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THE FAIRNESS OF DISCOUNTING: A MAJORITY RULE APPROACH

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S. P. A. Brown*
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* S.P.A. Brown is a Senior Economist at the Federal Reserve Bank of Dallas.

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Abstract

A model of majority rule is developed in which each of a finite number of generations votes on a redistribution of income between itself and the other generations. In voting, each generation expresses tastes for its own income and for the equality of income across generations. The model is then used to derive the conditions under which discounting is justified -- namely those conditions for which the majority rule exhibits a positive marginal rate of time preference. It is demonstrated that when each generation is wealthier than those preceding it, the parameters representing the taste for income equality must be relatively high for the majority rule to exhibit a positive marginal rate of time preference.

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1. Introduction

Economists have long been concerned that discounting may be unfair to future generations. Early writers were concerned that discounting could result in too little capital being transferred forward to future generations.1/ More recently, the literature in environmental and natural resource economics has drawn attention to the possibility that discounting could result in an insufficient preservation of natural resources and environmental amenities for future generations.2/

Not all economists agree that discounting is unfair. In his defense of the use of discounting to evaluate intergenerational transfers, Gordon Tullock (1964) argued that one need not worry about the effect of discounting when evaluating intergenerational transfers because future generations are going to be wealthier than the present generation.

I take the approach that the fairness of discounting is a property of an ethically appealing intergenerational social welfare function or social choice rule. Specifically, discounting is justified if the selected social welfare function or social choice rule exhibits a positive marginal rate of time preference for the intergenerational distributions in question. In the context of my approach, Tullock's argument can be seen as the adoption of an ethically appealing social welfare function or social choice rule that exhibits a positive marginal rate of time preference for all income distributions in which each generation is wealthier than those preceding it.3/

In this article, I develop a model of intergenerational majority rule and use it to derive conditions under which discounting is justified by the expectation that each generation is wealthier than those preceding it. The model extends inquiry along a line that Koichi Hamada (1973) first explored in an intragenerational context. The model is a simple majority rule in which the equality of income across generations is a public good and each generation votes on the amount of the public good to be provided through tax-financed redistribution. In voting, each generation ranks the alternative distributions on the basis of its own utility. I then use a revealed preference technique on the resulting majority rule to determine what individual taste parameters will result in a positive marginal rate of time preference when each generation is wealthier than those preceding it.

A model of majority rule has analytical and ethical appeal: Interpersonal utility comparisons, which most economists consider objectionable, are eliminated. Discounting is not imposed on the social choice rule because each generation receives equal weight under majority voting. Nevertheless, the majority rule evaluates the distribution of income and exhibits a positive marginal rate of time preference under specific distributional and taste assumptions.

Knowing what tastes must be assumed to yield a majority rule that exhibits a positive marginal rate preference provides insight into the consistency between current tastes and Tullock's justification for discounting. Furthermore, this knowledge may reveal how future generations will view intergenerational allocation decisions that the present generation has justified on the basis of efficient discounting.

The remainder of the discussion is organized as follows: In section 2, I develop a model of intergenerational majority rule over the distribution of income. In section 3, I derive the tastes for which the majority rule will exhibit a positive marginal rate of time preference for all distributions in which each generation is wealthier than those preceding it. Section 4 is the conclusion.

2. Intergenerational Majority Rule Over the Distribution of Income

The model is a simple majority rule in which the equality of income across generations is a public good and each of a finite number of non-overlapping generations votes on the amount of the public good to be provided through redistribution. Inquiry is simplified -- but the essence of the intergenerational problem is retained -- if each generation is represented as a single individual.4/ Although majority rule does not maximize the utility of all generations, in voting, each generation ranks the alternative distributions of income on the basis of its own utility.

Differences in the desired quantity of the public good are resolved in a majority rule that maximizes the utility of the generation whose most preferred amount of the public good is median. This median generation is a future generation.

