Federal Reserve Bank of Dallas
Globalization and Monetary Policy Institute
Working Paper No. 190
http://www.dallasfed.org/assets/documents/institute/wpapers/2014/0190.pdf

Technical Note on "Assessing Bayesian Model Comparison in Small Samples"*

Enrique Martínez-García
Federal Reserve Bank of Dallas

Mark A. Wynne
Federal Reserve Bank of Dallas

August 2014

Abstract

This technical note is developed as a companion to the paper 'Assessing Bayesian Model Comparison in Small Samples' (Globalization and Monetary Policy Institute working paper no. 189). Taking the workhorse open-economy model of Martínez-García and Wynne (2010) with nominal rigidities under monopolistic competition as our Data-Generating Process, we investigate with simulated data how Bayesian model comparison based on posterior odds performs when the model becomes arbitrarily close to a closed-economy and/or an economy with flexible prices and perfect competition. This technical note elaborates on three key technical points relevant for Martínez-García and Wynne (2014). First, we explain the building blocks of the open-economy model of Martínez-García and Wynne (2010). We also derive the equilibrium conditions (and the steady state) under producer-currency pricing. Second, we discuss the log-linearization of the equilibrium conditions around the deterministic steady state and our benchmark parameterization. The linear rational expectations model that results from the log-linearization is used to simulate the data under our benchmark parameterization. These simulated data is used in Martínez-García and Wynne (2014) to conduct their Bayesian model comparison exercises. Third, we describe the Bayesian estimation and model comparison techniques with special emphasis on the questions of: (a) how we elicit priors on the models themselves and the parameters of a given model, and (b) how we compute posterior model probabilities. Simultaneously, commentary is provided whenever appropriate to clarify the economic significance of the assumptions embedded in our workhorse open-economy model.

JEL codes: C11, C13, F41

---

* Enrique Martínez-García, Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201. 214-922-5262. enrique.martinez-garcia@dal.frb.org. Mark A. Wynne, Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201. 214-922-5159. Mark.A.Wynne@dal.frgb.org. We would like to thank Nathan Balke, María Teresa Martínez-García and Valentín Martínez Mira for helpful suggestions. Diego Vilán was a co-author in a related project and contributed to the early stages of development of this paper. We gratefully acknowledge the outstanding research assistance provided by Valerie Grossman, the help of Kuhu Parasrampuria, and the Federal Reserve Bank of Dallas support. The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

1 New Open Economy Macro (NOEM) Model

The NOEM model that we use in Martínez-García and Wynne (2014) is a variant of the workhorse model of Clarida et al. (2002) introduced in Martínez-García and Wynne (2010). This is a symmetric two-country model with a continuum of unit mass of households and consumption varieties, equally divided between the Home country and the Foreign country. We employ this framework because it integrates an open-economy New Keynesian Phillips curve that fleshes out the content of the *global slack hypothesis* into a stylized dynamic stochastic general equilibrium model, but also because it nests model specifications without nominal rigidities (monetary neutrality) and/or under autarky (closed-economies) as limiting cases.

We abstract from a number of relevant modelling features like capital and investment (see, e.g., Chari et al. (2002), and Martínez-García and Søndergaard (2008)), durable goods (see, e.g., Engel and Wang (2011)), and monopolistically competitive suppliers of labor (see, e.g., Clarida et al. (2002)) in order to assess Bayesian model comparison solely in the presence of effects from increased trade openness and monetary non-neutrality induced by nominal rigidities.

Households. The lifetime utility for the representative household in the Home country is additively separable in consumption, C_t, and labor, L_t, i.e.,

$$\sum_{\tau=0}^{+\infty} \beta^{\tau} \mathbb{E}_t \left[\ln\left(C_{t+\tau}\right) - \frac{1}{1+\varphi} \left(L_{t+\tau}\right)^{1+\varphi} \right], \tag{1}$$

and similarly the lifetime utility for the representative household in the Foreign country is additively separable in consumption, C_t^*, and labor, L_t^*, i.e.,

$$\sum_{\tau=0}^{+\infty} \beta^{\tau} \mathbb{E}_t \left[\ln\left(C_{t+\tau}^*\right) - \frac{1}{1+\varphi} \left(L_{t+\tau}^*\right)^{1+\varphi} \right], \tag{2}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. The inverse of the intertemporal elasticity of substitution is equal to 1 under the assumption of log-utility on consumption.

The Home household maximizes its lifetime utility subject to the sequence of budget constraints,

$$P_t C_t + \int_{\omega_{t+1} \in \Omega} Q_t\left(\omega_{t+1}\right) B_t\left(\omega_{t+1}\right) \leq B_{t-1}\left(\omega_t\right) + W_t L_t + D_t - T_t, \tag{3}$$

where W_t is the nominal wage in the Home country, P_t is the Home consumption price index (CPI), T_t is a nominal lump-sum tax from the Home government, and D_t are (per-period) nominal profits from all firms producing the Home varieties of goods. The Home budget constraint includes purchases of a portfolio of one-period Arrow-Debreu securities (contingent bonds) internationally traded and in zero net supply, $B_t\left(\omega_{t+1}\right)$. For simplicity, these contingent bonds are quoted in the unit of account of the Home country.

Similarly, the Foreign household maximizes its lifetime utility subject to the sequence of budget constraints,

$$P_t^* C_t^* + \frac{1}{S_t} \int_{\omega_{t+1} \in \Omega} Q_t\left(\omega_{t+1}\right) B_t^*\left(\omega_{t+1}\right) \leq \frac{1}{S_t} B_{t-1}^*\left(\omega_t\right) + W_t^* L_t^* + D_t^* - T_t^*, \tag{4}$$

where W_t^* is the nominal wage in the Foreign country, P_t^* is the Foreign consumption price index (CPI),

T_t^* is a nominal lump-sum tax from the Foreign government, and D_t^* are (per-period) nominal profits from all firms producing the Foreign varieties of goods. The Foreign budget constraint includes purchases of a portfolio of one-period Arrow-Debreu securities (contingent bonds) internationally traded and in zero net supply, $B_t^*(\omega_{t+1})$. The Home price of the contingent bonds that pay-off in state $\omega_{t+1} \in \Omega$ at time $t+1$ is denoted $Q_t(\omega_{t+1})$, while the corresponding Foreign price is $Q_t^*(\omega_{t+1}) = \frac{1}{S_t} Q_t(\omega_{t+1})$ and S_t is the nominal exchange rate.

The contingent bond market clearing conditions can be summarized as,

$$\frac{1}{2}\left[B_t(\omega_{t+1}) + B_t^*(\omega_{t+1})\right] = 0, \ \forall \omega_{t+1} \in \Omega. \tag{5}$$

Access to a full set of internationally-traded, one-period Arrow-Debreu securities completes the local and international asset markets recursively. In other words, a full set of internationally-traded, one-period Arrow-Debreu securities suffices to guarantee complete asset markets both within a country and internationally. In this setting under complete asset markets, households can perfectly share risks domestically and internationally. Hence, the intertemporal marginal rate of substitution is equalized across countries and in every state of nature, i.e.

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \frac{P_t}{P_{t+1}} = \beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-1} \frac{P_t^* S_t}{P_{t+1}^* S_{t+1}}. \tag{6}$$

We define the real exchange rate as $RS_t \equiv \frac{S_t P_t^*}{P_t}$, so by backward recursion the *perfect international risk-sharing condition* in (6) becomes,

$$RS_t = \upsilon \left(\frac{C_t^*}{C_t}\right)^{-1}, \tag{7}$$

where $\upsilon \equiv \frac{S_0 P_0^*}{P_0}\left(\frac{C_0^*}{C_0}\right)$ is a constant that depends on initial conditions. If the initial conditions correspond to the symmetric steady state, then the constant υ is equal to one.

We can also price a redundant one-period, uncontingent nominal bond for each country in its corresponding unit of account with the price of the contingent Arrow-Debreu securities and obtain a standard pair of stochastic Euler equations for both countries, i.e.

$$\frac{1}{1+i_t} = \int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1}) = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-1} \frac{P_t}{P_{t+1}}\right], \tag{8}$$

$$\frac{1}{1+i_t^*} = \int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1}) = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-1} \frac{P_t^*}{P_{t+1}^*}\right], \tag{9}$$

where i_t is the riskless, nominal interest rate in the Home country and i_t^* is the riskless, nominal interest rate in the Foreign country. The household's optimization problem also results in a pair of labor supply equations,

$$\frac{W_t}{P_t} = (C_t)(L_t)^{\varphi}, \tag{10}$$

$$\frac{W_t^*}{P_t^*} = (C_t^*)(L_t^*)^{\varphi}, \tag{11}$$

plus the appropriate transversality conditions on the portfolio of contingent bonds and the respective budget constraints of Home and Foreign households in (3) and (4).

