Federal Reserve Bank of Cleveland Working Paper Series

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Working Paper No. 24-26

November 2024

Suggested citation: Lamla, Michael J., Damjan Pfajfar, and Lea E. Rendell. 2024. "Inflation and Deflationary Biases in the Distribution of Inflation Expectations: Theory and Empirical Evidence from Nine Countries." Working Paper No. 24-26. Federal Reserve Bank of Cleveland. https://doi.org/10.26509/frbc-wp-202426.

Federal Reserve Bank of Cleveland Working Paper Series
ISSN: 2573-7953

Inflation and Deflationary Biases in the Distribution of Inflation Expectations: Theory and Empirical Evidence from Nine Countries*

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November 17, 2024

Abstract

We explore the consequences of losing confidence in the price stability objective of central banks by studying the resulting inflation and deflationary biases in medium-run inflation expectations. In a model with heterogeneous household perceptions of an occasionally binding zero-lower-bound constraint and of monetary policy objectives, we show that the estimated model-implied distribution of households' inflation expectations matches several characteristics of the empirical distribution when featuring *both* inflation and deflationary biases. We then directly identify these biases using unique individual-level survey data on medium-run inflation expectations across nine countries and over time. Both inflation and deflationary biases are important features of the distribution of medium-run inflation expectations.

Keywords: inflation bias, deflationary bias, confidence in central banks, effective lower bound, inflation expectations, microdata.

JEL classification: E31, E37, E58, D84.

*We thank Klaus Adam, Michael Ehrmann, Edith Liu, Taisuke Nakata, and Michael Weber for their comments and suggestions. We also thank seminar participants at the Swiss National Bank, Tilburg University, Bank for International Settlements, Federal Reserve Board, European Stability Mechanism, Virginia Commonwealth University, Macquarie University, Reserve Bank of Australia, CEBRA Annual Meeting (London), NY Fed-ECB Workshop on Expectations Surveys (New York), the CESifo SI workshop on Inflation Expectations (Venice), the SED Conference (Mexico City), EEA Annual Congress (Manchester), German Economic Association Meeting (Leipzig), EU@BBQ (Salzburg), ICARE (Essex), and the Workshop on Household Surveys in Macroeconomics (Hamburg) for their comments. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Cleveland or the Federal Reserve System. Previously, the paper was circulated as "Inflation and deflationary biases in inflation expectations." We are thankful to Ana Gil for sharing the data of the M&G YouGov Inflation Expectations Survey and for her support.

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1. Introduction

For the past four decades, central bank policymakers have been concerned with the potential existence of inflation bias due to a lack of confidence among economic agents that the central bank will achieve it's price stability objective in the medium to long run, in cases where central bankers were primarily focused on preventing high inflation. However, with the presence of the occasionally binding effective lower bound on interest rates (ZLB), the lack of confidence that the price stability objective will be achieved could manifest in lower-than-target long-run inflation expectations—deflationary bias—resulting in too low inflation rates, a concern that became equally important among policymakers in the aftermath of the financial crisis. During the period we study, a debate emerged over which bias may be the main concern, with the possibility that both biases may co-exist in individual expectations, which could have substantial implications for central banks.[1]

Despite the policy relevance, efforts to quantify both inflation and deflationary biases have been limited in the empirical literature due to identification issues, since relevant survey questions were not asked in standard surveys. The existing quantifications of the two biases mostly rely on model simulations or cross-country comparison of aggregate data. With regard to theory, the focus has been on homogeneous expectations and on modelling a single bias. The objective of this paper is to build a model that can exhibit both biases in the distribution of inflation expectations (heterogeneous expectations) and to empirically test for the potential co-existence of inflation and deflationary biases using both a model estimation and novel individual-level survey data that allow for a direct identification of the biases.

First, we develop a stylized model with a New Keynesian structure that features heterogeneous households and an occasionally binding ZLB that is the first in the literature to allow for a joint existence of both inflation and deflationary biases in the distribution of inflation expectations. Specifically, it allows for households' heterogeneous perceptions of the risk of the ZLB and of monetary policy objectives while they form their expectations based on local economic conditions. Households have a deflationary bias when they assign a positive probability to a shock that would push the economy to the ZLB (occasionally binding constraint)—that is, their inflation expectations are below the target level of inflation—although the economy is not at the ZLB at the time. Thus, this paper does not study liquidity traps or deflation expectations, but we focus on establishing evidence of the existence of deflationary bias as defined in this paragraph.[2] Conversely, households have an inflation bias when they perceive that the target level for the output gap is positive—their

[1] For example, Smaghi (2009) noted: "The debate between those who consider that inflation represents the main risk for advanced economies over the next few years and those who instead believe that deflation is the most immediate threat, has polarised, especially in the United States. It has also had an interesting echo here in Europe." The arguments put forward were on the one side the expansionary monetary policy stance that might lead to inflation rates above target and on the other side the fear of a negative shock that would lead to an economic slowdown and inflation rates below target.

[2] When the economy operates at the ZLB, a deflation bias occurs compared to deflationary bias that occurs when the economy operates away from the ZLB. Deflation bias was formalized in Krugman (1998) and Eggertsson (2006). Nakov (2008), Wiederhold (2015), and Nakata and Schmidt (2019) show that deflationary bias can also occur away from the ZLB in the presence of an occasionally binding ZLB constraint. Let us note here that only about 0.6 percent of long-run inflation expectations are negative, that is expect, deflation. In this paper we focus on deflationary expectations.

inflation expectations are above the target level of inflation. Households also have different perceptions of central bank conservatism (Rogoff, 1985)—weights associated with inflation and output stabilization—that affect the magnitude of both inflation and deflationary biases. In our model, households lose confidence in the central bank either because they believe that the economy could be pushed to the ZLB or because they believe that the central bank is targeting a positive output gap. Consequently, their inflation expectations deviate from the inflation target, resulting in inflation or deflationary bias. This model can generate the distribution of households' inflation expectations—or the disagreement about medium-run inflation expectations (Mankiw et al., 2004)—that matches several moments of the distribution (disagreement) of households' inflation expectations in survey data, including a positive bias for mean inflation expectations and a long right-hand tail. Reis (2020) called for models that can explain the main features of the disagreement about inflation expectations.

We estimate our model using the first household-level survey to measure both the respondents' confidence in achieving the central bank's price stability objective and medium-run inflation expectations.[3] The survey spans over nine different countries from the second quarter of 2013 to the fourth quarter of 2015, with approximately 85,000 observations.[4] Our sample period is ideally suited for testing the implications of our model, since it includes countries with interest rates close to the zero lower bound, where policymakers have articulated deflationary as well as inflationary fears.[5] Using a simulated methods of moments (SMM) approach, we show that our model is able to match the main features of the average distribution of medium-run inflation expectations in all nine countries well, where, on average, the inflation bias is about 2.5 percentage points and the deflationary bias is about 0.9 percent. Notably, significant differences across these countries exist. Italy and Spain have higher shares of consumers with deflationary bias compared to Germany and Austria. While these results represent the first estimate of both inflation and deflationary biases, in the model the sources of heterogeneity in expectations are exclusively linked to inflation and deflationary biases.

To account for some of the other potential sources of heterogeneity, we proceed with an analysis of micro data on expectations, where by we can control for some of these other sources of heterogeneity identified in the literature. Our analysis using micro data constitutes two separate analyses. First, we use the same data as in the estimation and directly identify the two biases. Aligning with the model, we are able to quantify the implications of the central bank's price objective for inflation expectations and provide an estimate of both inflation and deflationary biases. Our estimates

[3]Note that in our survey respondents are reminded of the inflation target when answering questions about confidence in the price stability objective.

[4]Sample: Austria, France, Germany, Hong Kong, Italy, Singapore, Spain, Switzerland, and the United Kingdom.

[5]Focusing on the ECB—since more than half of our sample is composed of EMU countries—President Draghi repeatedly pointed out in 2013 that "[m]edium to long-term inflation expectations continue to be firmly anchored in line with price stability," (see, e.g., Draghi, 2013a, 2013b, 2013c) and even mentioned that the ECB and other countries in the world had prevented deflationary risk "by adopting both standard and non-standard measures" (Draghi, 2013a). The narrative changed, however, in 2014, when Draghi (2014a) started to emphasize the risk of low inflation and how this may translate to lower inflation expectations and even a deflationary spiral in some countries. In Draghi (2014b) he stresses that "[w]e will do what we must to raise inflation and inflation expectations as fast as possible, as our price stability mandate requires of us."

show that households that lost confidence in the price stability objective have medium-run inflation expectations that are on average 1 percentage point higher. We provide evidence that the deflationary bias exists and is on average -0.54 percent for medium-run expectations. In addition, we confirm that countries in our sample that pursue inflation targeting experience both lower inflation expectations and lower dispersion of inflation expectations concurrently, confirming the results by Ehrmann (2021) using micro-level data on medium-run inflation expectations. However, we do not find evidence that inflation and deflationary biases are mitigated in inflation targeting countries. In fact, the deflationary bias may be even larger when the central bank pursues inflation targeting.[6]

Furthermore, by focusing on euro-area countries only, we can document a substantial heterogeneity in the perception of the European Central Bank's objective function.[7] Our empirical results suggest that Germany, Austria, and France have both inflation and deflationary biases, Spain has only a deflationary bias, and Italy has only an inflation bias. To further study the sources of this substantial variation among EMU member countries, we decompose the effects behind these empirical results by relying on the relationships in the model. Specifically, the model implies that when the perception of the target level of the output gap increases (or the probability of the ZLB decreases), the deflationary bias decreases and the inflation bias increases. However, when you increase the weight on output relative to inflation in the loss function, both inflation and deflationary bias increase. We show that the perception of the target level of the output gap is the highest in Italy. The differences among other countries are driven by the different relative weights attached to output and inflation. The perception of the weight on the output gap in the objective function is highest in Germany and Austria, and is the lowest in Spain.

Finally we conduct an analysis to provide further evidence on the model proposed mechanisms. Focusing on Germany we identify the biases directly using questions regarding the perceptions of the objective functions of the ECB and the probability of the effective lower bound in the future that we have been able to implement in the Bundesbank Online Panel of Households (BOP-HH). Thus, we are directly validating the main mechanisms in the model.[8] Using an instrumental variable approach, we indeed find that the mechanism that causes inflation bias in the model causally increases inflation expectations in Germany at that time.

Previous research either provides indirect evidence of inflation bias only (among others, Romer, 1993; Ireland, 1999) or identifies a bias toward inflation expectations, but cannot attribute it to losing confidence in the central bank's price objective (among others, Ehrmann et al., 2017 or Souleles, 2004). Authors relied on testing various implications of the Barro and Gordon (1983a, 1983b) and Kydland and Prescott (1977) models using either US time series data or a cross-country panel. Romer (1993) tests whether more open economies have lower average inflation, as unexpected monetary policy expansion leads to real exchange rate depreciation that mitigates the effects. He

[6]We also find that when more respondents have doubts about the price stability objective of the central bank, the dispersion of medium-run inflation expectations increases.