2.1 Individual utility

For n generations, the utility of each generation is described as a function of its own income and of the distribution of income across generations.

$$U_i = \gamma[\delta X^{-\rho} + (1-\delta)y_i^{-\rho}]^{-1/\rho} \quad \text{for } i = 1, 2, \ldots, n. \tag{1}$$

in which γ is a constant multiplier

δ is the intensity of desire for income equality,

X is a measure of the distribution of income across generations and is a public good as as conceived by Lester Thurow (1971),

$(1-\delta)$ is each generation's intensity of desire for its own income,

y_i is the income of the *i*th generation, and

$\rho = (1-\sigma)/\sigma$, where σ is the elasticity of substitution.5/

Individual utility is described with the common C.E.S. function to allow identification of the tastes for X and y_i in terms of the function's parameters.

2.2 Measuring the income distribution

A simple and general approach to measuring the equality of income is to take a weighted average of the incomes of all *n* generations:

$$X = \sum_{i=1}^{n} w_i y_i \tag{2}$$

in which $w_1 > w_2 > w_3 > \ldots > w_n$, $\sum_{i=1}^{n} w_i = 1$,

the income assignments are numbered such that $y_1 < y_2 < y_3 < \ldots < y_n$, and

the weights are invariant to any changes in income.6/

This measure of the income distribution has the properties usually associated with egalitarian ideals. In the absence of redistribution costs, a more equal distribution of a given amount of total income yields a higher value of X than does a less equal distribution of the same total income. Increasing the income of one generation without changing the income of another generation also increases X.

The measure also permits a trade-off between efficiency and equality. The exact trade-off is determined by the assignment of weights. X is affected equally by a one unit change in y_i and w_i/w_j unit change in y_j, provided that the rankings of y_i and y_j remain unaltered.

2.3 The price of equality -- redistribution

Increased provision of the public good requires a redistribution of income across generations. If each generation's endowment is taxed at the same rate to provide a pool from which income is distributed equally to all generations, the price that each generation faces for the public good does not vary as provision of the public good is changed.7/ Therefore, the opportunity locus facing each generation can be expressed follows:

$$P_i X + y_i = P_i X' + E_i \qquad \text{for } i = 1, 2, \ldots, n, \qquad (3)$$

in which P_i is the price of the public good to the *i*th generation, in terms of its own income,

X' is the endowment of the public good, and

E_i is generation *i*'s endowed income.

Figure 1 illustrates possible opportunity loci facing four different generations. Representative endowments of personal wealth for generations

A, B, C, and D are E_a, E_b, E_c, and E_d respectively. By definition, each generation has the same endowment of the public good, X'. The opportunity locus facing each generation passes through that generation's endowment, has a slope of $-P_i$ and terminates at the distribution in which all incomes are equal.8/ As shown, a generation with a greater endowment faces a higher price for the public good than does a generation with a smaller endowment.

A generation with less than the mean endowment could suffer reduced income as the provision of the public good is increased. Redistribution toward equality could reduce the average income. In the intragenerational context, the costs of redistribution are described as arising from a loss of market incentives to produce income. In this intergenerational model, redistribution toward equality reduces the average income because the shift of income from wealthier, future generations to the present involves foregoing some investment gains. A generation with an endowment below the income that each generation receives when income is distributed equally will receive greater income as provision of the public good is increased and, therefore, faces a negative price for the public good.

Interestingly, the price of the public good for each generation also reflects the weights used to construct the income distribution measure. The desire for a more equal distribution of income between generations is reflected by larger weights for low incomes and smaller weights for high incomes. In general, the more egalitarian the intent in assigning the weights, the smaller the value of X for a given distribution of income, and the lower the price of the public good.

2.4 Individual preferences over the distribution of income

In choosing the amount of the public good it most prefers, each generation maximizes its utility subject to the budget constraint that it faces. For each generation, maximization of (1) subject to (3) yields the following Kuhn-Tucker conditions for individual maximum utility:

$$\frac{\delta}{1-\delta}\left(\frac{y^*_i}{X^*_i}\right)^{1/\sigma} \geq P_i \text{, and} \tag{4}$$

$$\left[\frac{\delta}{1-\delta}\left(\frac{y^*_i}{X^*_i}\right)^{1/\sigma} - P_i\right](\mu^*-X^*_i) = 0, \tag{5}$$

in which y^*_i and X^*_i are the optimal quantities of y_i and X for the ith generation, and μ^* is the income that each generation receives when income is distributed equally.