C_t is a CES aggregator of Home and Foreign goods for the representative Home country household defined as,

$$C_t = \left[(1-\xi)^{\frac{1}{\sigma}} \left(C_t^H\right)^{\frac{\sigma-1}{\sigma}} + (\xi)^{\frac{1}{\sigma}} \left(C_t^F\right)^{\frac{\sigma-1}{\sigma}} \right], \tag{12}$$

where $\sigma > 0$ is the elasticity of substitution between the Home-produced consumption bundle C_t^H and the Foreign-produced consumption bundle C_t^F. Analogous preferences are assumed for the Foreign country representative household, except that C_t^* is defined as a CES aggregator of Home and Foreign goods in the following terms,

$$C_t^* = \left[(\xi)^{\frac{1}{\sigma}} \left(C_t^{H*}\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}} \left(C_t^{F*}\right)^{\frac{\sigma-1}{\sigma}} \right]. \tag{13}$$

The share of imported goods in the Home consumption basket and in the Foreign basket must satisfy $0 \leq \xi \leq \frac{1}{2}$. Clarida et al. (2002)—among others—make the assumption that the consumption baskets of both countries are identical, giving the same weight to Home-produced and Foreign-produced goods. In turn, differences in the basket of consumption goods across countries exist in this set-up under most parameterizations—except in the knife-edge case where $\xi = \frac{1}{2}$. Moreover, in the limiting case where ξ becomes arbitrarily close to 0 we approximate autarky, the solution of the closed-economy model where international spillovers arise solely through the exogenous covariance of shock innovations across countries.

The sub-indexes C_t^H and C_t^{H*} indicate respectively Home and Foreign consumption of the bundle of differentiated varieties produced in the Home country. Similarly, C_t^F and C_t^{F*} denote Home and Foreign consumption of the bundle of differentiated varieties produced in the Foreign country. These sub-indexes are defined as follows,

$$C_t^H = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_0^{\frac{1}{2}} C_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^F = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_{\frac{1}{2}}^1 C_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{14}$$

$$C_t^{H*} = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_0^{\frac{1}{2}} C_t^*(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^{F*} = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_{\frac{1}{2}}^1 C_t^*(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{15}$$

where $\theta > 1$ is the elasticity of substitution among differentiated varieties within a country. Similarly, output and labor are expressed as,

$$\frac{1}{2}Y_t = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_0^{\frac{1}{2}} Y_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad \frac{1}{2}Y_t^* = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_{\frac{1}{2}}^1 Y_t^*(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{16}$$

$$\frac{1}{2}L_t = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_0^{\frac{1}{2}} L_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad \frac{1}{2}L_t^* = \left[\left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \int_{\frac{1}{2}}^1 L_t^*(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{17}$$

where Y_t and Y_t^* denote the total output per-household produced by firms in the Home and Foreign countries respectively, while L_t and L_t^* refer to the per-household total labor employed.

The CPIs that correspond to this specification of consumption preferences are,

$$P_t = \left[(1-\xi)\left(P_t^H\right)^{1-\sigma} + \xi\left(P_t^F\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{18}$$

$$P_t^* = \left[\xi\left(P_t^{H*}\right)^{1-\sigma} + (1-\xi)\left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{19}$$

and,

$$P_t^H = \left[2\int_0^{\frac{1}{2}} P_t(h)^{1-\theta}\, dh\right]^{\frac{1}{1-\theta}},\ P_t^F = \left[2\int_{\frac{1}{2}}^1 P_t(f)^{1-\theta}\, df\right]^{\frac{1}{1-\theta}}, \tag{20}$$

$$P_t^{H*} = \left[2\int_0^{\frac{1}{2}} P_t^*(h)^{1-\theta}\, dh\right]^{\frac{1}{1-\theta}},\ P_t^{F*} = \left[2\int_{\frac{1}{2}}^1 P_t^*(f)^{1-\theta}\, df\right]^{\frac{1}{1-\theta}}, \tag{21}$$

where P_t^H and P_t^F are the price sub-indexes for the Home-produced and Foreign-produced bundles of varieties in the Home market. The Home and Foreign price of the Home-produced variety h is given by $P_t(h)$ and $P_t^*(h)$, respectively. This is similar for the sub-indexes P_t^{H*} and P_t^{F*} in the Foreign market and for the prices $P_t(f)$ and $P_t^*(f)$ of the Foreign-produced variety f.

Firms. Each firm supplies the Home and Foreign markets with its own differentiated variety under monopolistic competition. Clarida et al. (2002) make the assumption of producer currency pricing (PCP), which we also adopt here. Hence, firms set Home and Foreign prices (invoicing local sales and exports) in their local currency. The PCP assumption implies that the law of one price (LOOP) holds at the variety level (i.e. $P_t(h) = S_t P_t^*(h)$ and $P_t(f) = S_t P_t^*(f)$), so it follows that $P_t^H = S_t P_t^{H*}$ and $P_t^F = S_t P_t^{F*}$. However, the assumption of Home-product bias in consumption preferences (which introduces differences in the consumption baskets across countries) leads to deviations from purchasing power parity (PPP) in the model whenever $\xi \neq \frac{1}{2}$. For this reason, $P_t \neq S_t P_t^*$ and so the real exchange rate deviates from one (i.e., $RS_t \equiv \frac{S_t P_t^*}{P_t} \neq 1$). More specifically, the CPI-based real exchange rate (RER) can be expressed as,

$$\begin{aligned} RS_t &\equiv \frac{S_t P_t^*}{P_t} = \frac{S_t\left[\xi\left(P_t^{H*}\right)^{1-\sigma} + (1-\xi)\left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[(1-\xi)\left(P_t^H\right)^{1-\sigma} + \xi\left(P_t^F\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \\ &= \left[1-(1-2\xi)\left(\frac{1-\left(\frac{P_t^F}{P_t^H}\right)^{1-\sigma}}{(1-\xi)+\xi\left(\frac{P_t^F}{P_t^H}\right)^{1-\sigma}}\right)\right]^{\frac{1}{1-\sigma}}, \end{aligned} \tag{22}$$

where the second equality shows that the RER, RS_t, is equal to one only if $\xi = \frac{1}{2}$; otherwise, is a function of the Home terms of trade defined as the price of imports relative to the price of exports expressed in units of the domestic currency, i.e. $ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}} = \frac{P_t^F}{P_t^H}$.

We can relate the CPI of both countries in (18) – (19) to the terms of trade $ToT_t \equiv \frac{P_t^F}{P_t^H}$ using the LOOP as follows,

$$P_t = \left[(1-\xi)\left(P_t^H\right)^{1-\sigma} + \xi\left(P_t^F\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = P_t^H\left[(1-\xi)+\xi\left(\frac{P_t^F}{P_t^H}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{23}$$

$$\begin{aligned} P_t^* &= \left[\xi\left(P_t^{H*}\right)^{1-\sigma} + (1-\xi)\left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = P_t^{F*}\left[\xi\left(\frac{P_t^{H*}}{P_t^{F*}}\right)^{1-\sigma} + (1-\xi)\right]^{\frac{1}{1-\sigma}} \\ &= P_t^{F*}\left[\xi\left(\frac{S_tP_t^{H*}}{S_tP_t^{F*}}\right)^{1-\sigma} + (1-\xi)\right]^{\frac{1}{1-\sigma}} = P_t^{F*}\left[\xi\left(\frac{1}{\frac{P_t^F}{P_t^H}}\right)^{1-\sigma} + (1-\xi)\right]^{\frac{1}{1-\sigma}}, \end{aligned} \tag{24}$$

where the price sub-index for the Home-produced bundles of varieties in the Home market, P_t^H, and the price sub-index for the Foreign-produced bundles of varieties in the Foreign market, P_t^{F*}, correspond in this environment to the GDP deflator—or equivalently the Producer Price Index (PPI)—of the Home and Foreign countries respectively. Hence, the difference between the rate of CPI inflation and the GDP deflator inflation (or PPI inflation) is a function solely of the terms of trade, $ToT_t \equiv \frac{P_t^F}{P_t^H}$.

