10000 bootstrap replications.
Simulations/bootstrapping allowed for arbitrary cross sectional dep. of errors
> .
To change confidence interval coverage, use suboption ´bcialpha()´ of option ´
> bootstrap()´
-----------------------------------------------------------------------------
```

These results replicate the PB estimates in CKP. Comparing both of the bias-corrected PB estimates with the original (uncorrected) PB estimates, we see that the latter are smaller, which suggests a presence of a small downward bias for both inflation and income long-run coefficients.

In this illustrative example, we estimated the long-run relationships between consumption, disposable income and inflation. Regardless of the bias-correction option chosen, the coefficients estimated are quite similar. These coefficients range from 0.912 to 0.926 for disposable income, indicating an increase in disposable income is associated with almost one-to-one increase in consumption in the long-run. The null hypothesis of unit long-run coefficient on income can be rejected at 5 percent nominal level according to the uncorrected PB estimates using asymptotic standard errors, but the same null hypothesis can no longer be rejected at 5 percent nominal level when using bias-corrected estimates and bootstrapped confidence intervals robust to cross section dependence of residuals. For inflation, the estimated long run coefficients range from -0.134 to -0.120, and, with the exception of the uncorrected PB estimates, are insignificant at the 5 percent nominal level, which is consistent with neutral monetary policy in the long-run.

To change the nominal level for the bootstrap confidence intervals, the suboption `bcialpha(#)` can be used. Additionally, the option `errorcorrect` generates a variable with the error corrections terms, `residuals` generates a variable with fitted value residuals, and `fulldisplay` displays all the regression outputs used to calculate the error corrections coefficients. For illustration purposes, the following example computes bias-corrected PB estimates using stochastic simulations and computes 80 percent bootstrap confidence intervals. Bootstrapping is conditional on regressors (regressors are fixed across bootstrap replications), and re-sampling of errors does not allow for cross-section dependence. The estimation output shown below is curtailed at group $i = 1$ for brevity.

```
. xtpb con inc infl, lagorder(1) biascorrect(bootstrap) bootstrap(2000, bcialp
> ha(0.8) seed(123456)) errorcorrect(error_correct) residuals(residuals) fulld
> isplay
Bootstrapping progress:
----------10%
----------20%
----------30%
```

```
----------40%
----------50%
----------60%
----------70%
----------80%
----------90%
----------Complete!

Pooled Bewley Estimation of Long-Run Relationship in Dynamic Heterogenous Pane
> l
------------------------------------------------------------------------------
> -
Group variable: country                     Total number of observations= 791
                                            Number of groups = 24
                                            Obs per group: min = 32
                                                           max = 33
                                                           avg = 32.958333

Bias-corrected Pooled Bewley (PB) estimator using stochastic simulations
based on 2000 replications.
Inference below conducted based on asymptotic standard errors.
------------------------------------------------------------------------------
         con | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         inc |   .9164835   .0346838    26.42   0.000     .8485045    .9844625
        infl |  -.1599615   .0696367    -2.30   0.022     -.296447    -.023476
------------------------------------------------------------------------------

80 percent bootstrapped confidence intervals:
         inc |    .887293    .945674
        infl |  -.2727019  -.0472212
Bootstrap CIs are based on 2000 bootstrap replications.
Simulations/bootstrapping assumed no cross sectional dependence of errors.
To allow for arbitrary cross-sectional dependence, use suboption ´csrobust´ of
>  option ´bootstrap()´
To change confidence interval coverage, use suboption ´bcialpha()´ of option ´
> bootstrap()´
-----------------------------------------------------------------------------
Error correction estimation for panel variable group: 1
------------------------------------------------------------------------------
       D.con | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
error_corr~t |
         L1. |  -.0945969     .03584    -2.64   0.013    -.1680118    -.021182
             |
         inc |
         D1. |   .3109306   .0434062     7.16   0.000     .2220171    .3998441
             |
        infl |
         D1. |   .0295758   .0775029     0.38   0.706    -.1291817    .1883333
             |
       _cons |   .0147791   .0015504     9.53   0.000     .0116031     .017955
------------------------------------------------------------------------------
```

5 Conclusion

As argued by Chudik, Pesaran, and Smith (2023a), the main advantage of the PB estimator is that it can exhibit a better small sample performance in the relevant sample sizes of interest compared with the existing alternatives in the literature. This property, together with availability of bootstrapped confidence intervals robust to cross-sectional dependence of errors, makes `xtpb` a useful addition to the literature. However, none of the existing approaches in the literature for the estimation of long-run relationships in dynamic heterogeneous panels are, to the best of our knowledge, applicable to the case where $T/N \to 0$, which is an empirically a very important setting to consider in future work.

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