An evaluation of the Kuhn-Tucker conditions reveals that the amount of the public good that each generation prefers depends on the price that it faces for the public good and on the tastes for X and y_i. These tastes can be expressed in terms of preferred income distributions. If $\delta/(1-\delta) < P_i$ is true, then the relative intensity of desire for income equality is less than the price that the ith generation faces for the public good, and that generation most prefers an income distribution in which y_i is greater than X -- a distribution with some inequality. On the other hand, if $\delta/(1-\delta) \geq$

P_i is true, then the relative intensity of desire for income equality is greater than, or equal to, the price that the ith generation faces for the public good, and that generation most prefers a distribution in which y_i equals X -- a distribution with complete equality.

Because all generations with an endowment less than μ^* face a negative price for X, these generations favor a redistribution of income to complete equality as a corner solution. Each generation with an endowment greater than μ^* may favor complete equality as a corner solution, complete equality as an interior solution, or less than complete equality as an interior solution.

2.5. Majority rule and the median generation

Given single-peaked preferences over the amount of the public good to be provided, majority rule maximizes the utility of the generation whose most preferred amount of the public good is median.9/ Therefore, evaluated for this median generation, the Kuhn-Tucker conditions (4) and (5) express the outcome of the majority rule. This median generation is a future generation with an endowment that is median or greater.

Evaluation of the Kuhn-Tucker conditions (4) and (5) for the median voter reveals that the greater the relative intensity of desire for X the more equal the income distribution selected by majority rule. Specifically, if the relative intensity of desire for income equality is less than the price that the median voter faces for the public good, majority rule results in inequality because the median generation most prefers a distribution in which its own income is greater than X. On the other hand, if the relative intensity of desire for income equality is greater than, or equal to, the price that the median voter faces for the public good, majority rule results in a completely equal distribution of income because the median generation most prefers a distribution in which its own income equals X.10/

Because all generations have identical tastes, the voter with median preferences for the amount of the public good to be provided can be identified by the price it faces for the public good.11/ Given the normal assumption that the substitution effect dominates the income effect, the amount of the public good most preferred declines as larger P_i are examined. In this case, the voter with median preferences for the public good faces the median price and has the median endowment. If the income effect dominates the substitution effect at sufficiently high values of P_i, the preferred amount of the public good rises with income and price. In this case, the voter with median preferences for the public good faces a higher than median price and has an endowment that is greater than median.12/

3. Tastes and Marginal Rates of Time Preference under Majority Rule

To investigate the relationship between individual tastes and a majority rule that exhibits a positive marginal rate of time preference for all distributions in which each generation is wealthier than those preceding it, I examine three cases. The cases are as follows:

<u>Case 1</u>. $\delta/(1-\delta) < P_m$ at a zero discount rate.

<u>Case 2</u>. $\delta/(1-\delta) > P_m$ at a zero discount rate.

<u>Case 3</u>. $\delta/(1-\delta) = P_m$ at a zero discount rate.

The difference in these cases are the assumed tastes for X and y_i. Cases 2 and 3 represent more egalitarian tastes than does case 1 because in these cases there is a greater preference for X over y_i. For each of these cases, I derive the marginal rates of time preference that the majority rule associates with various distributions of income. I then inspect the distributions and their associated marginal rates of time preference to determine for what taste assumptions the majority rule will exhibit a positive marginal rate of time preference for all distributions in which each generation is wealthier than those preceding it.

The indifference curve in Figure 2 illustrates the first case -- the one in which tastes are least egalitarian. Points A, B and C in Figure 2 represent three consumption combinations of y_m and X that are on the same indifference curve for the median voter. A unique distribution of income is associated with each point along the indifference curve. The income distributions associated with these combinations form a social choice contour because the median voter is indifferent between the combinations of y_m and X along the curve.

By construction, a zero discount rate underlies the opportunity locus tangent to the indifference curve at point A. Majority rule yields some inequality at a zero discount rate. Because this distribution resulted from a zero discount rate, for this distribution of income, the marginal rate of time preference of the majority rule must be zero.