[7]This heterogeneity is highlighted in, e.g., Goldberg and Klein (2011) as well as in several speeches by central bank officials (see e.g., Cœuré, 2019).

[8]Notably, the data from the Bundesbank Online Panel of Households (BOP-HH) has was fielded in 2022 when inflation was high and the probability of ECB hitting the effective lower bound was small. Thus, one could expect only a small role for deflationary bias during this period in Germany.

finds strong evidence of the link between openness and inflation, supportive of time-inconsistency models. Ireland (1999) shows that there exists a cointegrating relationship between unemployment and inflation in the United States: The magnitude of inflation bias becomes higher when the natural rate of unemployment rises.[9] However, empirical analysis exploring a deflationary bias is scarce. Mertens and Williams (2021) study the effects of the ZLB on the distribution of inflation expectations and interest rates using options data. They find that the decrease in the natural rate of interest leads to a model-consistent effect on forecast densities for interest rates, but the effect on the densities of inflation expectations is more modest. We provide empirical evidence that both inflation and deflationary biases are present and sizable in households' inflation expectations. We demonstrate that different perceptions of monetary policy are also a source of heterogeneity in inflation expectations.

We contribute to the literature on optimal policy design and inflation expectations in the presence of an effective lower bound on interest rates. Krugman (1998) and Eggertsson (2006) show that when the economy operates at the zero lower bound inflation and inflation expectations are below target even if all agents are perfectly rational. Nakov (2008) and Nakata and Schmidt (2019), among others, point out that deflationary bias can also occur away from the ZLB in the presence of an occasionally binding ZLB constraint. Wiederhold (2015) studies the propagation of shocks and policy effectiveness when monetary policy is occasionally constrained by the ZLB. In his model, average inflation expectations slowly adjust to shocks, while individual inflation expectations are heterogeneous due to different perceptions of the likelihood of the ZLB. Our contribution to the theoretical literature is twofold: First, we show analytically how both inflation bias and deflationary bias interact within this class of models and that targeting a positive output gap reduces the magnitude of deflationary bias; second, if consumers have heterogeneous perceptions of monetary policy and the likelihood of the ZLB, then we can have the co-existence of both inflation and deflationary biases in consumers' distribution of inflation expectations.

Our paper is also related to the growing literature using survey data to better understand and explain inflation expectations formation processes. Several papers have shown that expectations are inconsistent with a full information rational expectations assumption (Coibion and Gorodnichenko, 2015a); that there are substantial informational frictions present when forming inflation expectations (Coibion and Gorodnichenko, 2012); that individuals possibly use different models to forecast inflation expectations (Branch, 2004; Pfajfar and Santoro, 2010); and that they rely on their lifetime inflation experiences (Ehrmann and Tzamourani, 2012; Malmendier and Nagel, 2015), recent shopping experiences (D'Acunto et al., 2021), and gasoline prices (Coibion and Gorodnichenko, 2015b). Coibion et al. (2020) explore the relevance of steering inflation expectations as a policy tool with interest rates close to the ZLB. However, we add an additional dimension of heterogeneity

[9] Also using the US time series data, Ruge-Murcia (2003a) empirically compares different models that result in an inflation bias. His empirical test suggests that the data prefer the restrictions from the model with asymmetric preferences over those arising from the standard Barro-Gordon model. Ruge-Murcia (2004) performs a cross-country evaluation of the asymmetric preference models by evaluating whether the bias is proportional to the conditional variance of the unemployment rate and finds supporting evidence for the United States and France. Finally, Surico (2008) presents evidence that the inflation bias was positive and significant in the 1960s and '70s, while it was not significant in the subsequent period.

that is due to the perceptions of monetary policy. Andrade et al. (2019) show that heterogeneity in expectations arises because of the existence of differential perceptions of forward guidance.[10] Note that previous studies document that households rarely, at least in normal times, expect deflation Gorodnichenko and Sergeyev (2021). However, we focus on deflationary bias (that occurs away from the ZLB) and not deflation bias (that occurs at the ZLB); so we are investigating whether a lack of confidence in the inflation target is associated with below target expectations.

In contrast to the majority of papers on inflation expectations, which rely on one-year-ahead expectations only, we have the advantage of focusing on medium-run inflation expectations because our survey contains five-year-ahead inflation expectations. This is an important distinction, since, recently, many seminal papers have shown that short-run expectations are influenced by shopping experiences and oil prices, and that they may not be updated frequently when households rely on their life-time experiences. However, medium-run inflation expectations should be less affected by transitory movements in the economy and current experiences and more affected by perceptions of central bank (and fiscal) policies. The results in our paper show that there is substantial heterogeneity in medium-run inflation expectations due to perceptions of monetary policy.[11]

Although the questions in the survey we use are specifically about confidence in the central banks to achieve its medium-run price objective, our analysis is also related to the literature on trust in central banks.[12] The relevance of trust for economic development and particularly for central banks has long been established, but only a few papers that address this topic particularly with respect to the implications for inflation and inflation expectations. Ehrmann et al. (2013) investigate the determinants of trust in the ECB. They rightly note that "If low public trust in central banks is associated with higher household inflation expectations, then swings in public trust in the ECB also directly affect its ability to deliver on its mandate, although the empirical relevance of this proposition has yet to be tested." This is, of course, where we can contribute as well.[13]

Our paper is structured as follows: Section 2 presents the model and discusses the analytical results. Section 3 describes the data and model estimation, Section 4 presents the results using the M&G YouGov micro data, while Section 5 directly tests the main mechanism in the model using a special survey conducted as part of the Bundesbank BOP-HH. Section 6 concludes.

[10] Dräger et al. (2016) also show that central bank communication can affect consumers' understanding of key macroeconomic relationships. Coibion et al. (2022) and Coibion et al. (2023) show how information about monetary policy affects inflation expectations in a large-scale randomised controlled trial.

[11] We control for sociodemographic characteristics and income expectations to capture any heterogeneity due to economic status and lifetime experiences. Controlling for income expectations explains a significant part of the upward bias in inflation expectations and is a proxy for the perceptions of the future economic outlook (Ehrmann et al., 2017; Das et al., 2019). In addition, we also control for national and regional economic conditions in our regressions as another potential source of heterogeneity of inflation expectations.

[12] See Appendix A for definitions of credibility, reputation, confidence, and trust in the context of this discussion.

[13] Bursian and Faia (2018) explore the interaction between trust and macroeconomic outcomes by endogenizing the level of trust within a DSGE model. They test the validity of their model using aggregate data and conclude that trust affects short-run inflation expectations. Christelis et al. (2020) and Mellina and Schmidt (2018) also examine the effect of trust on short-run inflation expectations and show that losing trust increases inflation expectations.

2. Stylized Model

This section presents the stylized model. After defining the equilibrium, we derive some analytical results that serve as a basis for the empirical analysis in the next section.

2.1. Private Sector

The private sector of the economy has the standard New Keynesian structure formulated as presented in detail in Woodford (2003) and Galí (2008), and is most similar to Nakata and Schmidt (2019) implementation with the ZLB. However, in contrast to other papers in the literature, we assume that households have heterogeneous perceptions of monetary policy objectives and of the probability of a large shock that would push the economy to the ZLB. Households are also boundedly rational, as their forecasts are based on local economic conditions and they do not take into account what other households are doing in the economy. Thus, we abstract from the general equilibrium effects outlined in Angeletos and Lian (2018). The objective of the model is to show that both deflation and inflation bias co-exist in the distribution of inflation expectations. To do so we opted for the simplest possible model that provides the required features, abstracting from additional behavioral features that have been discussed in the literature.

There is a continuum of households with a mass one in the economy; we index them by $i \in [0, 1]$. Otherwise, infinitely lived households are identical, supplying labor in a perfectly competitive labor market and consuming a basket of differentiated goods produced by firms. The structure of the household problem is thus similar to Wiederhold (2015).[14]

Firms are identical, form rational expectations based on all relevant information (E_t), and maximize profits subject to nominal frictions, as in Calvo (1983). To derive closed-form results, we put all model equations except the ZLB constraint in semi-loglinear form. The New Keynesian Phillips curve, see eq. (1), represents the equilibrium condition for firms:

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1}, \tag{1}$$

where π_t is the inflation rate in t, y_t denotes the output gap, and i_t is the level of the nominal interest rate. The slope of the New Keynesian Phillips curve, κ, equals:

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}(\sigma^{-1}+\eta), \tag{2}$$

[14] The derivation of the household problem under these conditions can be found in Wiederhold (2015). Specifically, our model is most similar to his case with imperfect information, where it is assumed that households set wages and form their expectations based on local conditions. However, our timing assumptions are slightly different, since the time of the forecast in period t, our households do not observe the realizations of the endogenous variables. Furthermore, there are differences in the specification of heterogeneous perceptions of the demand shock, as can be seen below. Note that in order to derive eq. (3) it is convenient to assume that households can trade state-contingent claims in period -1 with one another, so it is not necessary to track the dynamics of the wealth distribution in periods $0 \leqslant t \leqslant T-1$.

where $\alpha \in (0, 1)$ is the share of firms in a given period that cannot re-optimize their price, $\theta > 1$ denotes the price elasticity of demand for differentiated goods, and $\eta > 0$ is the inverse Frisch elasticity of labor supply.

Households are heterogeneous, since they form expectations rationally based on local economic conditions (indexed using subscript i) including their perceptions of the probability of the ZLB and the parameters of the central bank's objective function as defined below, and not based on aggregate conditions. We denote these expectations as $E_{t,i}$. Thus, the household-specific consumption Euler equation (eq. (4)) for periods $0 \leqslant t \leqslant T-1$ is:

$$y_{t,i} = E_{t,i} y_{t+1,i} - \sigma(i_{t,i} - E_{t,i}\pi_{t+1,i} - r^*) + \tau_t, \tag{3}$$

τ_t is an exogenous shock (see below). The natural real rate of interest, r_t, equals $r^* + \frac{1}{\sigma}\tau_t$. $\sigma > 0$ is the intertemporal elasticity of substitution in consumption, $\beta \in (0, 1)$ is the subjective discount factor, and the deterministic steady state of the natural real rate, r^*, is $\frac{1}{\beta} - 1$. To obtain the aggregate output for periods $0 \leqslant t \leqslant T-1$ we integrate eq. (3) across households:

$$y_t = \int_{i=0}^{1} E_{t,i} y_{t+1,i} - \sigma \int_{i=0}^{1} (i_{t,i} - E_{t,i}\pi_{t+1,i} - r^*) + \tau_t, \tag{4}$$

We implement the demand shock, τ_t, as a two-state Markov process. These processes are commonly used in the effective lower bound literature—for example, Eggertsson and Woodford (2003), Wiederhold (2015), and Nakata and Schmidt (2019)—to intuitively describe the underlying mechanisms and transmission processes of the shocks. We assume that τ_t takes the value of either τ_H or τ_L where, for simplicity, we refer to $\tau_H > -\sigma r^*$ as the high state and $\tau_L < -\sigma r^*$ as the low state. The transition probabilities are given by:

$$Prob_i(\tau_{t+1} = \tau_L | \tau_t = \tau_H) = p_{H,i}, \tag{5}$$

$$Prob(\tau_{t+1} = \tau_L | \tau_t = \tau_L) = p_L. \tag{6}$$

$p_{H,i}$ represents the perception of household i of the likelihood of switching to the low state in the next period when the economy is currently in a high state and will be referred to as the *frequency* of the low state. $p_{H,i} \in \{p_{H,H}, p_{H,L}\}$ where $p_{H,H} > p_{H,L}$ and each household either perceives a high probability $p_{H,H}$ or a low probability $p_{H,L}$ of the shock pushing them to the low state. The mass ω of households perceive $p_{H,i} = p_{H,H}$ and the mass $1 - \omega$ of households perceive $p_{H,i} = p_{H,L}$. We denote the average as $p_H = \omega p_{H,H} + (1 - \omega)p_{H,L}$, which is also the true frequency of the low state. Thus, on average households are correct. This assumption allows us to solve the model analytically and simplifies the firms' problem. p_L denotes the likelihood of staying in the low state when the economy is currently in a low state and will be referred to as the *persistence* of the low state. This probability is known and is the same for all households and corresponds to the actual probability. The assumption that there is heterogeneity in $p_{H,i}$ and not in p_L is only for simplicity and expositional reasons; one could straightforwardly extend the model to include heterogeneity in p_L. Firms know the true transition probabilities.