Given household's preferences, we derive the demand for any Home variety h and for any Foreign variety f as,

$$\begin{aligned} Y_t(h) &= \frac{1}{2}C_t(h) + \frac{1}{2}C_t^*(h) \\ &= \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}\left\{\left(\frac{P_t^H}{P_t}\right)^{-\sigma}\left[(1-\xi)C_t + \xi\left(\frac{1}{RS_t}\right)^{-\sigma}C_t^*\right]\right\},\ \forall h \in \left[0,\frac{1}{2}\right], \end{aligned} \tag{25}$$

$$\begin{aligned} Y_t^*(f) &= \frac{1}{2}C_t(f) + \frac{1}{2}C_t^*(f) \\ &= \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta}\left\{\left(\frac{P_t^F}{P_t}\right)^{-\sigma}\left[\xi C_t + (1-\xi)\left(\frac{1}{RS_t}\right)^{-\sigma}C_t^*\right]\right\},\ \forall f \in \left(\frac{1}{2},1\right]. \end{aligned} \tag{26}$$

Firms maximize profits subject to a partial adjustment rule on nominal prices at the variety level à la Calvo (1983). In each period, every firm receives with probability $0 < \alpha < 1$ a signal to maintain their prices and with probability $1-\alpha$ a signal to re-optimize. Hence, in the limiting case where α becomes arbitrarily close to 0, we approximate the flexible price allocation and, therefore, the allocation solution that would arise were nominal rigidities (the origin of the monetary non-neutrality) to be absent.

A re-optimizing Home firm in any given period chooses a price $\widetilde{P}_t(h)$ optimally to maximize the expected discounted value of its corresponding profits, i.e.,

$$\sum\nolimits_{\tau=0}^{+\infty}\mathbb{E}_t\left\{(\alpha\beta)^\tau\left(\frac{C_{t+\tau}}{C_t}\right)^{-1}\frac{P_t}{P_{t+\tau}}\left[\widetilde{Y}_{t,t+\tau}(h)\left(\widetilde{P}_t(h) - (1-\phi)MC_{t+\tau}\right)\right]\right\}, \tag{27}$$

subject to the constraint of always satisfying the demand given by (25) at the chosen price $\widetilde{P}_t(h)$ for as long as that price remains unchanged. The demand schedule $\widetilde{Y}_{t,t+\tau}(h)$ indicates the total consumption demand of variety h at time $t+\tau$ whenever the prevailing prices are unchanged since time t, i.e. whenever prices are $P_{t+\tau}(h) = \widetilde{P}_t(h)$. Analogously, a re-optimizing Foreign firm in any given period chooses a price $\widetilde{P}_t^*(f)$

optimally to maximize the expected discounted value of its corresponding profits, i.e.,

$$\sum\nolimits_{\tau=0}^{+\infty} \mathbb{E}_t \left\{ (\alpha\beta)^{\tau} \left(\frac{C^*_{t+\tau}}{C^*_t} \right)^{-1} \frac{P^*_t}{P^*_{t+\tau}} \left[\widetilde{Y}^*_{t,t+\tau}(f) \left(\widetilde{P}^*_t(f) - (1-\phi) MC^*_{t+\tau} \right) \right] \right\}, \tag{28}$$

subject to the constraint of always satisfying the demand given by (26) at the chosen price $\widetilde{P}^*_t(f)$ for as long as that price remains unchanged. The demand schedule $\widetilde{Y}^*_{t,t+\tau}(f)$ indicates the total consumption demand of variety f at time $t+\tau$ whenever the prevailing prices are unchanged since time t, i.e. whenever prices are $P_{t+\tau}(f) = \widetilde{P}^*_t(f)$.

The government of each country raises lump-sum taxes from its local households in order to subsidize labor employment. We introduce the time-invariant labor subsidy ϕ as proportional to the nominal marginal cost. Firms produce their own varieties subject to a linear-in-labor technology. Labor is assumed to be immobile across countries. However, we assume perfectly competitive local labor markets—instead of monopolistically competitive suppliers of labor as in Clarida et al. (2002)—and homogeneity of the labor input in each national labor market. These assumptions ensure that wages equalize within a country. Hence, the (before-subsidy) nominal marginal costs are given by,

$$MC_t \equiv \left(\frac{W_t}{A_t} \right), \ MC^*_t \equiv \left(\frac{W^*_t}{A^*_t} \right), \tag{29}$$

where MC_t and MC^*_t are respectively the Home and Foreign (before-subsidy) nominal marginal costs. Home and Foreign nominal wages are denoted by W_t and W^*_t, while Home and Foreign productivity shocks are A_t and A^*_t respectively. The stochastic process for aggregate productivity in each country evolves according to the following bivariate autoregressive process,

$$\begin{aligned} \begin{pmatrix} \ln A_t \\ \ln A^*_t \end{pmatrix} &= \begin{pmatrix} \delta_a & 0 \\ 0 & \delta_a \end{pmatrix} \begin{pmatrix} \ln A_{t-1} \\ \ln A^*_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon^a_t \\ \varepsilon^{a*}_t \end{pmatrix}, & (30) \\ \begin{pmatrix} \varepsilon^a_t \\ \varepsilon^{a*}_t \end{pmatrix} &\sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_a & \rho_{a,a^*}\sigma^2_a \\ \rho_{a,a^*}\sigma^2_a & \sigma^2_a \end{pmatrix} \right). & (31) \end{aligned}$$

The Home and Foreign productivity shock innovations are labeled ε^a_t and ε^{a*}_t respectively. We assume a common volatility $\sigma^2_a > 0$, a common autoregressive parameter $-1 < \delta_a < 1$ and allow the cross-correlation of innovations between the two countries to be $-1 < \rho_{a,a^*} < 1$.

Given the inherent symmetry of the Calvo-type pricing scheme, the price sub-indexes P^H_t and P^{F*}_t evolve according to the following pair of equations,

$$\begin{aligned} \left(P^H_t\right)^{1-\theta} &= \alpha \left(P^H_{t-1}\right)^{1-\theta} + (1-\alpha)\left(\widetilde{P}_t(h)\right)^{1-\theta} = \left(S_t P^{H*}_t\right)^{1-\theta}, & (32) \\ \left(P^{F*}_t\right)^{1-\theta} &= \alpha \left(P^{F*}_{t-1}\right)^{1-\theta} + (1-\alpha)\left(\widetilde{P}^*_t(f)\right)^{1-\theta} = \left(\frac{P^F_t}{S_t}\right)^{1-\theta}. & (33) \end{aligned}$$

The price sub-indexes, P^{H*}_t and P^F_t, follow from the LOOP condition. The optimal pricing rule of the re-optimizing Home firms at time t is given by,

$$\widetilde{P}_t(h) = \left(\frac{\theta}{\theta-1}(1-\phi)\right) \frac{\sum_{\tau=0}^{+\infty} (\alpha\beta)^{\tau} \mathbb{E}_t \left[\left(\frac{1}{P_{t+\tau}C_{t+\tau}}\right) \widetilde{Y}_{t,t+\tau}(h)\, MC_{t+\tau}\right]}{\sum_{\tau=0}^{+\infty} (\alpha\beta)^{\tau} \mathbb{E}_t \left[\left(\frac{1}{P_{t+\tau}C_{t+\tau}}\right) \widetilde{Y}_{t,t+\tau}(h)\right]}, \tag{34}$$

while the optimal pricing rule of the re-optimizing Foreign firms at time t is,

$$\widetilde{P}_t^*(f) = \left(\frac{\theta}{\theta-1}(1-\phi)\right) \frac{\sum_{\tau=0}^{+\infty} (\alpha\beta)^{\tau} \mathbb{E}_t \left[\left(\frac{1}{P_{t+\tau}^*C_{t+\tau}^*}\right) \widetilde{Y}_{t,t+\tau}^*(f)\, MC_{t+\tau}^*\right]}{\sum_{\tau=0}^{+\infty} (\alpha\beta)^{\tau} \mathbb{E}_t \left[\left(\frac{1}{P_{t+\tau}^*C_{t+\tau}^*}\right) \widetilde{Y}_{t,t+\tau}^*(f)\right]}. \tag{35}$$

Even absent nominal rigidities, the market structure of monopolistic competition on the supply-side introduces a mark-up between prices and marginal costs, $\frac{\theta}{\theta-1} > 1$. This mark-up term is constant and a function of the elasticity of substitution across varieties within a country, $\theta > 1$.[1]

We choose an identical optimal labor subsidy $\phi = \frac{1}{\theta}$ in both countries to neutralize the monopolistic competition mark-up wedge $\frac{\theta}{\theta-1}$, as indicated before. The labor subsidy is funded with lump-sum taxes raised on the local households, and ensures the allocation attained by the model under flexible prices replicates the one of an economy with flexible pricing and perfectly competitive firms. The government budget constraint of each country then becomes,

$$\frac{1}{2}\left[T_t + \phi MC_t\right] = 0, \tag{36}$$

$$\frac{1}{2}\left[T_t^* + \phi MC_t^*\right] = 0, \tag{37}$$

which shows that the labor subsidy is financed with the non-distortionary, lump-sum taxes on Home and Foreign Households, T_t and T_t^*. The government in each country has no other tax instruments, does not borrow to fund its operations and does not consume or invest. Hence, the government budget constraint is also balanced in every period after collecting the revenues from the households and paying-off the labor subsidies.