Increasing the discount rate increases the price of X for the median voter.13/ And with the median voter facing a higher price for X, majority rule results in a smaller provision of X and a less equal distribution of income.14/ The difference between point B and point A in Figure 2 illustrates the effect of increasing the discount rate. A positive discount rate is required to make the distribution of income associated with point B the majority choice. Consequently, for this distribution of income, the marginal rate of time preference is positive.

Point C in Figure 2 is associated with a more equal distribution of income than is point A. A negative discount rate is required to make the distribution associated with point C the majority choice. Consequently, for this distribution of income, the marginal rate of time preference is negative.

Figure 3 presents an index of income equality and corresponding marginal rates of time preference. In Figure 3, income equality is measured by the index X/μ, where μ is the mean income. At its maximum value of one, the index denotes complete income equality. Points A, B and C in Figure 3(a) represent, respectively, the same distributions of income as points A, B and C in Figure 2.

If tastes are not very egalitarian, the majority rule does not exhibit a positive marginal rate of time preference for all distributions in which each generation is wealthier than those preceding it, as is shown in Figure 3(a). In case 1, zero and negative marginal rates of time preference are exhibited for some distributions in which each generation is wealthier than those preceding it.

If tastes are sufficiently egalitarian, the majority rule will exhibit a positive marginal rate of time preference for all distributions in which each generation is wealthier than those preceding it. As shown in Figure 3, cases 2 and 3 do represent sufficiently egalitarian tastes. For these cases, the median voter most prefers income equality when confronted with redistribution at a zero discount rate. In Figures 3(b) and 3(c), the effects of sufficiently egalitarian tastes are evidenced as a zero marginal rate of time preference at a completely equal distribution of income. For all distributions in which there is inequality, a positive marginal rate of time preference is exhibited.15/

These three cases indicate the potential importance of egalitarian tastes to Tullock's justification of discounting. If assumed tastes are not sufficiently egalitarian, the majority rule exhibits zero and negative marginal rates of time preference for some income distributions in which each generation is wealthier than those preceding it. Only if the assumed tastes are sufficiently egalitarian does the majority rule exhibit a positive marginal rate of time preference for all income distributions in which each generation is wealthier than those preceding it.

4. Conclusion: Egalitarian Tastes and The Fairness of Discounting

An intergenerational majority rule over the distribution of income is, of course, hypothetical. Most decisions regarding the intergenerational distribution of income must be made before most of the voters would be able to cast their ballots. Nevertheless, hypothetical majority rule may be useful as a guide to public policy. The present generation could be concerned with how future generations will view the fairness of current decisions.

The more egalitarian the tastes of future generations, the more fair they will consider increased consumption by the poorest generation -- which we assume is the present generation. Because future generations are our offspring, their tastes will be shaped by our tastes. The more egalitarian are the present generation's tastes, the more income it can justify to future generations in claiming for itself.

Discounting can be an expression of egalitarian tastes if each generation is wealthier than those preceding. Discounting gives greater weight to changes in the income of earlier generations than it does to changes in the income of later generations. In the model presented here, egalitarian tastes are required for discounting to be justified by the simple expectation that each generation is wealthier than those preceding it. Only if tastes are strongly egalitarian does the majority rule exhibit a positive marginal rate of time preference for all distributions in which each generation is wealthier than those preceding it.

Post Script: The Logic of Tullock's Position

The above exercises may have a direct bearing on the logical foundation of Professor Tullock's position. He argued that because future generations will be wealthier than the present generation, a positive social rate of discount is justified. This argument appeals to some underlying -- albeit unknown -- social choice rule. Given that egalitarian tastes underlie a simple majority rule that exhibits a positive marginal rate of time preference, it is tempting to conclude that Professor Tullock is very egalitarian. A more circumspect conclusion, however, would note that other social choice rules may not have the same implications.