In principle, any demand shock, like τ_t, or any supply shock that would push the equilibrium to the ZLB would be sufficient for our results to hold. Thus, the agents in the economy can have potentially different perceptions of which shock causes the central bank to be pushed to the ZLB.

2.2. Monetary Policy

Society's welfare at time t is represented by the expected discounted sum of future utility flows,

$$V_t = u(\pi_t, y_t) + \beta E_t V_{t+1}, \tag{7}$$

where society's contemporaneous utility function, $u(\pi_t, y_t)$, is given by the standard quadratic function of inflation and the output gap,

$$u(\pi, y) = -\frac{1}{2}(\pi^2 + \bar{\lambda}(y)^2). \tag{8}$$

As shown by Woodford (2003), this objective function can be derived using a second-order approximation to the household's preferences. In this case we can further set $\bar{\lambda}$ to $\frac{\kappa}{\theta}$.

The form of households' perception of the central bank's objective function is similar to society's but potentially has important differences—as advocated by the time-inconsistency and rules versus discretion debates initiated by Barro and Gordon (1983a) and Kydland and Prescott (1977). As we mentioned before, households have to forecast the real rate in order to make consumption decisions while they observe only local conditions; that is, when forecasting, they use their perceptions of $\{p_{H,i}, y_i^*, \lambda_i\}$ and not the actual economy-wide values of $\{p_H, y^*, \lambda\}$. We assume that households do not observe the economy-wide output, inflation, and interest rates, but they perceive that the central bank reacts to their local output and inflation. The other way to think about it is that they forecast the economy-wide output and inflation to be the same as their local inflation and output and that all agents in their local economy have the same $\{p_{H,i}, y_i^*, \lambda_i\}$. In this paper, households' perceived utility is,

$$V_{t,i}^H = u_i^H(\pi_{t,i}, y_{t,i}) + \beta E_{t,i} V_{t+1,i}^H, \tag{9}$$

where the household perception of the contemporaneous utility, $u_i^H(\pi_{t,i}, y_{t,i})$, is—compared to society's contemporaneous utility—augmented for the possibility that the central bank may be inclined to push the output gap above the natural level:

$$u_i^H(\pi_{t,i}, y_{t,i}) = -\frac{1}{2}(\pi_i^2 + \lambda_i(y_i - y_i^*)^2). \tag{10}$$

Although households' perceptions of the objective function resemble society's perceptions of the objective function, there are potentially three differences. Note that Backus and Driffill (1985) and Barro (1986) considered the possibility that households have asymmetric information on monetary policy. Recently, Barthélemy and Mengus (2018) document the importance of central bank signalling of their *type* in the presence of liquidity traps, that is, their objective function. We build on this literature and consider the following departures from the full information rational expectations case with common information that the central bank optimally minimizes society's objective function.

First, $y_i^* \geqslant 0$ represents household i's perceptions of the central bank's desired level of the output gap, which, if positive, can lead to inflation bias, as proven in proposition 2.[15] We assume that $y_i^* \in \{y_H^*, y_L^*\}$ where $y_H^* > y_L^* \geqslant 0$ and each household either perceives a high value y_H^* or a low value y_L^* for y_i^*. The mass ξ of households perceive $y_i^* = y_H^*$ and the mass $1 - \xi$ of households perceive $y_i^* = y_L^*$. We denote the average as $y^* = \xi y_H^* + (1 - \xi) y_L^*$.

Second, perceptions of the relative weight the central bank assigns to the stabilization of the output gap, $\lambda_i > 0$, differe across households and may not be equal to society's optimal $\bar{\lambda}$. $\lambda_i \in \{\lambda_H, \lambda_L\}$, where $\lambda_H > \lambda_L \geqslant 0$ and each household either perceives a high value λ_H or a low value λ_L for λ_i. The mass ζ of households perceive $\lambda_i = \lambda_H$ and the mass $1 - \zeta$ of households perceive $\lambda_i = \lambda_L$. We denote the average as $\lambda = \zeta \lambda_H + (1 - \zeta) \lambda_L$.

Third, as mentioned above, households perceive that the central bank considers only their local conditions. In this stylized model we assume that households observe only their local conditions that are realized when $\{p_{H,i}, y_i^*, \lambda_i\}$ are the true values of the corresponding aggregate parameters and that they do not observe the national, economy-wide output, inflation, and interest rates. The fact that households do not update their information set with economy-wide variables is a restrictive assumption, but not necessarily implausible given the biases that have been identified in the literature on inflation expectations. For the purpose of this paper, we resort to this simplification, as it transparently conveys the main mechanisms in the model and allows a clearer mapping of the theoretical model to our empirical strategy given the questions asked in the M&G YouGov survey.

Integrating across households, we arrive at the following average perception of the central bank's contemporaneous utility function (which equals the true behavior of the central bank):

$$u^H(\pi_t, y_t) = -\frac{1}{2}(\pi^2 + \lambda(y - y^*)^2). \tag{11}$$

Also, households know that the central bank is subject to the ZLB constraint,[16]

$$i_t \geqslant 0. \tag{12}$$

We assume that the average households' perception is correct and the central bank behaves accordingly. Thus, $u^H(\pi_t, y_t) = u^{CB}(\pi_t, y_t)$. The central bank behaves with discretion; the commitment option is not available and all agents in the economy know that. The central bank chooses the output gap, inflation rate, and the nominal interest rate in each period t to maximize its objective function, subject to the behavioral constraints of the private sector, while taking as given the policy functions at time $t + 1$. Therefore, the central bank is maximizing the following objective:

[15] There are alternatives and possibly empirically more plausible formulations of inflation bias that build on asymmetric preferences of the central bank or recession aversion. See, for example, Gerlach (2003), Cukierman and Gerlach (2003), and Ruge-Murcia (2003a). The effects are isomorphic.

[16] For simplicity, we consider a ZLB instead of an effective lower bound that is lower than zero. Results in this paper remain unchanged if we consider a lower bound $i_t < 0$.

$$V_t^{CB} = \max_{\pi_t, y_t, i_t} u^{CB}(\pi_t, y_t) + \beta E_t V_{t+1}^{CB}, \tag{13}$$

subject to the ZLB constraint in eq. (12) and the private-sector equilibrium conditions detailed in eqs. (1) and (4). However, households form their decisions based on their local conditions and perceptions of $\{p_{H,i}, y_i^*, \lambda_i\}$.

We define the Markov perfect equilibrium as a set of time-invariant value and policy functions $\{V_t^{CB}(\cdot), y(\cdot), \pi(\cdot), i(\cdot)\}$ that solves the central bank's problem described in the preceding text, together with society's value function $V(\cdot)$ that is consistent with $y(\cdot)$ and $\pi(\cdot)$. In addition, a Markov perfect equilibrium has to satisfy the equilibrium conditions for every household i in this economy. Armenter (2018), Nakata (2018), and Nakata and Schmidt (2019), among others, point out that there are potentially four Markov perfect equilibria in this economy. The equilibrium that is the most relevant to our current study is the *standard* Markov perfect equilibrium. The *standard* Markov perfect equilibrium fluctuates around a positive nominal interest rate and zero inflation and output. The other potentially interesting equilibrium is the *deflationary* Markov perfect equilibrium that fluctuates around a zero nominal interest rate and negative inflation and output.

The *standard* Markov perfect equilibrium over time in periods $0 \leqslant t \leqslant T-1$ is given by a vector $y_H, \pi_H, i_H, y_L, \pi_L, i_L$ that solves the following system of linear equations for periods $0 \leqslant t \leqslant T-1$ as well as an analogous problem for household i:

$$y_H = [(1-p_H)y_H + p_H y_L] + \sigma[(1-p_H)\pi_H + p_H\pi_L - i_H + r^*] + \tau_H, \tag{14}$$

$$\pi_H = \kappa y_H + \beta[(1-p_H)\pi_H + p_H\pi_L], \tag{15}$$

$$0 = \lambda(y_H - y^*) + \kappa\pi_H, \tag{16}$$

$$y_L = [(1-p_L)y_H + p_L y_L] + \sigma[(1-p_L)\pi_H + p_L\pi_L - i_L + r^*] + \tau_L, \tag{17}$$

$$\pi_L = \kappa y_L + \beta[(1-p_L)\pi_H + p_L\pi_L], \tag{18}$$

$$i_L = 0, \tag{19}$$

and satisfies the non-negativity of the nominal interest rate in the high state and non-positivity of the Lagrange multiplier on the ZLB constraint in the low state:

$$i_H > 0, \tag{20}$$

$$\lambda(y_L - y^*) + \kappa\pi_L < 0, \tag{21}$$

x_k denotes the value of variable $x \in \{\pi, p, y, i\}$ in the k state where $k \in \{H, L\}$. In this system of equations we can also immediately observe that in this *standard* Markov perfect equilibrium, agents have expectations that are a weighted average of the two states, i.e., they take into account the possibility of switching to the low state.

Proposition 1. *The standard Markov perfect equilibrium over time in periods* $0 \leqslant t \leqslant T-1$ *exists if and only if*

$$p_L \leqslant p_L^*(\Theta_{(-p_L)}),$$
$$p_H \leqslant p_H^*(\Theta_{(-p_H)}),$$
$$p_{H,i} \leqslant p_{H,i}^*(\Theta_{(-p_{H,i})}),$$

where i) for any parameter x, $\Theta_{(-x)}$ denotes the set of parameter values excluding x, and ii) the cutoff values $p_L^*(\Theta_{(-p_L)})$, $p_H^*(\Theta_{(-p_H)})$, and $p_{H,i}^*(\Theta_{(-p_{H,i})})$ are given in Appendix B.1.