Monetary Policy. We model monetary policy in the Home and Foreign countries according to Taylor (1993)-type rules on the short-term nominal interest rates, i_t and i_t^*, i.e.,

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\rho_i} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{1+\psi_\pi} \left(\frac{Y_t}{\overline{Y}_t}\right)^{\psi_x}\right]^{1-\rho_i} e^{\varepsilon_t^m}, \tag{38}$$

$$\frac{1+i_t^*}{1+\bar{i}^*} = \left(\frac{1+i_{t-1}^*}{1+\bar{i}^*}\right)^{\rho_i} \left[\left(\frac{\Pi_t^*}{\overline{\Pi}^*}\right)^{1+\psi_\pi} \left(\frac{Y_t^*}{\overline{Y}_t^*}\right)^{\psi_x}\right]^{1-\rho_i} e^{\varepsilon_t^{m*}}, \tag{39}$$

[1] Monopolistic competition introduces a mark-up over marginal costs that is a function of the elasticity of substitution across varieties within a country, θ. The mark-up is the only place where the parameter θ shows up in the model up to a first-order approximation, so the optimal labor subsidy ϕ which neutralizes the mark-up distortion also makes the pair (ϕ, θ) irrelevant for the characterization of the steady state and the dynamics of the economy.

where ε_t^m and ε_t^{m*} are the Home and Foreign monetary policy shocks. The monetary shocks in each country are given by,

$$\left(\begin{array}{c} \varepsilon_t^m \\ \varepsilon_t^{m*} \end{array} \right) \sim N\left(\left(\begin{array}{c} 0 \\ 0 \end{array} \right), \left(\begin{array}{cc} \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{array} \right) \right). \tag{40}$$

We assume cross-correlation in the innovations between the two countries $-1 < \rho_{m,m^*} < 1$ and a common volatility $\sigma_m^2 > 0$.

The policy parameters $\psi_\pi > 0$ and $\psi_x > 0$ represent the sensitivity of the monetary policy rule to movements in inflation and the output gap respectively, while $0 < \rho_i < 1$ represents the smoothing parameter, and $\bar{i}$ and $\bar{i}^*$ are the steady state Home and Foreign nominal interest rates. $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ are the (gross) CPI inflation rates, while $\overline{\Pi}$ and $\overline{\Pi}^*$ are the corresponding steady state inflation rates. Y_t and Y_t^* define the per-household output levels, while $\overline{Y}_t$ and $\overline{Y}_t^*$ are the potential per-household output levels that monetary policy tracks—potential output being defined as the output level that would prevail if all frictions could be eliminated, that is, in a frictionless economy with perfectly competitive firms and flexible prices. The ratios $\frac{Y_t}{\overline{Y}_t}$ and $\frac{Y_t^*}{\overline{Y}_t^*}$ define the output gap for the Home and Foreign country. These indexes of monetary policy take the form of the standard policy rule postulated by Taylor (1993) once they are log-linearized.

Steady State We characterize a deterministic, zero-inflation steady state assuming that $\overline{\Pi} = \overline{\Pi}^* = 1$ and $S = 1$. Moreover, we define a symmetric steady state where $\overline{P} = \overline{P}^H = \overline{\widetilde{P}}(h)$ and $\overline{P}^* = \overline{P}^{F*} = \overline{\widetilde{P}}^*(f)$, so that $\overline{RS} = \overline{TOT} = 1$ and $\overline{C}^* = \overline{C}$. With the optimal labor subsidy in place, the mark-up distortion from monopolistic competition is eliminated and does not affect the steady state. The steady state is thus determined by the interest rates $1 + \bar{i} = 1 + \bar{i}^* = \frac{1}{\beta}$ and the following allocation of resources: $\overline{Y} = \overline{L} = \overline{Y}(h) = \overline{L}(h) = \overline{C} = 1$ and $\overline{Y}^* = \overline{L}^* = \overline{Y}^*(f) = \overline{Y}^*(f) = \overline{C}^* = 1$. Steady state real wages are equal to $\frac{\overline{W}}{\overline{P}} = \frac{\overline{W}^*}{\overline{P}^*} = 1$, the CPIs equalize across countries (i.e. $\overline{P} = \overline{P}^*$), and the consumption of each variety is given in the Home country by $\overline{C}(h) = 2\overline{C}^H$, $\overline{C}^H = (1-\xi)\overline{C}$, $\overline{C}(f) = 2\overline{C}^F$, $\overline{C}^F = \xi\overline{C}^*$ and in the Foreign country by $\overline{C}^*(h) = 2\overline{C}^{H*}$, $\overline{C}^{H*} = \xi\overline{C}^*$, $\overline{C}^*(f) = 2\overline{C}^{F*}$, $\overline{C}^{F*} = (1-\xi)\overline{C}^*$.

2 Solution Method and Parameterization

2.1 Solution Method

We derive the deterministic, zero-inflation steady state, and then log-linearize the equilibrium conditions around that steady state. This local approximation is accurate for arbitrarily small exogenous shocks that are bounded within a neighborhood of the steady state. We use this log-linear approximation of the workhorse NOEM model discussed in the previous section and summarized in Table 1 as our Data-Generating Process (DGP) (which we refer to as model M_1) in Martínez-García and Wynne (2014). We solve the corresponding linear rational expectations model based on the generalized Schur decomposition method (see, e.g., Villemot (2011)), as implemented by the software package Dynare described in Adjemian et al. (2011).

For model comparison, we consider different nested variants of the log-linearized NOEM model (model M_1) that we report here in Table 2.A (the M_2 International Real Business Cycle model, $\alpha \to 0$), in Table 2.B (the M_3 closed-economy New Keynesian model, $\xi = 0$), and in Table 2.C (the M_4 closed-economy Real Business Cycle model, $\xi = 0$ and $\alpha \to 0$). We simulate the full model over $11,000$ periods, and drop the first $1,000$ observations of each series to exclude any effect of the initial conditions on the simulation. We retain $10,000$ periods of this single simulation to explore the effect of sample size, but we also select three shorter sub-samples of 160 observations each. The short sub-samples correspond to 40 years of quarterly data and are meant to capture a time series of international macro data of a length that is large by the standards of applied work but not implausible. The long sample corresponds to an unrealistic time series of $2,500$ years of quarterly data that suffices to illustrate the asymptotic properties of the Bayesian model comparison method that we investigate in this paper.

Whenever it is pertinent to simulate the model under different parameterizations, we maintain invariant the realization of the exogenous shocks and the strategy to select a long sample and three shorter sub-samples solely varying the relevant structural parameters of the model.

Table 1 - New Open-Economy Macro (NOEM) Model (M_1)