NOTES

1. Those who have addressed this issue include F. P. Ramsey (1928), A.C. Pigou (1932), Stephen Marglin (1963) and Gordon Tullock (1964).
2. Those who have addressed this more recent issue include Robert Solow (1974), Talbot Page (1977), Martin J. Bailey (1979) and Robert A. Becker (1982).
3. In fairness to Professor Tullock, it should be noted that he was not attempting to develop a social welfare function, but merely pointing out that individuals would be unlikely to prefer a redistribution of income away from themselves to those with greater wealth.
4. This approach eliminates simultaneous consideration of the intergenerational and intragenerational aspects of the income distribution. In taking this approach, I ignore the possibility that the poorest individual to ever exist may be a member of the wealthiest generation.
5. The results obtained in a single good model can be generalized to a multiple good world. An efficiently composed bundle of goods for a given generation is obtained when the same discount rate is applied to all of the goods in that generation's bundle. S. P. A. Brown (1979) discusses this issue in further detail.
6. This construction of X also has the desirable property of making it linear in the y_i. Given the single parameter redistribution formula used below, a linear opportunity locus is obtained for each generation.

7. A single parameter redistribution formula is used because it results in non-cyclic voting. Given a restriction that the tax rate not exceed one, this redistribution formula yields a linear mapping between y_i and X for each generation. Single-peaked preferences result because each generation's utility function is convex to the origin.

   Under the single parameter redistribution formula the price of X is strictly a function of constants. Each generation receives an income of $y_i = t\mu^* + (1-t)E_i$; where t is the tax rate at which each generation's endowment is taxed to provide a pool for redistribution and μ^* is the income that each generation would receive when income is distributed equally. The amount of the public good corresponding to a given tax rate is $X = t\mu^* + (1-t)X'$; where X' is the endowment of the public good. The price of the public good for the ith generation is the net amount of its own wealth that it must sacrifice for one unit of the public good. Mathematically, the price of X for generation i is $P_i = -(\partial y_i/\partial t)/(\partial X/\partial t) = -(\mu^*-E_i)/(\mu^*-X')$.

   This construction assumes that the loss in total income resulting from redistribution is proportional to t. Also note that generation i faces a net tax rate of $t(1 - \mu^*/E_i)$ on its endowment.

8. At tax rates exceeding one, the relative ranking of income assignments is reversed, violating a necessary restriction that the relative ranking of each generation's income be preserved. This restriction does not impair the overall analysis because inquiry is directed at determining the fairness of discounting when each generation is wealthier than those preceding it.

Also note that redistribution toward the wealthy must end when the income of the poorest generation goes to zero. This occurs at a tax rate of $1 - \mu^*/(\mu^*-E_1)$.

9. Duncan Black (1958) has shown that single-peaked preferences result in a consistent majority rule in which the social choice is that of the median voter.

10. If the median voter has an endowment below μ^*, majority rule results in complete equality of income -- regardless of assumed tastes.

11. Under the budget constraint, the effects of differing endowments are incorporated as differing prices.

12. If $\sigma > 1$, then $\partial X^*/\partial P_i < 0$ for all P_i. If $\sigma < 1$, then $\partial X^*/\partial P_i$ is negative if P_i is sufficiently low for the generations with the greatest endowments and $\delta/(1-\delta)$ is sufficiently high. An elasticity of substitution of less than one can result in a negative relationship between X^* and P_i because $\partial X^*/\partial P_i$ is the rate of change in the price expansion path about the point (μ^*,μ^*) -- not a point for which y_i equals zero.

13. The chain rule is used to determine the sign of $\partial P_m/\partial r$. Because $\partial P_m/\partial \mu^*$ and $\partial \mu^*/\partial r$ are both negative, their product is positive.

    For the median generation, the derivative of price with respect to μ^* is $(X'-E_m)/(X'-\mu^*)^2$. Because the denominator of this expression is positive, the sign of $\partial P_i/\partial \mu^*$ depends on the sign of the numerator. It is reasonable to assume that the median generation's own income endowment (E_m) is greater than the endowment of the public good (X'). Therefore, the numerator and $\partial P_m/\partial \mu^*$ are negative.

That $\partial\mu^*/\partial r$ is negative is best explained intuitively. If the only cost of redistribution is foregone investment opportunities, μ^* is calculated as: $\mu^* = [\Sigma E_i(1+r)^{-i+1}] / [\Sigma(1+r)^{i+1}]$, where r is the discount rate and summation here is from 1 to n and over i. As the discount rate is increased, the amount of income transferred to earlier generations for a each dollar taken from a later generation is reduced. Because the incomes of earlier generations rise more slowly for a given reduction of later generation income, the income gap between generations is closed more slowly as t is increased. Therefore, more income must be taken from later generations to equalize the distribution of income and μ^* is reduced.