Proof. See Appendix B.1. □

The three conditions guarantee the non-positivity of the Lagrange multiplier in the low state and the non-negativity of the nominal interest rate in the high state for all agents populating the economy. When the frequency of the low state, p_H, is high, the central bank reduces the nominal interest rate aggressively to mitigate the deflationary bias. Thus, for the policy rate to be positive in the high state, p_H must be sufficiently low. With $p_L > p_L^*(\Theta_{(-p_L)})$, inflation and output in the low state are positive when they satisfy the consumption Euler equation and the Phillips curve. When the persistence of the low state, p_L, is high, inflation and output in a current low state are largely dependent on private-sector expectations of output and inflation in the next period's low state. Thus, positive inflation and output in the low state can be self-fulfilling. However, such positive inflation and output cannot be an equilibrium because the central bank would have incentives to raise the nominal interest rate from zero in the low state. This incentive manifests itself in the positive Lagrange multiplier in the low state when inflation and output are positive. Let us point out that the condition on $p_{H,i}$ ensures that the Markov perfect equilibrium exists for all possible combinations of perceptions on the probability of the low state and perceptions on the monetary policy in this economy. Strictly speaking the second condition (p_H) is redundant when ensured that the last condition holds ($p_{H,i}$).

The other three possible Markov perfect equilibria in this framework are: the *deflationary* Markov perfect equilibrium, which is briefly discussed in the preceding text; the *ZLB-free* Markov perfect equilibrium, where the ZLB constraint does not bind in either state; and the *topsy-turvy* Markov perfect equilibrium, where the ZLB binds in the high state but not in the low state. Finding evidence of the latter two equilibria in the data is less likely. Most of the existing literature does not focus on them as they are empirically less plausible, given the experience of various countries in which state of the economy the ZLB binds. For the purpose of this paper, we do not consider these three equilibria and focus on the equilibrium in the normal state of the economy where the deflationary bias can occur.

We focus on the standard Markov perfect equilibrium, as this one seems more relevant for the set of countries we consider in the empirical sections of this paper. In particular, we focus on the high state of the standard Markov perfect equilibrium. Note that a defining feature of the high state is that y_H is positive, whereas in the low state y_L is negative in most cases.[17] Also, all these

[17] Proposition 2 states that this is always the case as long as y^* is not too high. Summary statistics are in Table D.1.

economies have positive long-run inflation expectations, and most surveys indicate that economic agents expect the central bank to eventually raise interest rates. Furthermore, all countries lowered the interest rate after the end of 2015, which marks the end of the time sample for our empirical analysis. All three factors support our decision for the high state, although the threat that central banks in these countries have limited policy space was definitely present at the time in most countries in the sample.[18] Mertens and Williams (2021) also find empirical support in the United States for this type of equilibrium over the deflationary (liquidity trap) equilibrium.

In $t \geqslant T$ the economy is in the non-stochastic steady state with zero inflation. In this equilibrium all households have the same post-transfer wealth in period T, as they traded state-contingent claims in period -1. In this equilibrium in period T, the Euler equation, the New Keynesian Phillips curve, and the optimal policy rule are satisfied with $p_{H,i} = y_i^* = p_L = 0$ and $\lambda_i = \bar{\lambda}$. Thus, only one state exists with $\pi_t = y_{t,i} = y_t = 0$.

2.3. Analytical Results

In this section we first focus on the economy-wide analytical results and then look at individual household properties. When the conditions for the existence of the equilibrium hold, it is possible to show that depending on the values of y^*, we can observe either inflation or deflationary bias. Conditional on y^*, the signs of the endogenous variables can be determined. The propositions for individual household i are identical to propositions 2-5 if we replace the economy-wide averages of $\{p_H, y^*, \lambda\}$ with individual levels $\{p_{H,i}, y_i^*, \lambda_i\}$.

Proposition 2. *When the conditions for the existence of the equilibrium hold, we can observe either inflation bias or deflationary bias depending on the values of y^*. For any $\lambda \geqslant 0$ and $y^* \geqslant 0$:*[19]

- $\pi_H \leqslant 0$ *iff* $y^* \leqslant \widetilde{y^*}$ *and* $\pi_H > 0$ *otherwise, where* $\widetilde{y^*} \equiv -\beta p_H r_L (\kappa C)^{-1}$
- $\pi_L \leqslant 0$ *iff* $y^* \leqslant \widehat{y^*}$ *and* $\pi_L > 0$ *otherwise, where* $\widehat{y^*} \equiv B r_L (D \kappa \lambda)^{-1}$
- $y_H > 0$
- $y_L \leqslant 0$ *iff* $y^* \leqslant \overline{y^*}$ *and* $y_L > 0$ *otherwise, where* $\overline{y^*} \equiv (1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda(-\lambda\left[(1-\beta)C + (1-\beta p_L)\right]\kappa)^{-1} r_L$
- $E_H \pi \leqslant 0$ *iff* $y^* \leqslant \widehat{y^*}$ *and* $E_H \pi > 0$ *otherwise, where* $\widehat{y^*} \equiv p_H r_L (\kappa^2 + \lambda)(\kappa\lambda[C + p_H])^{-1}$
- $E_L \pi \leqslant 0$ *iff* $y^* \leqslant \breve{y^*}$ *and* $E_L \pi > 0$ *otherwise, where* $\breve{y^*} \equiv [\beta\lambda(p_L - p_H) - p_L(\kappa^2 + \lambda)] r_L (\kappa\lambda[C + p_L])^{-1}$

Proof. See Appendix B.2. □

In this proposition we observe the interaction between the shock and the perceived target for the output gap, y^*. As long as $y^* < min\{\widehat{y^*}, \overline{y^*}\}$, we observe that output and inflation are below

[18] See the discussion in footnote 2.

[19] We use the following definitions: $B = \kappa^2 + \lambda(1 - \beta(1 - p_H))$, $C = \frac{(1-p_L)}{\sigma\kappa}(1 - \beta p_L + \beta p_H) - p_L$, $D = -1 - C$, and $E(\lambda) = A(\lambda)D - B(\lambda)C$.

target values in the low state, as the ZLB constraint is binding and monetary policy cannot offset the shock. However, higher y^* reduces these effects, and if $y^* \geqslant \widehat{y^*}$, inflation becomes positive. In the high state, firms lower prices because of a positive probability of τ_L (low state) that leads to a reduction in the expected marginal costs of production. This raises the expected real interest rate that incentivizes households to postpone their consumption plans. These anticipation effects are mitigated by the central bank's lowering of nominal rates. In the literature, this effect is usually referred to as deflationary bias and alone causes inflation in the high state to be negative. With $y^* > 0$ we have an additional effect that, in equilibrium, raises inflation and leads to inflation bias if $y^* \geqslant \widetilde{y^*}$. Since the central bank would like to stabilize the output gap around y^*, this raises inflation because of a tradeoff between inflation and output gap stabilization. Output in the high state is positive, irrespective of the value of y^*. To be precise, both effects of y^* and r_L lead to higher output (see propositions 4 and 5). These mechanisms are consistent with those described in the inflation bias and deflationary bias literature. Inflation expectations also exhibit inflation and deflationary biases. As outlined in proposition 2 average inflation expectations are either in the deflationary bias region or inflation bias region depending on the average value of y^*. The $E_H\pi$ is negative and thus in the deflationary bias region if $y^* < \widehat{y^*}$, while they are in the inflation bias region if $y^* > \widehat{y^*}$. Similarly, average inflation expectations in the low state, $E_L\pi$, can be either in the deflationary bias region or inflation bias region, depending on y^* being below or above the threshold $\widetilde{y^*}$.

As a special case, when $y^* = 0$ and $\lambda > 0$ only deflationary bias exists, that is, $\pi_H < 0$, $y_H > 0$, $i_H < r_H$, $\pi_L < 0$, $y_L < 0$, $E_H\pi < 0$, and $E_L\pi < 0$. A second special case occurs when setting $\lambda = 0$, or, in other words, if a conservative central banker is appointed (Rogoff, 1985), there are no inflation or deflationary biases, although inflation expectations feature deflationary bias ($\pi_H = 0$, $E_H\pi < 0$, and $E_L\pi < 0$). We now further establish several results on how the degree of conservatism affects endogenous variables in both states.

Proposition 3. *How the degree of conservatism affects endogenous variables depends on the values of y^*. Higher conservatism (lower λ) reduces the absolute distance of inflation from 0 irrespective of y^*.*

(a) For any $\lambda \geqslant 0$ and $y^* \leqslant \widetilde{y^*}$: $\frac{\partial \pi_H}{\partial \lambda} \leqslant 0$, $\frac{\partial \pi_L}{\partial \lambda} \leqslant 0$, $\frac{\partial y_L}{\partial \lambda} \leqslant 0$, $\frac{\partial y_H}{\partial \lambda} < 0$, $\frac{\partial E_H\pi}{\partial \lambda} \leqslant 0$, and $\frac{\partial E_L\pi}{\partial \lambda} \leqslant 0$.

(b) For any $\lambda \geqslant 0$ and $y^* > \widetilde{y^*}$: $\frac{\partial \pi_H}{\partial \lambda} > 0$, $\frac{\partial \pi_L}{\partial \lambda} > 0$, $\frac{\partial y_L}{\partial \lambda} > 0$, $\frac{\partial y_H}{\partial \lambda} > 0$, $\frac{\partial E_H\pi}{\partial \lambda} > 0$, and $\frac{\partial E_L\pi}{\partial \lambda} > 0$.

Proof. See Appendix B.3. □

This proposition states that since the central bank cares relatively more about inflation, both inflation and deflationary biases that can occur in the high state will be lower, and inflation will move closer to zero in the high state. When deflationary bias prevails ($y^* \leqslant \widetilde{y^*}$) and inflation in the high state is negative, then a lower value of λ increases inflation. However, when inflation bias prevails ($y^* \geqslant \widetilde{y^*}$) and inflation in the high state is positive, then a lower value of λ decreases inflation in this state. This holds for both inflation and inflation expectations in the high state.

Proposition 4 details the effect of the y^* on inflation, output, and inflation expectations:

Proposition 4. *For any* $y^* \geqslant 0$*:* $\frac{\partial \pi_H}{\partial y^*} > 0$*,* $\frac{\partial \pi_L}{\partial y^*} > 0$*,* $\frac{\partial y_H}{\partial y^*} > 0$*,* $\frac{\partial y_L}{\partial y^*} > 0$*,* $\frac{\partial E_H \pi}{\partial y^*} > 0$*, and* $\frac{\partial E_L \pi}{\partial y^*} > 0$*.*

Proof. See Appendix B.4. □

Higher y^* increases expectations of inflation and in turn inflation in both states. This is a standard result in the inflation bias literature. Note that, in the case where deflationary bias is present, an increase in y^* means that the deflationary bias is reduced or even that expectations are now in the region of inflation bias.

Next, we turn our attention to the effect of p_H on π_H and $E_H \pi$.