	Home Economy
Phillips curve	$\widehat{\pi}_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}\right) + ...$ $\left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right) \left[(1-\xi) \left(\varphi + \left(\frac{\sigma-(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \right) \widehat{x}_t + \xi \left(\varphi + \left(\frac{\sigma+(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \right) \widehat{x}^*_t \right]$
Output gap	$(1-2\xi) \left(\mathbb{E}_t \left[\widehat{x}_{t+1} \right] - \widehat{x}_t \right) \approx$ $(1-\xi) \left(\sigma - (\sigma-1)(1-2\xi) \right) \left[\widehat{r}_t - \widehat{\overline{r}}_t \right] - \xi \left(\sigma + (\sigma-1)(1-2\xi) \right) \left[\widehat{r}^*_t - \widehat{\overline{r}}^*_t \right]$
Output	$\widehat{y}_t = \widehat{\overline{y}}_t + \widehat{x}_t$
Monetary policy	$\widehat{i}_t \approx \rho_i \widehat{i}_{t-1} + (1-\rho_i) \left[(1+\psi_\pi) \widehat{\pi}_t + \psi_x \widehat{x}_t \right] + \widehat{\varepsilon}^m_t$
GDP deflator	$\widehat{\pi}^H_t \approx \widehat{\pi}_t - \xi \Delta \widehat{tot}_t$
Fisher equation	$\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t \left[\widehat{\pi}_{t+1} \right]$
Natural interest rate	$\widehat{\overline{r}}_t \approx (1-\xi) \left(\frac{\sigma-(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1} \right] - \widehat{\overline{y}}_t \right) + \xi \left(\frac{\sigma+(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \left(\mathbb{E}_t \left[\widehat{\overline{y}}^*_{t+1} \right] - \widehat{\overline{y}}^*_t \right)$
Potential output	$\widehat{\overline{y}}_t \approx \left(1 + (\sigma-1) \left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1} \right) \right) \widehat{a}_t - (\sigma-1) \left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1} \right) \widehat{a}^*_t$
	Foreign Economy
Phillips curve	$\widehat{\pi}^*_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}^*_{t+1}\right) + ...$ $\left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right) \left[\xi \left(\varphi + \left(\frac{\sigma+(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \right) \widehat{x}_t + (1-\xi) \left(\varphi + \left(\frac{\sigma-(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \right) \widehat{x}^*_t \right]$
Output gap	$(1-2\xi) \left(\mathbb{E}_t \left[\widehat{x}^*_{t+1} \right] - \widehat{x}^*_t \right) \approx$ $-\xi \left(\sigma + (\sigma-1)(1-2\xi) \right) \left[\widehat{r}_t - \widehat{\overline{r}}_t \right] + (1-\xi) \left(\sigma - (\sigma-1)(1-2\xi) \right) \left[\widehat{r}^*_t - \widehat{\overline{r}}^*_t \right]$
Output	$\widehat{y}^*_t = \widehat{\overline{y}}^*_t + \widehat{x}^*_t$
Monetary policy	$\widehat{i}^*_t \approx \rho_i \widehat{i}^*_{t-1} + (1-\rho_i) \left[(1+\psi_\pi) \widehat{\pi}^*_t + \psi_x \widehat{x}^*_t \right] + \widehat{\varepsilon}^{m*}_t$
GDP deflator	$\widehat{\pi}^{F*}_t \approx \widehat{\pi}^*_t + \xi \Delta \widehat{tot}_t$
Fisher equation	$\widehat{r}^*_t \equiv \widehat{i}^*_t - \mathbb{E}_t \left[\widehat{\pi}^*_{t+1} \right]$
Natural interest rate	$\widehat{\overline{r}}^*_t \approx \xi \left(\frac{\sigma+(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1} \right] - \widehat{\overline{y}}_t \right) + (1-\xi) \left(\frac{\sigma-(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \left(\mathbb{E}_t \left[\widehat{\overline{y}}^*_{t+1} \right] - \widehat{\overline{y}}^*_t \right)$
Potential output	$\widehat{\overline{y}}^*_t \approx -(\sigma-1) \left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1} \right) \widehat{a}_t + \left(1 + (\sigma-1) \left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1} \right) \right) \widehat{a}^*_t$
	International Relative Prices and Trade
Terms of trade	$\widehat{tot}_t \approx \left(\frac{1}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \left(\widehat{y}_t - \widehat{y}^*_t \right)$
Real exchange rate	$\widehat{rs}_t \approx (1-2\xi) \widehat{tot}_t$
Real trade balance	$\widehat{tb}_t \equiv \widehat{y}_t - \widehat{c}_t \approx \xi \left(\frac{\sigma+(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2} \right) \left(\widehat{y}_t - \widehat{y}^*_t \right)$
	Exogenous, Country-Specific Shocks
Productivity shock	$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}^*_t \end{pmatrix} \approx \begin{pmatrix} \delta_a & 0 \\ 0 & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_a & \rho_{a,a^*} \sigma^2_a \\ \rho_{a,a^*} \sigma^2_a & \sigma^2_a \end{pmatrix} \right)$
Monetary shock	$\begin{pmatrix} \widehat{\varepsilon}^m_t \\ \widehat{\varepsilon}^{m*}_t \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_m & \rho_{m,m^*} \sigma^2_m \\ \rho_{m,m^*} \sigma^2_m & \sigma^2_m \end{pmatrix} \right)$

Table 2.A - International Real Business Cycle (IRBC) Model (M_2)	
Home Economy	
Output gap	$\widehat{x}_t \approx 0$
Output	$\widehat{y}_t \approx \left(1+(\sigma-1)\left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1}\right)\right)\widehat{a}_t-(\sigma-1)\left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1}\right)\widehat{a}_t^*$
Monetary policy	$\widehat{i}_t \approx \rho_i \widehat{i}_{t-1}+(1-\rho_i)\left[(1+\psi_\pi)\widehat{\pi}_t+\psi_x\widehat{x}_t\right]+\widehat{\varepsilon}_t^m$
GDP deflator	$\widehat{\pi}_t^H \approx \widehat{\pi}_t-\xi\Delta\widehat{tot}_t$
Fisher equation	$\widehat{r}_t \equiv \widehat{i}_t-\mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$
Natural interest rate	$\widehat{r}_t \approx (1-\xi)\left(\frac{\sigma-(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2}\right)\left(\mathbb{E}_t\left[\widehat{y}_{t+1}\right]-\widehat{y}_t\right)+\xi\left(\frac{\sigma+(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2}\right)\left(\mathbb{E}_t\left[\widehat{y}_{t+1}^*\right]-\widehat{y}_t^*\right)$
Foreign Economy	
Output gap	$\widehat{x}_t^* \approx 0$
Output	$\widehat{y}_t^* \approx -(\sigma-1)\left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1}\right)\widehat{a}_t+\left(1+(\sigma-1)\left(\frac{2\xi(1-\xi)}{\varphi\left(\sigma-(\sigma-1)(1-2\xi)^2\right)+1}\right)\right)\widehat{a}_t^*$
Monetary policy	$\widehat{i}_t^* \approx \rho_i \widehat{i}_{t-1}^*+(1-\rho_i)\left[(1+\psi_\pi)\widehat{\pi}_t^*+\psi_x\widehat{x}_t^*\right]+\widehat{\varepsilon}_t^{m*}$
GDP deflator	$\widehat{\pi}_t^{F*} \approx \widehat{\pi}_t^*+\xi\Delta\widehat{tot}_t$
Fisher equation	$\widehat{r}_t^* \equiv \widehat{i}_t^*-\mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]$
Natural interest rate	$\widehat{r}_t^* \approx \xi\left(\frac{\sigma+(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2}\right)\left(\mathbb{E}_t\left[\widehat{y}_{t+1}\right]-\widehat{y}_t\right)+(1-\xi)\left(\frac{\sigma-(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2}\right)\left(\mathbb{E}_t\left[\widehat{y}_{t+1}^*\right]-\widehat{y}_t^*\right)$
International Relative Prices and Trade	
Terms of trade	$\widehat{tot}_t \approx \left(\frac{1}{\sigma-(\sigma-1)(1-2\xi)^2}\right)\left(\widehat{y}_t-\widehat{y}_t^*\right)$
Real exchange rate	$\widehat{rs}_t \approx (1-2\xi)\,\widehat{tot}_t$
Real trade balance	$\widehat{tb}_t \equiv \widehat{y}_t-\widehat{c}_t \approx \left(1-(1-\xi)\left(\frac{\sigma-(\sigma-1)(1-2\xi)}{\sigma-(\sigma-1)(1-2\xi)^2}\right)\right)\left(\widehat{y}_t-\widehat{y}_t^*\right)$
Exogenous, Country-Specific Shocks	
Productivity shock	$\begin{pmatrix}\widehat{a}_t\\ \widehat{a}_t^*\end{pmatrix} \approx \begin{pmatrix}\delta_a & 0\\ 0 & \delta_a\end{pmatrix}\begin{pmatrix}\widehat{a}_{t-1}\\ \widehat{a}_{t-1}^*\end{pmatrix}+\begin{pmatrix}\widehat{\varepsilon}_t^a\\ \widehat{\varepsilon}_t^{a*}\end{pmatrix}$ $\begin{pmatrix}\widehat{\varepsilon}_t^a\\ \widehat{\varepsilon}_t^{a*}\end{pmatrix} \sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix},\begin{pmatrix}\sigma_a^2 & \rho_{a,a^*}\sigma_a^2\\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2\end{pmatrix}\right)$
Monetary shock	$\begin{pmatrix}\widehat{\varepsilon}_t^m\\ \widehat{\varepsilon}_t^{m*}\end{pmatrix} \sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix},\begin{pmatrix}\sigma_m^2 & \rho_{m,m^*}\sigma_m^2\\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2\end{pmatrix}\right)$