Formal proof that $\partial\mu^*/\partial r$ is negative is somewhat involved. In the interest of conserving space, the proof has been confined to an appendix that is available from the author.

14. For the median voter, $\partial(X^*_m/y^*_m)/\partial P_m < 0$.

15. If tastes are very egalitarian, however, positive marginal rates of time preference can also be associated with an equal distribution of income, as is shown in Figure 3(b). Nevertheless, once a high enough value of r is reached, higher marginal rates of time preference are associated with less equal distributions of income. In the third case, this threshold value occurs at a zero marginal rate of time preference.

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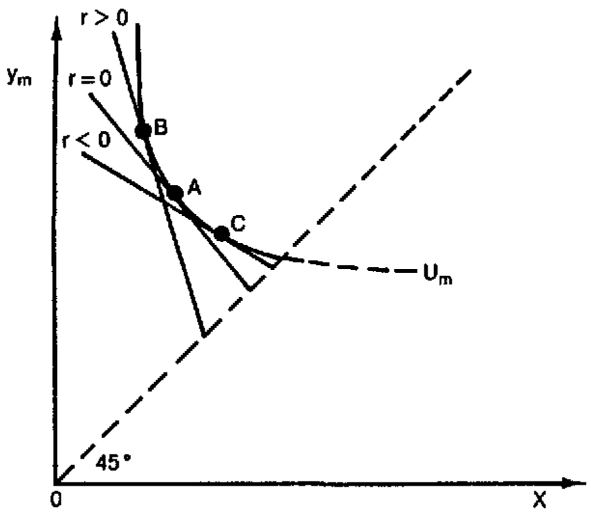
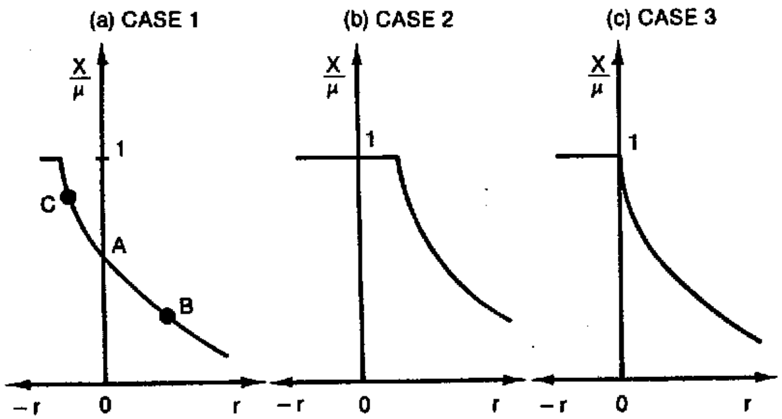


Figure 2 A utility contour of the median voter



(μ is the mean income)

Figure 3 Discount rates and optimal income distributions

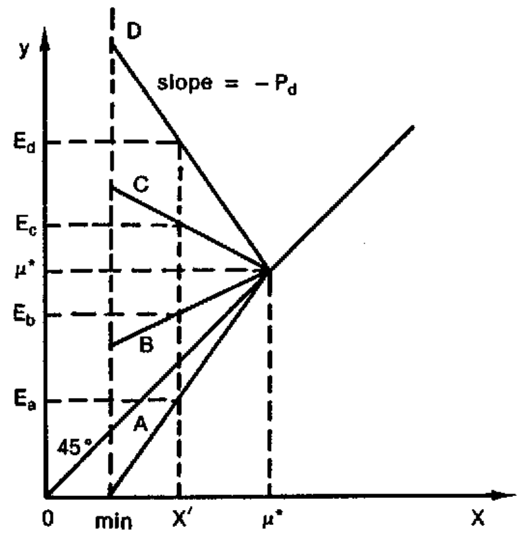


Figure 1 Four possible budget constraints