Proposition 5. *For any* p_H *that satisfies conditions for the existence of equilibrium* $\frac{\partial \pi_H}{\partial p_H} < 0$ *and* $\frac{\partial E_H \pi}{\partial p_H} < 0$*.*

Proof. See Appendix B.5. □

As we can see in proposition 5, the effect of p_H on inflation and inflation expectations is always negative. Thus, when there is a higher probability of the shock, which would push the economy to the ZLB, the inflation expectations are lower and the deflationary bias is higher.

Corollary 1 summarizes the results for p_H and $E_H \pi$:

Corollary 1. *We can observe the following effects on* π_H *and* $E_H \pi$ *depending on the level of* y^**:*

(a) For any $y^* \leqslant \widetilde{y^*}$*:*

- *(i)* $\frac{\partial \pi_H}{\partial p_H} < 0$ *and* $\frac{\partial \pi_H}{\partial \lambda} \leqslant 0$ *and* $\frac{\partial \pi_H}{\partial y^*} > 0$*.*
- *(ii)* $\frac{\partial E_H \pi}{\partial p_H} < 0$ *and* $\frac{\partial E_H \pi}{\partial \lambda} \leqslant 0$ *and* $\frac{\partial E_H \pi}{\partial y^*} > 0$*.*

(b) For any $y^* > \widetilde{y^*}$*:*

- *(i)* $\frac{\partial \pi_H}{\partial p_H} < 0$ *and* $\frac{\partial \pi_H}{\partial \lambda} > 0$ *and* $\frac{\partial \pi_H}{\partial y^*} > 0$*.*
- *(ii)* $\frac{\partial E_H \pi}{\partial p_H} < 0$ *and* $\frac{\partial E_H \pi}{\partial \lambda} > 0$ *and* $\frac{\partial E_H \pi}{\partial y^*} > 0$*.*

Proof. See proofs of the propositions in the preceding text. □

This corollary states that there are two regions of y^*. In the first region, when $y^* \leqslant \widetilde{y^*}$, π_H is nonpositive, while the threshold for the sign of $E_H \pi$ is $\widehat{y^*}$. As Collorary 1 shows, the three variables in question have the same effect on both π_H and $E_H \pi$. The effect of p_H on both variables is negative. Thus, a higher p_H further increases the deflationary bias. Higher conservatism and higher y^* decrease the inflation bias in both inflation and inflation expectations, as formulated in propositions 3 and 4. When y^* is larger than $\widetilde{y^*}$, we have the case where inflation bias exists and both a higher p_H and a higher degree of conservatism decrease the amount of inflation bias, while higher y^* increases the inflation bias.

The next corollary deals with the properties of individual household inflation expectations.

Corollary 2. *Households have heterogeneous inflation expectations:*

(a) *Households with a combination of perceptions* $\{p_{H,H}, \lambda_H, y_L^*\}$ *have the lowest inflation expectations and households with a combination of perceptions* $\{p_{H,L}, \lambda_H, y_H^*\}$ *have the highest inflation expectations in both states.*

(b) *Both inflation and deflationary biases are present in the cross-section of inflation expectations iff* $y_L^* < \widehat{y^*}(\lambda_H, p_{H,H})$ *and* $y_H^* > \widehat{y^*}(\lambda_H, p_{H,L})$.

(c) *The cross-sectional dispersion of inflation expectations is increasing: (i) when the difference between* $(p_{H,H} - p_{H,L})$ *is increasing; (ii) with higher* λ_H*; (iii) when the difference between* $y_H^* - y_L^*$ *is larger.*

Proof. See proofs of the propositions in the preceding text. □

Households form heterogeneous expectations in our model, as their perceptions of $\{p_{H,i}, y_i^*, \lambda_i\}$ are potentially different. Specifically, there are eight different combinations of perceptions that result in eight different expectations of inflation in each state. Households with a combination of perceptions $\{p_{H,H}, \lambda_H, y_L^*\}$ have the lowest inflation expectations, while households with a combination of perceptions $\{p_{H,L}, \lambda_H, y_H^*\}$ have the highest inflation expectations in both states. The value of y_i^*, in combination with other perceptions, determines whether inflation expectations of household i exhibit inflation bias or deflationary bias. When $y_L^* < \widehat{y^*}(\lambda_H, p_{H,H})$ and $y_H^* > \widehat{y^*}(\lambda_H, p_{H,L})$, some households have inflation bias, while other households have deflationary bias. Thus, in the cross-section, inflation expectations will be distributed around the inflation target of the central bank, some higher and some lower. The cross-sectional variance will depend on the perceived variance of the probability of the low state across households as well as the perceived variance of the y_i^* across households. Similarly it will also depend on the variance of the λ_i perceptions.

3. Data and Model Estimation

3.1. The M&G YouGov Survey

Our main dataset consists of individual-level data across nine countries and 11 survey waves from the second quarter of 2013 to the fourth quarter of 2015, with a total of 84,735 observations.[20] We believe that, as indicated before, this time period is well suited to test for the existence of deflationary biases, because the countries in our dataset are close to the zero lower bound and policymakers have expressed deflationary concerns during our sample period. In addition, it is in line with our theoretical model, which requires a non-zero probability for the low state equilibrium.[21] To analyze the data, we combine countries and survey waves to form a panel of data across individuals,

[20]The M&G YouGov survey was conducted in the following countries: Austria, France, Germany, Hong Kong, Italy, Singapore, Spain, Switzerland, and the United Kingdom. We are using all data available to us from this survey. Surveys were conducted at the beginning of each quarter.

[21]The results are likely to depend on the time sample, as it is likely that in any pre-2008 sample, one would not find the deflationary bias in inflation expectations. However, it is also likely that in the next decades, the deflationary bias would become more important. One could perceive the change in long-run goals of the FOMC in August 2020 as a "preparation" for such events.

countries, and time. The M&G YouGov Inflation Expectations Survey data are collected by YouGov, an online research center focusing on the perceptions and opinions of individuals across the world.[22]

The survey includes information, among others, on inflation expectations, confidence in the central bank's price objective, trust in the government, and general characteristics of the individual. To measure inflation expectations, the survey asks the participants to provide their short-run inflation expectations (what they expect inflation to be 12 months from the date of the survey) and their medium-run inflation expectations (what they expect inflation to be five years from the date of the survey) using the following question: "What annual rate of inflation do you expect for [the country] in [time] from now?" Answers are recorded with up to one decimal place or as "Don't know,".[23] Note that the households are required to provide inflation expectations for their respective countries and not for the EMU. This is in line with our model that assumes that households forecast their local economic conditions. For the model estimation we use the distribution of medium-run inflation expectations.

3.2. Estimation

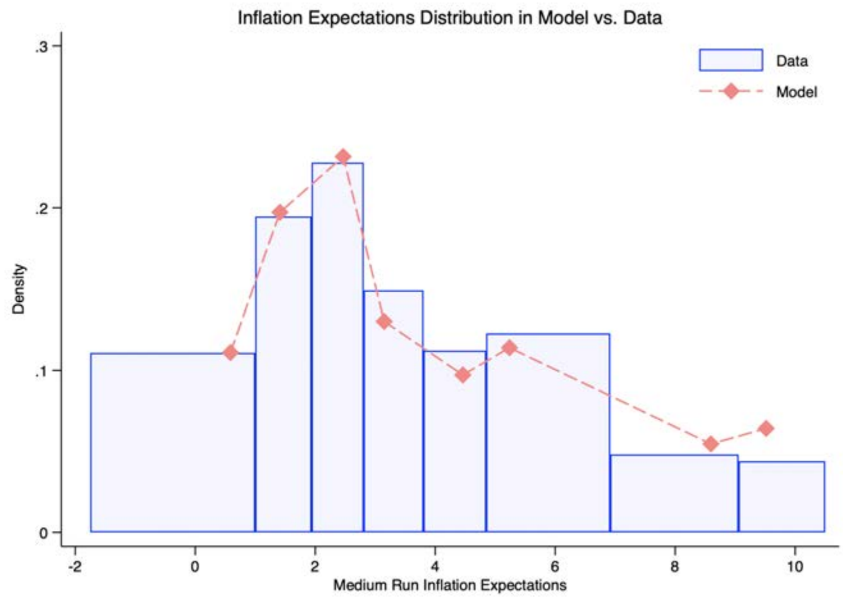
The objective of this subsection is to estimate the distribution of inflation expectations and to evaluate how well the model can fit the empirical distribution of inflation expectations from the M&G YouGov survey. Since output growth was positive in all countries in our sample, we consider that our sample is best described by a *standard* Markov perfect equilibrium in a high state, given that the high state is characterized by positive growth, while in the low state output growth is negative. Furthermore, monetary authorities in all countries in the sample decreased interest rates after the end of the sample. In the low state, the equilibrium features interest rates at the effective lower bound. This assumption is implemented in both the estimation of the model and for the analysis using micro data.

We first estimate the overall distribution of inflation expectations across nine countries and then focus on estimating country-specific distributions. We proceed with calibrating the model's structural parameters and focusing the estimation on the parameters in the model that describe the heterogeneity of inflation expectations. In particular, we estimate the following nine parameters in the model: $p_{H,H}, p_{H,L}, y_H^*, y_L^*, \lambda_H, \lambda_L, \omega, \xi$, and ζ. To estimate the model we use simulated methods

[22]YouGov conducts surveys using active sampling: It predetermines who is allowed to participate in the survey in order to maximize the representativeness of the sample. Each survey is anonymous and takes under 10 minutes to complete, and YouGov provides a monetary incentive for completing the survey. The data are statistically weighted to correspond to the national population profile (over the age of 18). These weights are calculated based on age, gender, social class, region, party identity, and the readership of individual newspapers. YouGov's results have been shown to be comparable in accuracy to those of other major polling entities and have a high predictive accuracy for actual outcomes in national and regional elections. YouGov conducts its surveys according to the Market Research Society's guidelines.

[23]Inflation expectations are truncated by -5 percent to +30 percent. While truncation is common practice to remove outliers in consumer surveys, there is no consensus on the optimal truncation. In comparison, the National Bank of New Zealand truncates its consumer survey sample by -2 percent to +15 percent, while the University of Michigan Survey of Consumers (MSC) first truncates all responses larger than 95 percent in absolute value and, second, truncates reported expectations at -10 percent and +50 percent when reporting mean inflation expectations. To further safeguard our analysis using micro data in Section 4 against extreme observations, we also apply Huber robust regressions as done in Coibion et al. (2022). Results are presented in Table E.1.

Figure 1. Distribution of Inflation Expectations in the Model and Data



of moments (SMM) on the inflation expectations data where the data are adjusted for the inflation target.[24] The estimated SMM parameters for the overall distribution of inflation expectations across all nine countries in the sample are in Table 1, while the calibrated parameters are in Table C.1 in Appendix C.

Figure 1 presents the empirical distribution of inflation expectations and the model estimate of the distribution of inflation expectations. As we can see in Figure 1, the model can match several features of the empirical distribution of inflation expectations. In particular, it is able to match that the modal/median inflation expectations exhibit a positive inflation bias. Furthermore, it is able to match the long right tail that is a frequent feature of distributions of inflation expectations. In addition, the model can match the variance of inflation expectations. This simple model of the economy is thus able to reproduce the main features of the distribution of inflation expectations. These stylized facts can be observed in various individual-level datasets of (long-run) inflation expectations across various countries as we can confirm later on by showing the estimates for various countries in our sample.