Table 2.B - New Keynesian (NK) Closed-Economy Model (M_3)	
	Home Economy
Phillips curve	$\widehat{\pi}_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}\right) + \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)(1+\varphi)\,\widehat{x}_t$
Output gap	$\mathbb{E}_t\left[\widehat{x}_{t+1}\right] - \widehat{x}_t \approx \widehat{r}_t - \widehat{\overline{r}}_t$
Output	$\widehat{y}_t = \widehat{\overline{y}}_t + \widehat{x}_t$
Monetary policy	$\widehat{i}_t \approx \rho_i \widehat{i}_{t-1} + (1-\rho_i)\left[(1+\psi_\pi)\,\widehat{\pi}_t + \psi_x \widehat{x}_t\right] + \widehat{\varepsilon}_t^m$
GDP deflator	$\widehat{\pi}_t^H = \widehat{\pi}_t$
Fisher equation	$\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$
Natural interest rate	$\widehat{\overline{r}}_t \approx \mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}\right] - \widehat{\overline{y}}_t$
Potential output	$\widehat{\overline{y}}_t \approx \widehat{a}_t$
	Foreign Economy
Phillips curve	$\widehat{\pi}_t^* \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^*\right) + \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)(1+\varphi)\,\widehat{x}_t^*$
Output gap	$\mathbb{E}_t\left[\widehat{x}_{t+1}^*\right] - \widehat{x}_t^* \approx \widehat{r}_t^* - \widehat{\overline{r}}_t^*$
Output	$\widehat{y}_t^* = \widehat{\overline{y}}_t^* + \widehat{x}_t^*$
Monetary policy	$\widehat{i}_t^* \approx \rho_i \widehat{i}_{t-1}^* + (1-\rho_i)\left[(1+\psi_\pi)\,\widehat{\pi}_t^* + \psi_x \widehat{x}_t^*\right] + \widehat{\varepsilon}_t^{m*}$
GDP deflator	$\widehat{\pi}_t^{F*} = \widehat{\pi}_t^*$
Fisher equation	$\widehat{r}_t^* \equiv \widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]$
Natural interest rate	$\widehat{\overline{r}}_t^* \approx \mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}^*\right] - \widehat{\overline{y}}_t^*$
Potential output	$\widehat{\overline{y}}_t^* \approx \widehat{a}_t^*$
	International Relative Prices and Trade
Terms of trade	$\widehat{tot}_t \approx \widehat{y}_t - \widehat{y}_t^*$
Real exchange rate	$\widehat{rs}_t \approx (1-2\xi)\,\widehat{tot}_t$
Real trade balance	$\widehat{tb}_t \equiv \widehat{y}_t - \widehat{c}_t \approx 0$
	Exogenous, Country-Specific Shocks
Productivity shock	$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_a & 0 \\ 0 & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix}\right)$
Monetary shock	$\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{pmatrix}\right)$

Table 2.C - Real Business Cycle (RBC) Closed-Economy Model (M_4)	
	Home Economy
Output gap	$\widehat{x}_t \approx 0$
Output	$\widehat{y}_t \approx \widehat{a}_t$
Monetary policy	$\widehat{i}_t \approx \rho_i \widehat{i}_{t-1} + (1-\rho_i)\left[(1+\psi_\pi)\widehat{\pi}_t + \psi_x \widehat{x}_t\right] + \widehat{\varepsilon}_t^m$
GDP deflator	$\widehat{\pi}_t^H = \widehat{\pi}_t$
Fisher equation	$\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$
Natural interest rate	$\widehat{r}_t \approx \mathbb{E}_t\left[\widehat{y}_{t+1}\right] - \widehat{y}_t$
	Foreign Economy
Output gap	$\widehat{x}_t^* \approx 0$
Output	$\widehat{y}_t^* \approx \widehat{a}_t^*$
Monetary policy	$\widehat{i}_t^* \approx \rho_i \widehat{i}_{t-1}^* + (1-\rho_i)\left[(1+\psi_\pi)\widehat{\pi}_t^* + \psi_x \widehat{x}_t^*\right] + \widehat{\varepsilon}_t^{m*}$
GDP deflator	$\widehat{\pi}_t^{F*} = \widehat{\pi}_t^*$
Fisher equation	$\widehat{r}_t^* \equiv \widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]$
Natural interest rate	$\widehat{r}_t^* \approx \mathbb{E}_t\left[\widehat{y}_{t+1}^*\right] - \widehat{y}_t^*$
	International Relative Prices and Trade
Terms of trade	$\widehat{tot}_t \approx \widehat{y}_t - \widehat{y}_t^*$
Real exchange rate	$\widehat{rs}_t \approx (1-2\xi)\,\widehat{tot}_t$
Real trade balance	$\widehat{tb}_t \equiv \widehat{y}_t - \widehat{c}_t \approx 0$
	Exogenous, Country-Specific Shocks
Productivity shock	$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_a & 0 \\ 0 & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix}\right)$
Monetary shock	$\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{pmatrix}\right)$

2.2 Parameterization

The parameterization of the NOEM model (model M_1) in Martínez-García and Wynne (2014), whose references we discuss here, is compactly summarized in Table 3. We fix the values of most (policy and structural) parameters, but we also consider a range of values for some key parameters that can influence the degree of openness of these economies as well as how close the monetary policy is to being optimal. We use this range of parameter values to generate simulated data from the NOEM model (our DGP process) along dimensions of the parameter space that make the implicit distribution of the endogenous variables increasingly closer to that arising from a closed-economy model and/or an economy with flexible pricing. We exploit that to investigate the power of conventional Bayesian techniques to help us select the correct DGP model from the observable data.

Structural (non-policy) parameters. We set the intertemporal discount factor β at 0.99 to attain an average yearly interest rate of 4% in steady state (i.e., we choose β to imply that $\left(\frac{1}{\beta}\right)^4 = 1.041$). We adopt the standard value of 0.75 for the degree of price stickiness α, implying an expected price duration of four quarters, to be consistent with the average duration in Chari et al. (2002) and the standard parameterization in the NOEM literature. The evidence surveyed by Taylor (1999) and more recently by Klenow and Malin

(2010) is consistent with the view that prices change on average closer to once a year. This parameter determines the degree of nominal rigidities of the economy, so we also consider an interval of values for α that spans its theoretical range (that is, the interval between 0 and 1).

We set the share of imported goods in the consumption basket ξ at 0.06 as our benchmark in order to obtain an average import share of 6% for the U.S. based on the U.S. and European trade data documented by Chari et al. (2002). This parameter determines the degree of openness of the economy, so we investigate the power of Bayesian model comparison over an interval of values of ξ that spans its theoretical range (that is, the interval between 0 and $\frac{1}{2}$).

Estimates of the Frisch elasticity of labor supply, $\frac{1}{\varphi}$, based on micro data are commonly below 0.5. The classical study of Pencavel (1986) reports a range of estimates going from 0 to 0.45, while Canzoneri et al. (2007) discuss a similar range from 0.05 to 0.35. In turn, macro studies often impose a value well-above 0.5—e.g., Rotemberg and Woodford (1998a) and Rotemberg and Woodford (1998b) argue for a value as high as 9.5. As a compromise, we set the Frisch elasticity of labor supply $\frac{1}{\varphi}$ at 0.5 so as not to depart too much from the micro estimates.

The elasticity of intratemporal substitution between Home and Foreign goods, σ, is also greatly debated. Based on empirical estimates of trade models, it is generally noted that plausible values of the U.S. elasticity of intratemporal substitution lie between 1 and 2. Here we borrow from the work of Backus et al. (1994) and Chari et al. (2002) by setting the elasticity σ to be equal to 1.5.

In the dynamics of the flexible price model (either open or closed to trade), the intertemporal discount factor, β, and the Calvo price stickiness parameter, α, are not present, as there is no Phillips curve relationship under monetary neutrality. In the dynamics of the closed economy model (with or without nominal rigidities), the share of imported goods in the consumption basket, ξ, is zero and hence drops out from the set of relevant parameters.

Policy parameters. We assume a partial-adjustment Taylor rule mechanism which introduces *intrinsic* inertia into the original rule proposed by Taylor (1993), as it is the most standard policy specification in the NOEM literature. We adopt the policy parameters estimated for the U.S. by Rudebusch (2006) under an analogous partial-adjustment specification. According to those estimates, we set the smoothing parameter ρ_i that determines the *intrinsic* inertia at 0.78, the response to inflation $(1+\psi_\pi)$ is chosen to be 1.33 and the response to the output gap ψ_x is equal to 1.29. We assume that the policy parameters are identical in both countries. We investigate Bayesian model comparison for a range of values of the policy parameter $(1+\psi_\pi)$, where ψ_π goes from 0 to 6. As ψ_π increases, this generates endogenous dynamics increasingly close to those arising from a flexible price model where monetary policy has no real effects.