As we have shown analytically in the model, the distribution of inflation expectations exhibit both deflationary bias and inflation bias. Inflation expectations in the high state are equal to $E_H\pi = (1 - p_H)\pi_H + p_H\pi_L$. Thus $E_H\pi$ can be written as:

[24]The details of the estimation procedure are explained in Appendix C. Note that the data are recentered, so that the no inflation bias and the no deflation bias, which in the model is 0, coincide with the inflation target in the respective country. The inflation target in their respective country is presented to the respondents as part of the survey.

Table 1: Estimated Model Parameters

Parameters	All	AUT	FR	DE	HK	IT	SGP	ESP	CH	GBR
Mean Exp.	3.44	3.47	2.95	3.36	4.96	2.96	4.40	2.92	2.82	3.11
ζ (prob. λ_H)	0.36	0.30	0.35	0.40	0.28	0.44	0.18	0.40	0.35	0.27
ω (prob. p_H)	0.46	0.37	0.71	0.43	0.49	0.66	0.49	0.49	0.60	0.37
ξ (prob. y_H)	0.33	0.35	0.17	0.33	0.38	0.20	0.24	0.36	0.23	0.35
y_H^*	0.14	0.18	0.16	0.14	0.12	0.13	0.14	0.16	0.17	0.20
y_L^*	0.08	0.06	0.07	0.06	0.09	0.06	0.06	0.08	0.05	0.11
$p_{H,H}$	0.19	0.15	0.17	0.14	0.24	0.13	0.16	0.22	0.14	0.35
$p_{H,L}$	0.13	0.06	0.01	0.08	0.08	0.04	0.01	0.16	0.04	0.18
λ_H	0.0160	0.0070	0.0110	0.0110	0.0185	0.0205	0.0145	0.0080	0.0095	0.0055
λ_L	0.0050	0.0020	0.0040	0.0035	0.0020	0.0020	0.0045	0.0035	0.0025	0.0020
Total Loss	0.0057	0.0034	0.0013	0.0049	0.0035	0.0029	0.0028	0.0030	0.0034	0.0053

$$E_H\pi = \underbrace{-\frac{p_{H,i}(\kappa^2+\lambda_i)}{E(\lambda)_i}r_L}_{\text{deflationary bias}} \underbrace{-\frac{C_i + p_{H,i}}{E(\lambda)_i}\kappa\lambda_i y_i^*}_{\text{inflation bias}} \tag{22}$$

where $E(\lambda)$ is defined in footnote 19. As proposition 2 states, this equation will exhibit a net deflationary bias if $y^* \leqslant \widehat{y^*}$ and a net inflation bias otherwise. As we can see in Table 2, for the overall sample we find that about 69 percent of individuals predominantly have an inflation bias—where the mean inflation bias is about 2.5 percentage points—and 31 percent of the participants have a deflationary bias—where the mean deflationary bias is about 0.9 percentage point. Thus, the overall distribution exhibits a mean net inflation bias of about 1.5 percentage points.

Table 2: Estimated Inflation and Deflationary Biases

	All	AUT	FR	DE	HK	IT	SGP	ESP	CH	GBR
Mean Deflationary Bias	0.88	0.61	0.87	0.76	1.09	0.62	1.19	0.60	1.05	0.84
Mean Inflation Bias	2.47	2.13	3.55	2.22	1.85	2.73	2.94	3.62	2.43	1.72
Mean Net Bias	1.44	1.47	0.95	1.36	0.96	0.96	1.40	0.92	0.82	1.11
% with Def. Net Bias	30.8	24.1	58.9	28.8	30.4	52.8	37.2	64.0	46.2	24.1
% with Inf. Net Bias	69.2	75.9	41.1	71.2	69.6	47.2	62.8	36.0	53.8	75.9

3.3. Differences across Countries

Figure C.1 in Appendix C reports the details of the individual countries' estimates and shows that our model is flexible enough to account for several features of the distribution that appear across individual countries in our dataset, despite the distributions being quite different in several moments across our set of countries. For example, France and Singapore have dramatically different distributions of medium-run inflation expectations as can be seen in Figure C.1, but our model is able to match key moments from both of them. All countries have on average inflation bias, which is in line with a positive bias commonly observed in households' survey data. Our estimation suggests that Austria and the United Kingdom have the lowest proportions of individuals with predominant deflationary bias (24 percent) and the highest share of those with inflation bias. In contrast, in Spain 64 percent of agents have predominant deflationary bias and only 36 percent have predominantly inflation bias. However, the mean deflationary bias is about the same in Austria and Spain (0.6 percentage point). The highest mean deflationary bias is in Singapore, Hong Kong, and Switzerland, where it exceeds 1 percentage point. We observe the highest mean inflation bias in France and Spain where it exceeds 3 percentage points. This suggests that there is a lot of heterogeneity in inflation expectations across countries in our sample.

4. Micro Evidence on Inflation and Deflationary Biases Using the M&G YouGov Survey

The analysis in the previous section postulates that all heterogeneity in inflation expectations is due to either inflation bias or deflationary bias. As we stated in the introduction, there are several other potential sources of heterogeneity of long-run inflation expectations, although, in the long-run, perceptions about monetary and fiscal policy should be the most important ones. However, in the analysis using micro data we can use other variables in the survey to control for household characteristics, macroeconomic conditions, and perceptions about fiscal policy as potential sources of heterogeneity. One shortcoming is that we can only estimate the net average bias when using the micro data in different parts of the distribution in the analysis in this section. For Germany, we can go one step further and show that the mechanism at work in the model causally impacts inflation expectations in Germany in August 2022.

4.1. Additional Data from the M&G YouGov Survey

Data on inflation expectations from this survey are explained in Section 3.1. To measure confidence in the central bank's price objective, the survey asks the following question for inflation targeting countries: "How confident, if at all, are you that the [country's central bank or ECB if member of EMU] is currently pursuing the correct policies in order to meet its target of price stability (i.e., inflation around [target]) over the medium term (i.e., the next 3 - 5 years)?" For non-inflation targeting countries the reference to the inflation target is removed and the question reads "...correct policies to achieve price stability (i.e., inflation of around 2 percent)...". Since existing surveys mainly ask about trust in the central bank in general, having a question that is rather specific about the inflation target is a great advantage.[25] The individual then chooses between "Not at

[25] Unfortunately, we are not able to be more specific regarding the inflation target, as for instance the ECB has had an asymmetric target (until July 2019) of close to but below 2 percent, while the survey question is stated in a symmetric manner "around 2 percent." That being said, it is not clear that consumers perceive temporary deviation from the threshold slightly above 2 percent as indication of breaching the promised inflation target. Similarly, the mandate of the Swiss National Bank also specifies an upper bound on inflation. Using a follow-up survey, we found that there was no difference in respondents' inflation expectations when the question was rephrased as "less than 2 percent" instead of "around 2 percent."

all," "Not very," "Fairly confident," "Very confident," or "Don't know." For our main specification we decided to condense this variable into a confidence dummy variable for which we combine the categories "Fairly confident," and "Very confident" into 0 and "Not at all," and "Not very" into 1; we assign missing values to "Don't know" answers.[26]

Our dataset allows us to also infer the respondent's level of trust in the government. This is another advantage of our data, since we are able to disentangle the attitude toward the government from the confidence in the central bank. To measure trust in the government, the survey asks the individual the following question: "To what extent do you agree or disagree with the following statement? 'I think that the government is currently following the right economic policies for [Country]." The participants then choose between the following answers: "Strongly disagree," "Tend to Disagree," "Neither agree nor disagree," "Tend to Agree," "Strongly agree," and "Don't know."

With respect to the ordering of the questionnaire, the survey starts with the questions on inflation expectations, followed by the question on confidence in the central bank and the question on the government's economic policy. We prefer this ordering, since stating the inflation target (and expressing (lack of) confidence in the central bank) after the inflation expectations question does not prime the answers on expected inflation in a specific direction. In terms of socioeconomic characteristics this survey has information on the participants' gender, age, expected income growth, and region in which they are currently living.[27] Note that income expectations can explain a large portion of the bias in inflation expectations (Ehrmann et al., 2017; Das et al., 2019). With that we are able to control for the most relevant socioeconomic characteristics identified in the literature.[28]

The macroeconomic variables we use are the annualized CPI inflation rate, the short-run interest rate, the output gap, and the unemployment rate.[29] In our main specification we use regional and time fixed effects. With regional fixed effects we take account of any region-specific characteristic within a country. In addition, we also provided a regression in the robustness section with region times time fixed effects that account for any regional-specific development at each point in time. With that we make sure we are not missing any macroeconomic control variable that might affect the results.

We classify a country as inflation-targeting based on the central bank's mission, as stated on the country's central bank website. Those countries that state a specific number as their inflation target are labeled as inflation targeting in our sample. This includes all of our countries, except Singapore and Hong Kong.[30] The latter two countries are pursuing a currency board as their monetary policy strategy, which can also be a viable option to control domestic inflation, and thus,

[26]Results for the full measure are qualitatively identical and presented in Appendix E.

[27]The question capturing income expectations reads as follows: "Do you expect your net income to increase, decrease or be about the same in 12 months time?" Respondents can chose the following answer options: 'increase,' 'stay about the same,' 'decrease,' 'don't know,' or 'not applicable' if they have no income.

[28]For the UK we have additional socioeconomic characteristics that we can control for, like employment status, social grade, martial status, and number of children in the household. Notably, adding those control variables does not alter our results. Results are presented in Table E.3 in Appendix E.

[29]Most of the data are taken from the OECD Economic Outlook No. 106. Summary statistics and further details on the macroeconomics variables are provided in Table D.1 in Appendix D.

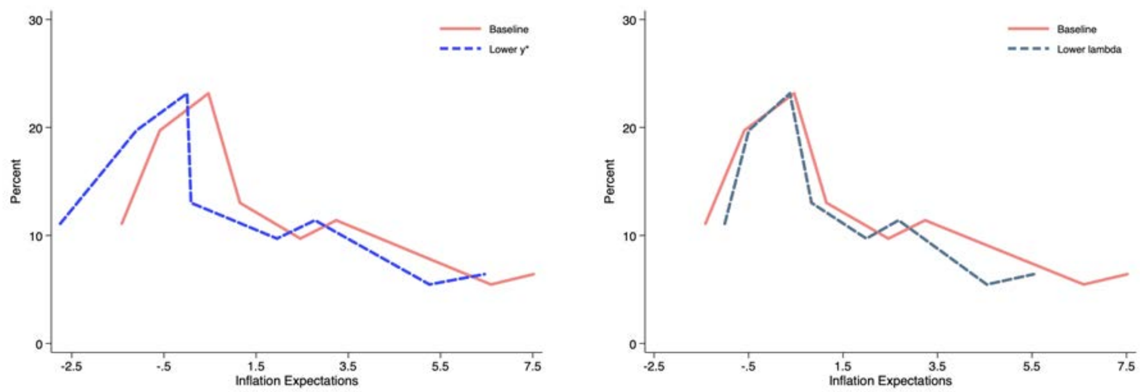
[30]We also classify the ECB as an inflation targeting central bank, since it states the explicit inflation target, although the ECB has repeatedly clarified that it is not an inflation targeting central bank.

broadly speaking, compatible with our theoretical framework. In fact, the Monetary Authority of Singapore states that its monetary policy "is centered on managing the trade-weighted exchange rate with the objective to ensure price stability over the medium term as a basis for sustainable economic growth." The Monetary Authority of Singapore also defines its price stability objective as, on average, a core inflation rate of just under 2 percent.[31]

4.2. Identification and Relation to the Model

This section explains the empirical strategy used for the M&G YouGov survey data. We assume households that take part in this survey use the above model to forecast inflation and that they know the correct structural parameters (all the fixed parameters in the model), but they form their own perceptions of the three non-structural parameters as described in the model $\{p_{H,i}, \lambda_i, y_i^*\}$ as in line with our model estimation. These parameters influence inflation expectations—see propositions 2-5—and lead to heterogeneous expectations across households (see also Corollary 2).

Figure 2. Distribution of Inflation Expectations in the Model



(a) Reducing y^*

(b) Reducing λ

Notes: The solid red lines represent our baseline simulation based on the estimates of the parameters for the full dataset. The dashed blue line in panel (a) represents a simulation with lower y^* and the dashed teal line in panel (b) represents a simulation with lower λ.

In the M&G YouGov survey we do not directly observe $\{p_{H,i}, \lambda_i, y_i^*\}$, but we utilize the following question: "How confident, if at all, are you that the central bank is currently pursuing the correct policies in order to meet its target of price stability (that is, inflation around [target]) over the medium term (that is, the next 3 - 5 years)?" According to the model, two possibilities lead economic agents to reply that they are not confident that the central bank is pursuing the right policies to meet its inflation target. The first possibility leads to higher than target inflation expectations and is due to the perception of y^* being positive, $y_i^* > 0$ (and this effect dominates the effect of $p_{H,i}$). Figure 2 shows that increasing y^* shifts the whole distribution of inflation expectations to the right.

[31] See the homepage of the MAS: https://www.mas.gov.sg/monetary-policy and https://www.mas.gov.sg/monetary-policy/Singapores-Monetary-Policy-Framework/faqs/section-1.

Thus, the central bank would not be pursuing the right policies to achieve its inflation objective. The second possibility is when the agents perceive a positive probability of the shock, $p_{H,i}$, that could push the economy to the ZLB (and this effect dominates the y_i^* effect). Higher $p_{H,i}$ pushes the whole distribution of inflation expectations to the left. Once again, in this case, the central bank is not pursuing the right policies to meet its inflation objective—inflation is too low—and thus the respondents answer that they are not confident that the central bank will achieve its inflation objective. To differentiate between these two possibilities, we proceed with inference in different parts of the distribution, as detailed in Table 3. We conjecture that for expectations below the inflation target the dominant reason for no confidence is the probability of ZLB, $p_{H,i}$, while for those with expectations above the inflation target the reason for no confidence is the perception of y^* being positive.[32]

Table 3: Empirical Strategy

Inflation Expectations	**below target**	**around target**	**above target**
Necessary condition	$y_i^* < \widehat{y_i^*}$	$y_i^* \approx \widehat{y_i^*}$	$y_i^* > \widehat{y_i^*}$
Dominating perception	$p_{H,i}$	-	y_i^*
No Confidence households	lower $E_i\pi$	same $E_i\pi$	higher $E_i\pi$

4.3. Empirical Strategy and Results

We use the following regression to study inflation and deflationary bias:

$$\pi^e_{i,j,t} = \alpha + \beta NC_{i,j,t} + \delta_A D^A + \Phi_A D^A NC_{i,j,t} + \delta_B D^B + \Phi_B D^B NC_{i,j,t} + \Gamma Z_{i,j,t} + \mu_j + \nu_t + \varepsilon_{i,j,t}, \quad (23)$$

where the subscripts i, j, t denote individual i, region j (within a specific country), and time t. π^e represents medium-term expectations (5 years ahead). NC (not confident) is a dummy variable that captures whether the individual is not confident that the central bank will achieve its inflation target. The vector Z contains several control variables, including individual characteristics as well as macroeconomic control variables. μ and ν are region and time fixed effects, α is the coefficient estimate of the constant term, and ε is the i.i.d. error term. For the error term we use two-way clustering in region and time.

The threshold variables represent expectations below 1.5 percent (D^B) and above 2.5 percent (D^A), respectively. The reference region of expectations to which we compare the two regions is 1.5 percent to 2.5 percent. In effect, we calculate the average effects of being not confident in three regions of inflation expectations. This follows from the model-guided empirical strategy laid out in the previous section and in Table 3 and the results are discussed in Table 4. Note that using

[32] Note that λ has to be positive in order for these effects to exist and the exact size of these effects will depend on the perceptions of λ (as can be seen from propositions above and in Figure 2).

dummy variables for different regions of inflation expectations is in the same spirit as, for instance, conducting a quantile regression, but with a more educated specification of the thresholds.

Table 4: Medium-Run Inflation Expectations

Group	**Mean**	**Median**	**Obs.**
Below Confidence	0.92	1	4100
Below No Confidence	0.73	1	4558
Mid Confidence	2.00	2	4685
Mid No Confidence	2.00	2	3751
Above Confidence	5.67	4	11143
Above No Confidence	7.10	5	18895
Full Sample	4.70	3	47132

Notes: The table shows mean and median medium-run inflation expectations for different sub-groups. "Below group" is the group having expectations strictly below 1.5 percent, the "mid group" has expectations truncated at 1.5 percent and 2.5 percent, and the "above group" has expectations strictly above 2.5 percent. Within these groups we differentiate between people who have confidence in the central bank and those who report having no confidence in the price stability objective of the central bank.

We control for socioeconomic characteristics, such as gender, age, and income and for the macroeconomic situation in the country, proxied by the short-term interest rate, the output gap, and the inflation rate. The literature on survey inflation expectations commonly identifies the bias in inflation expectations. As shown, for example, in Ehrmann et al. (2017) and Souleles (2004), sociodemographic characteristics, income expectations, and macroeconomic conditions explain most of this bias. Finally, we control for trust in the government. Since there might be a lack of trust in institutions in general, we need to make sure that we are able to separate the lack of confidence in central banks from the attitude toward other policy making bodies. Being able to decompose those two improves our identification.

Since our model allows for the possibility of both inflation and deflationary biases, we are particularly interested in the estimated coefficients of the interaction terms Φ_A and Φ_B. If both coefficient estimates have a positive sign, this would mean that losing confidence in the price stability objective would lead to higher inflation expectations (inflation bias) independent of the level of inflation expectations. This would indicate that an inflation bias is dominating the empirical distribution of expectations. From our model, however, we expect the above interaction term coefficient estimate (Φ_A) to be positive (inflation bias) and the below interaction term coefficient estimate (Φ_B) to be negative (deflationary bias). This implies that losing confidence and having high expectations are associated with even higher inflation expectations, while losing confidence and having low inflation expectations are associated with even lower (deflationary) expectations. Lastly, if both interaction terms are not significant or negative, this would imply that an inflation bias is not present in the data.

As in Table 4, our reference threshold is 1.5 percent to 2.5 percent, which is an ad hoc threshold indicating a moderate deviation of expectations from the inflation target. We report a number of robustness checks and alternative definitions of these thresholds in Table E.1. In one specification, we re-estimate the same equation with thresholds of 1 percent and 3 percent. In another, we calculate the country-specific thresholds that may not be necessarily centered around the official target. Furthermore, since the ECB used to interpret its inflation target asymmetrically with close to 2 percent being the maximum, we provide an estimation for EMU member countries using a threshold of 1 percent to 2 percent. Results remain qualitatively identical.[33]

Table 5: Medium-Run Expectations

	(1)	(2)	(3)	(4)
Not Confident	1.025***	1.013***	-0.423***	-0.566***
	(0.0543)	(0.0539)	(0.0363)	(0.1398)
Gov. Mistrust	0.178***	0.220***	0.127***	0.126**
	(0.0548)	(0.0549)	(0.0493)	(0.0494)
Inflation Target		-1.922***	-0.650***	-0.084
		(0.1729)	(0.1780)	(0.1777)
Below			-1.337***	-1.189***
			(0.0260)	(0.0709)
Below × Not Confident			-0.123***	-0.127***
			(0.0343)	(0.0342)
Above			3.439***	4.105***
			(0.0845)	(0.0869)
Above × Not Confident			1.462***	1.497***
			(0.0768)	(0.0779)
Not Confident × Inflation Target				0.150
				(0.1459)
Below × Inflation Target				-0.165**
				(0.0730)
Above × Inflation Target				-0.748***
				(0.1055)
Observations	45,396	45,396	45,396	45,396
R-Squared	0.073	0.075	0.276	0.277
MacroVars	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes

Notes: Two-way (region time) clustered standard errors in parentheses. "Above" represents dummy variable denoting a threshold of 2.5 percent and "Below" a threshold of 1.5 percent respectively. Medium-run inflation expectations are 5 years ahead. All regressions include regional and time fixed effects. Errors are two-way clustered over time and region. Below × Not Confident and Above × Not Confident represent interaction terms between "Not Confident" and the "Above" or "Below" threshold. Not Confident × Inflation Target denotes the interaction term between "Inflation Target" and "Not Confident". Above × Inflation Target and Below × Inflation Target are the interaction terms between "Inflation Targeting" and the "Above" and "Below" threshold variables. "FE" denotes fixed effects. "Socio" stands for control variables on socioeconomic characteristics and "MacroVar" for macroeconomic control variables. *** $p < 0.01$ ** $p < 0.05$ * $p < 0.1$

[33]The ECB, as of July 2019, interprets its threshold symmetrically. Furthermore, it is not clear that consumers perceive temporary deviation from the threshold slightly above 2 percent as an indication of breaching the promised inflation target. Hence, using the band 1.5 percent to 2.5 percent may not be in line with official statements but is in line with a pragmatic reaction of consumers. Robustness to this is presented in Section 7. See also Table E.5 for results using a threshold of 1 percent to 2 percent.

As already indicated in the previous section and in Figure C.3 and Table 4, there seems to be support in our data for the conjecture that a lack of confidence in the price stability objective is linked to both aggregate inflation bias and both inflation and deflationary biases in households with inflation expectations below or above a certain inflation (target) level.

Table 4 contains the means of the three groups separated across households that have confidence in the central bank and those that do not. As we can observe, there is little surprise that in the above group, inflation expectations increase further if people lose confidence. Hence, this shows that the classic result of inflation bias holds. However, taking into account the below group, we see that a deflationary bias could be possible as well. From the table we observe that inflation expectations of the households that still have confidence in the central bank have slightly higher mean expectations than those that do not have confidence in the central bank meeting its price objective. Summary statistics are provided in Appendix D.