By definition, current output and potential output are the same object in the flexible price economy, so there is no need for monetary policy to respond to the output gap as there will be no gap in that case. Hence, the policy parameter ψ_x becomes irrelevant for those specifications that assume flexible prices. Since the monetary policy rule responds solely to country-specific macro aggregates (inflation and the output gap) in the open-economy case, the policy rule does not have to change in the specification of the closed-economy case.

Parameters of the shock processes. For the parameterization of the VAR(1) productivity shock process, we follow Kehoe and Perri (2002) and Chari et al. (2002) which use data for the U.S. and a foreign aggregate

Table 3 - Model Parameterization			
Structural parameters		**Range**	**Value**
Non-policy parameters			
Intertemporal discount factor	β	$(0, 1)$	0.99
Inverse of the Frisch elasticity of labor supply	φ	$\mathbb{R}^+$	2
Elasticity of substitution btw. Home and Foreign bundles	σ	$\mathbb{R}^+$	1.5
Share of imports in the consumption basket	ξ	$(0, 0.5)$	0.06, range: $(0, 0.5)$
Calvo price stickiness parameter	α	$(0, 1)$	0.75, range: $(0, 1)$
Policy parameters			
Policy smoothing parameter	ρ_i	$(0, 1)$	0.78
Sensitivity to deviations from inflation target	ψ_π	$\mathbb{R}^+$	0.33, range: $(0, 6)$
Sensitivity to deviations from potential output target	ψ_x	$\mathbb{R}^+$	1.29
Shock parameters			
Persistence of the productivity shock	δ_a	$(-1, 1)$	0.95
Volatility of the productivity shock	σ_a	$\mathbb{R}^+$	0.7
Correlation btw. Home and Foreign productivity innovations	ρ_{a,a^*}	$(-1, 1)$	0.25
Volatility of the monetary policy shock	σ_m	$\mathbb{R}^+$	0.38
Correlation btw. Home and Foreign monetary innovations	ρ_{m,m^*}	$(-1, 1)$	0.5

Note: The elasticity of substitution between Home and Foreign bundles, θ, and the optimal labor subsidy for firms, ϕ, are among the parameters of the model that can affect the steady state as well as the log-linearized dynamics. However, these two parameters drop out entirely whenever an optimal labor subsidy is chosen where $\phi = 1/\theta$, as it happens in our specification of the model. For that reason, we do not include them in the table or discuss them further for parameterization or estimation purposes.

that bundles together 15 European countries, Canada and Japan in their estimates. Based on their work, we set the parameter δ_a at 0.95, the volatility σ_a is set at 0.7 and the correlation between domestic and foreign productivity innovations ρ_{a,a^*} at 0.25. We adopt the parameter values estimated for the U.S. by Rudebusch (2006) in setting $\sigma_m = 0.38$ for the volatility of the monetary shock in both countries. We complete the description of the parameters of the shock processes of the model by choosing the correlation between Home and Foreign monetary innovations ρ_{m,m^*} to be 0.5 as in Chari et al. (2002). Monetary and productivity innovations are assumed to be uncorrelated with each other, and we also rule out by construction the presence of spillovers between monetary and productivity shocks or across countries.

3 Bayesian Estimation and Model Comparison: Technical Aspects

In Martínez-García and Wynne (2014), Bayesian estimation and model comparison is implemented with the software package Dynare (see, e.g., Adjemian et al. (2011)). Model comparison over a collection of alternative specifications is based on posteriors odds tests and the estimation of a posterior density for each model. We compute the marginal density with a Laplace approximation. We assume a uniform prior over the four nested models considered in our comparison exercise.

Eliciting subjective priors. We adopt the subjective theory of Bayesian inference for the structural parameters of the model. What this entails is that we use prior distributions for the parameters that are *informative* to incorporate other sources of information and to reflect our current views on the parameters of the model (subjective priors) as it is conventionally done in the Bayesian estimation of NOEM models, rather than imposing e.g. *non-informative* priors.[2]

We only consider independent prior densities of the beta, gamma, inverse gamma, normal, and uniform distributions as well as the degenerate distribution that puts mass one on a single value. We choose among these priors because they are widely used in the Bayesian estimation literature. We assume that the prior mean is equal to the true value of the parameter in our parameterization, while we choose the shape and dispersion of the prior distribution to reflect the degree of uncertainty often associated with those parameters.

We maintain these prior distributions over the structural and policy parameters invariant in all our subsequent exercises of Bayesian model comparison, taking them as given. For those parameterizations for which we have simulated data over a range of values, we shift the prior mean with its true value at each point of the interval considered but keep the prior distribution itself and its dispersion otherwise unchanged for the purpose of Bayesian estimation and model comparison. By keeping the priors on the parameters invariant across modelling specifications, we make Bayesian comparisons and the implementation of Bayesian estimation more straightforward. All our prior distributions for the model parameters are summarized in Table 4.

As is conventionally done, we use a degenerate prior for the intertemporal discount factor β and fix it at 0.99 targeting an average yearly interest rate of 4% as in our benchmark parameterization. We choose a tight prior to recognize that the share of imported goods in the consumption basket, ξ, should not deviate too much from its prior average. We use a Beta distribution with a small standard deviation of 0.01. However, we investigate a range of values for this parameter over the $\left(0, \frac{1}{2}\right)$-interval and accordingly we set the prior mean to correspond to the true value used to simulate the data in each case.

We adopt the Gamma distribution centered around 2 for φ, but we impose a wide standard deviation of 2 to encompass the wide range of values considered as plausible in the NOEM literature. We also adopt the Gamma distribution centered around 1.5 for σ, with a wide standard deviation of 1. We adopt the Beta distribution centered around 0.75 for the Calvo parameter, α, and make the distribution tight with a standard deviation 0.07. We also explore a range of values for this parameter over the $(0, 1)$-interval and accordingly we set the prior mean to correspond to the true value used to simulate the data in each case.

We investigate a range of values for the policy parameter ψ_π from 0 to 6 and accordingly we set the prior mean to correspond to the true value used to simulate the data in each case. The prior mean of the sensitivity to deviations from potential output ψ_x is maintained at 1.29. We impose an Inverse Gamma distribution for both of them and select fairly wide priors with a standard deviation of 2 for each. Having imposed *intrinsic* inertia on the monetary policy rule, the parameter ρ_i ought to be positive and high in order to match the parsimonious interest rate movements that we observe in the actual data. We reflect this by selecting a Beta distribution centered around its parameterized value of 0.78 with a prior standard deviation equal to 0.1.

We adopt a Beta prior distribution for the persistence of the productivity shock, δ_a, with a prior mean of 0.95 and a prior standard deviation of 0.05—as there seems to be broad agreement that productivity is pretty persistent. The prior means of the productivity shock and monetary shock volatilities, σ_a and σ_m, are set at 0.7 and 0.38, respectively. We select an Inverse Gamma distribution to represent the prior distribution

[2] *Non-informative* priors are often improper in the sense that they are only defined up to a constant. While this does not pose a problem for Bayesian estimation *per se*, the undefined constant can create problems for comparison across models. We avoid this issue altogether by instead adopting *informative*, subjective priors for the structural parameters as part of our estimation strategy.

Table 4 - Prior Distributions				
Structural parameters	**Prior Density**	**Domain**	**Prior Mean**	**Prior Std. Dev.**
Non-policy parameters				
β	Fixed	–	0.99	–
φ	Gamma	$\mathbb{R}^+$	2	2
σ	Gamma	$\mathbb{R}^+$	1.5	1
ξ	Beta	$(0, 0.5)$	0.06, range: $(0, 0.5)$	0.01
α	Beta	$(0, 1)$	0.75, range: $(0, 1)$	0.07
Policy parameters				
ρ_i	Beta	$(0, 1)$	0.78	0.1
ψ_π	InvGamma	$\mathbb{R}^+$	0.33, range: $(0, 6)$	2
ψ_x	InvGamma	$\mathbb{R}^+$	1.29	2
Shock parameters				
δ_a	Beta	$(0, 1)$	0.95	0.05
σ_a	InvGamma	$\mathbb{R}^+$	0.7	2
ρ_{a,a^*}	Beta	$(0, 1)$	0.25	0.18
σ_m	InvGamma	$\mathbb{R}^+$	0.38	2
ρ_{m,m^*}	Beta	$(0, 1)$	0.5	0.22

Note: This table reports only the prior mean and prior standard deviation for each model parameter. For any plausible choice of these two moments of the prior there is a mapping onto the prior distribution parameters v and s that matches both of them and fully characterizes the prior distribution itself. For the Normal distribution, the mean is μ=v and the variance is σ^2=s^2. For the Beta distribution, the mean is μ=$v/(v+s)$ and the variance is σ^2=$vs/((v+s)^2(v+s+1))$. For the Gamma distribution, the mean is μ=vs and the variance is σ^2=vs^2. For the Uniform distribution, the upper and lower bound of the support are v and s respectively, while the mean is μ=$(v+s)/2$ and the variance is σ^2=$(v-s)^2/12$. For the Inverse Gamma distribution, the mean is μ=s/(v-1) and the variance is σ^2=$s^2/((v-1)^2(v-2))$.

of both volatility parameters and impose a large standard deviation of 2 on both monetary and productivity shock innovations leaving it up to the data to determine the contribution of each to explain the endogenous volatility of the observed data.

Finally, we select the Beta prior distribution for the cross-country correlation of innovations ρ_{a,a^*} and ρ_{m,m^*}. We choose rather diffuse priors for these cross-country correlations because these parameters of the shock processes can be crucial for the dynamics of the model, but their values are often greatly debated in the literature. We center ρ_{a,a^*} at 0.25 with a standard deviation of 0.18, and ρ_{m,m^*} at 0.5 with a standard deviation of 0.22.

Computing posterior model probabilities. We have a collection of $k \geq 2$ models each of which is fully-described with a parameterized joint probability density over the vector of observable variables Z, i.e.,

$$M_i = \{f_i(z \mid \theta_i) : \theta_i \in \Theta_i\}, \ \forall i = 1, ..., k, \tag{41}$$

where θ_i is the vector of unknown parameters of model i, Θ_i is the corresponding parameter space, $f_i(z \mid \theta_i)$ is its parameterized probability density function, and z is a given realization of the vector of observable variables Z. The log-likelihood function for model M_i is,

$$l_i(\theta_i) = \ln f_i(z^n \mid \theta_i) = \sum\nolimits_{j=1}^{n} \ln f_i(z_j \mid \theta_i), \ \forall i = 1, ..., k, \tag{42}$$

defined over n observations of the endogenous observable variables, i.e. $z^n = (z_1, ..., z_n)$.

For all models $i = 1, ..., k$, we assign prior probabilities to each, $\Pr(M_i)$, and prior probabilities to the parameters θ_i that characterize each model specification, $f_i(\theta_i)$. The posterior probability for any model M_i can be calculated using Bayes' Theorem as,

$$\Pr(M_i \mid Z^n = z^n) = \frac{f_i(z^n \mid M_i)\Pr(M_i)}{\sum_{p=1}^{k} f_p(z^n \mid M_p)\Pr(M_p)} = \frac{m_i \Pr(M_i)}{\sum_{p=1}^{k} m_p \Pr(M_p)}, \tag{43}$$

where

$$m_i \equiv f_i(z^n \mid M_i) = \int_{\theta_i} f_i(z^n \mid \theta_i) f_i(\theta_i) d\theta_i, \; \forall i = 1, ..., k, \tag{44}$$

is the marginal likelihood.

The Bayesian posterior odds for model M_1 versus the competing model M_i for any $i = 2, ..., k$ $(k \geq 2)$ is the product of the prior odds $\frac{\Pr(M_1)}{\Pr(M_i)}$ times the Bayes Factor B_{1i}, i.e.,

$$\frac{\Pr(M_1 \mid Z^n = z^n)}{\Pr(M_i \mid Z^n = z^n)} = B_{1i}\frac{\Pr(M_1)}{\Pr(M_i)}, \tag{45}$$

where the Bayes factor B_{1i} is the quotient of the marginal likelihoods of both models, i.e.

$$B_{1i} = \frac{m_1}{m_i}. \tag{46}$$

Then, it is possible to write the posterior model probability in terms of Bayes Factors as follows,

$$\begin{aligned}\Pr(M_i \mid Z^n = z^n) &= \frac{B_{i1} m_1 \Pr(M_i)}{\sum_{p=1}^{k} B_{p1} m_1 \Pr(M_p)} \\ &= \frac{e^{\ln B_{i1}} \Pr(M_i)}{\sum_{p=1}^{k} e^{\ln B_{p1}} \Pr(M_p)},\end{aligned} \tag{47}$$

and from here to obtain that,

$$\begin{aligned}\Pr(M_i \mid Z^n = z^n) &= \frac{e^{\ln B_{i1} + \ln \Pr(M_i)}}{\sum_{p=1}^{k} e^{\ln B_{p1} + \ln \Pr(M_p)}} \\ &= \frac{e^{\ln m_i + \ln \Pr(M_i) - \ln m_1}}{\sum_{p=1}^{k} e^{\ln m_p + \ln \Pr(M_p) - \ln m_1}} \\ &= \frac{e^{\ln m_i + \ln \Pr(M_i)}}{\sum_{p=1}^{k} e^{\ln m_p + \ln \Pr(M_p)}}.\end{aligned} \tag{48}$$

In order to compute the posterior model probabilities, we only need to specify the prior model probabilities $\Pr(M_i)$ and obtain the log-marginal densities (under the Laplace approximation in our case) $\ln m_i$ from the estimation of each model variant $i = 1, ...k$. Under our assumption of a uniform prior over the four nested models under consideration, the prior model probabilities drop out from (48) and we only need to recover the log-marginal densities in order to compute the posterior model probabilities.

The log-marginal densities are a standard by-product of Bayesian estimation with the software package Dynare (see, e.g., Adjemian et al. (2011)) under the Laplace approximation, so no further transformations are needed. Then, in Martínez-García and Wynne (2014) we simply apply the formula derived in equation (48) to the collection of models that we aim to compare in order to obtain the corresponding posterior model probabilities.

References

Adjemian, S., H. Bastani, M. Juillard, F. Mihoubi, G. Perendia, M. Ratto, and S. Villemot (2011). *Dynare: Reference Manual, Version 4*.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1994). Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? *American Economic Review 84*(1), 84–103.

Calvo, G. A. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics 12*(3), 383–398.

Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2007). The Cost of Nominal Rigidity in NNS Models. *Journal of Money, Credit and Banking 39*(7), 1563–1586.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002). Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates? *Review of Economic Studies 69*(3), 533–563.

Clarida, R., J. Galí, and M. Gertler (2002). A Simple Framework for International Monetary Policy Analysis. *Journal of Monetary Economics 49*(5), 879–904.

Engel, C. and J. Wang (2011). International Trade in Durable Goods: Understanding Volatility, Cyclicality, and Elasticities. *Journal of International Economics 83*(1), 37–52.

Kehoe, P. J. and F. Perri (2002). International Business Cycles with Endogenous Incomplete Markets. *Econometrica 70*(3), 907–928. May.

Klenow, P. J. and B. A. Malin (2010). Microeconomic Evidence on Price-Setting. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3, Chapter 6, pp. 231–284. Elsevier.

Martínez-García, E. and J. Søndergaard (2008). The Real Exchange Rate in Sticky Price Models: Does Investment Matter? *Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute Working Paper no. 17*(July).

Martínez-García, E. and M. A. Wynne (2010). The Global Slack Hypothesis. Federal Reserve Bank of Dallas *Staff Papers, 10*. September.

Martínez-García, E. and M. A. Wynne (2014). Assessing Bayesian Model Comparison in Small Samples. *Globalization and Monetary Policy Institute Working Paper no. 189*. August.

Pencavel, J. (1986). Labor Supply of Men: A Survey. In O. Ashenfelter and R. Layard (Eds.), *Handbook of Labor Economics. Volume I*, Chapter 1, pp. 3–102. Elsevier Science Publishers BV.

Rotemberg, J. J. and M. Woodford (1998a). An Optimization-Based Econometric Model for the Evaluation of Monetary Policy. In B. S. Bernanke and J. J. Rotemberg (Eds.), *NBER Macroeconomics Annual 1997, vol. 12*, Volume 12, pp. 297–346. Cambridge, Massachusetts, USA: MIT University Press.

Rotemberg, J. J. and M. Woodford (1998b). Interest Rate Rules in an Estimated Sticky Price Model. In J. B. Taylor (Ed.), *Monetary Policy Rules*. Chicago, Illinois, USA: University of Chicago Press.

Rudebusch, G. D. (2006). Monetary Policy Inertia: Fact or Fiction? *International Journal of Central Banking 2*(4), 85–135.

Taylor, J. B. (1993). Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Taylor, J. B. (1999). Staggered Price and Wage Setting in Macroeconomics. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics, Vol. 1B*, Volume 1B, Chapter 15, pp. 1009–1050. Amsterdam: Elsevier/North Holland.

Villemot, S. (2011). Solving Rational Expectations Models at First Order: What Dynare Does. *Dynare Working Paper Series no. 2, CEPREMAP, April.*