Moving to the regression results, estimations of eq. (23) are reported in Table 5. In column (1) of this table, the main variable of interest is the indicator for households not confident in the price stability objective. These estimates represent the average effect for the whole distribution of inflation expectations and test whether on average across all households there is inflation or deflationary bias. We can show that the coefficient estimate of being not confident in the central bank's price stability objective is highly statistically significant and has a positive sign. Losing confidence in the central bank's price stability objective, on average, is associated with the classic result of an inflation bias. Notably, we not only provide empirical evidence of this result, but we are now in a position to exactly quantify inflation bias, resulting a 1.0 percentage point higher medium-run inflation expectations. Given that inflation targets are around 2 percent in our sample of countries, this number represents a sizable bias.

In column (2) we add an indicator for inflation targeting countries. Our main variable of interest remains highly significant. Looking at the relevance of inflation targeting, the corresponding coefficient estimate is negative, implying that inflation targeting reduces medium-run expectations by 1.9 percent. Hence, also for our sample, we can confirm that inflation targeting reduces inflation expectations and, on average, brings them closer to the target level.

In column (3) we dig deeper into the heterogeneity of medium-run inflation expectations and test the simultaneous existence of inflation and deflationary biases. Therefore, as described above, we add the dummy variables "Above" and "Below" and the corresponding interaction terms of Above and Below with being not confident (denoted as Above×Not Confident and Below×Not Confident). Those interaction terms are of particular interest here.

Looking at the estimation results in column (3) (for 1.5 percent and 2.5 percent as thresholds), we observe that the coefficient estimates of Above×Not Confident and Below×Not Confident are statistically significant and have opposite signs. The estimate for Above×Not Confident is positive, implying that having inflation expectations above 2.5 percent and being not confident is associated with an inflation bias for medium-term expectations of 1.46-0.42=1.04 percent. In contrast, the coefficient estimate of Below×Not Confident is negative, implying that for individuals who have inflation expectations below 1.5 percent, losing confidence is associated with even lower inflation expectations and a deflationary bias for medium-term expectations: -0.12-0.42=-0.54 percent. Thus, we provide the first empirical evidence of the simultaneous co-existence of inflation and deflationary biases on medium-run inflation expectations when losing confidence in the central bank's objective of keeping inflation close to target level.

In column (4), we explore how controlling for inflation targeting affects the results for inflation and deflationary biases. For this purpose we interact inflation targeting with our threshold variables. Since inflation targeting should reduce the variability of inflation expectations, we expect the interaction term with "Above" to be negative and the interaction term with "Below" to be positive.

We can observe that there is little change in the coefficient estimates of our main variables of interest between columns (3) and (4). This estimation leads to very similar results for both inflation and deflationary biases: 1 percent and -0.69 percent, respectively. We observe that inflation targeting shifts the whole distribution of inflation expectations to the left, as both coefficient estimates of Above×Inflation Target and Below×Inflation Target are negative and significant. The coefficient on the interaction with above is notably quite large. This means that the dispersion of inflation expectations is lower in the inflation targeting countries. However, our sample has only two countries where the central banks are not pursuing inflation targeting, according to our definition, so these results may be affected by the selection of countries in our sample. Notably, this is something all empirical studies investigating the effect of inflation targeting have in common.

Table 6: EMU Medium-Run Expectations

	(1) EMU	(2) Germany	(3) Austria	(4) France	(5) Spain	(6) Italy
Not Confident	-0.216***	-0.389***	-0.079	0.007	-0.109	-0.091
	(0.0386)	(0.0669)	(0.0853)	(0.0704)	(0.0840)	(0.0926)
Gov. Mistrust	0.007	-0.257***	0.050	-0.045	0.064	0.148**
	(0.0297)	(0.0616)	(0.1023)	(0.0533)	(0.0550)	(0.0610)
Below	-1.284***	-0.925***	-1.174***	-1.048***	-1.206***	-1.274***
	(0.0320)	(0.0534)	(0.1340)	(0.0570)	(0.0716)	(0.0550)
Below × Not Confident	-0.213***	-0.318***	-0.483***	-0.396***	-0.174*	0.018
	(0.0410)	(0.0804)	(0.1829)	(0.0745)	(0.0902)	(0.0760)
Above	4.159***	3.319***	3.389***	4.712***	5.365***	4.328***
	(0.1030)	(0.1587)	(0.2301)	(0.2702)	(0.2113)	(0.1900)
Above × Not Confident	0.898***	1.059***	0.887***	0.781***	-0.372	1.539***
	(0.1110)	(0.1819)	(0.2498)	(0.2919)	(0.2571)	(0.2320)
Observations	25,677	6,340	2,726	4,401	6,073	6,137
R-Squared	0.291	0.237	0.232	0.345	0.331	0.305
MacroVars	Yes	Yes	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Two-way (region time) clustered standard errors in parentheses. "Above" represents dummy variable denoting a threshold of 2.5 percent and "Below" a threshold of 1.5 percent respectively. Medium-run inflation expectations are 5 years ahead. All regressions include regional and year fixed effects. Errors are two-way clustered over time and region. Below × Not Confident and Above × Not Confident represent interaction terms between "Not Confident" and the "Above" or "Below" threshold. "FE" denotes fixed effects. "Socio" stands for control variables on socioeconomic characteristics and "MacroVar" for macroeconomic control variables. *** $p < 0.01$ ** $p < 0.05$ * $p < 0.1$

We test whether there are differences in the perception of monetary policy across EMU countries by running the same regression as in Table 1 column (3) but focusing on these countries only. To explore the different perceptions of y^*, we estimate the specification for each country separately. With that setup, we can analyze the variation of country-specific inflation and deflationary biases and compare the perceptions of the central bank's objective within a monetary union. Table 6 contains the estimation results. In the first column we replicate the specification of Table 5 column (3) for the EMU countries in our sample. The other columns estimate the same specification for the individual member countries.

Column (1) confirms the simultaneous existence of inflation and deflationary biases for the EMU countries. The coefficients of Above×Not Confident and Below×Not Confident are positive and negative, respectively, and confirm, together with the not confident coefficient estimate, the existence of inflation and deflationary biases (-0.22+0.90=0.68 and -0.22-0.21=-0.43). These estimates are a touch smaller than for our full sample of countries, but they remain highly significant. In columns (2)-(6), we replicate the same specification for each EMU member state individually. We observe that Germany, Austria, and France have an inflation bias as well as a deflationary bias. Spain has a deflationary bias only, while Italy only has an inflation bias. In terms of size, Italy has the strongest relationship to losing confidence in the price stability objective, resulting in a high inflation bias followed by Austria, France, and Germany. Regarding the deflationary bias, Germany and Austria have the highest coefficient estimate, closely followed by France and then Spain. Overall, it is remarkable that despite having the same experience with the ECB, the relationships to losing confidence across the member countries of the monetary union are quite different in terms of size and propensity for inflation and deflationary biases.[34]

Notably, our model allows us to explore the drivers of this observed heterogeneity within a monetary union. From our model, given the form of the objective function in eq. (10), the differences in perceptions can be either due to λ_i, y_i^*, or $p_{H,i}$. As our results imply, perceptions in different countries can vary so we can compare the average perceptions of households for a given country pair. We can determine if the differences in perceptions are due to λ on the one hand or y^* and p_H on the other hand, as they have different effects on inflation and deflationary biases. As indicated in Table 3, we can distinguish between y^* and p_H as the predominant factor only based on the position in the distribution of inflation expectations. Most strikingly, we can say that perceptions of y^* in Italy are significantly higher than in any other EMU country in the sample. Thus, in Italy, deflationary bias is not present, while there is a high inflation bias due to perceptions that the ECB is targeting a positive output gap. This could potentially be driven by Italy's experience with inflation which has been among the highest in Europe. Differences in perceptions among other countries are mostly guided by different perceptions of the weight associated with the output gap in the ECB's objective function (λ). The perceptions of λ are highest in Austria and Germany, followed by France, and the lowest in Spain, where inflation bias is not significant. Thus, we can

[34]Note that these results have limited comparability with the ones obtained using the model estimation. Model estimation results assume that all deviations in inflation expectations from the target level are due to either inflation bias or deflationary bias. Results in this section identify the effect of not being confident in the price stability objective on inflation expectations in different parts of the distribution.

argue that, in Austria and Germany, households are worried most about the ECB not pursuing a clear hierarchical mandate, where the inflation target is the primary goal. It is possible that these different perceptions are driven by different historical experiences. Robustness for these results is examined in Appendix E.12, where, among other exercises, we entertain the possibility that there are differences with respect to the perceptions of the inflation target.

4.4. Inflation Targeting and Inflation/Deflationary Bias

In this section we test the implications of central bank design for inflation expectations. One reason for introducing inflation targeting is to better influence inflation via inflation expectations. Our sample consists of both inflation and non-inflation targeting countries, although only two out of the nine countries are not inflation targeting countries. We have already shown that the dispersion of inflation expectations is lower in inflation targeting countries than in other countries in our sample. Here we also test whether there is a difference in the inflation and deflationary biases across these two groups of countries. We test the difference in sizes of these biases by implementing a triple interaction term between above (below), non-confident, and inflation targeting. Since inflation targeting, according to our model, should reduce both biases, we expect a negative coefficient for the above interaction term and a positive coefficient for the below interaction term.

Table 7 presents the results. Column (1) provides the estimation of our main table for comparison. The results in column (2) suggest that there is no statistical difference in inflation and deflationary biases among inflation and non-inflation targeting central banks, although we confirm the results from Table 5 that the dispersion of inflation expectations is smaller in inflation targeting countries in our sample.

However, in column (3) of Table 7 we perform an additional check—that is also potentially an interesting robustness check for our results overall—where we assign a country-specific threshold for the reference group, as in Section 7.8. After adjusting the reference group in column (3) of Table 7, we do not find a significant difference in the inflation bias among these two groups of countries; however, the deflationary bias is twice as large among inflation targeting countries, 0.72 percent, compared to other countries in our sample.

Overall, while we find no evidence that inflation targeting reduces inflation bias, we show that it increases deflationary bias; that is, participants expect inflation that is lower than inflation target. Notably, we face the same empirical limitations other studies have in analyzing the implications of inflation targeting. We have only a few countries that are not inflation targeting countries in our sample and consequently, they may not be an optimal comparison group.

In Appendix E, we conduct several types of robustness checks to solidify our main results. We check the robustness of the specification in Table 5 column (3). Table E.1 in the Appendix contains a set of robustness exercises we executed, including (i) the effect on short-run expectations, (ii) the role of trust in the government, (iii) the role of higher-order fixed effects and (iv) bootstrap standard errors, (v) estimation results from Huber regressions that take into account that there may be outliers, (vi) regressions that investigate whether there is a selection bias and various alternative thresholds, including (vii) country-specific thresholds and (viii) wider no biases intervals, (ix) regressions with a full measure of confidence, and (x) regressions with additional sociodemographic controls for countries with available data. It also reports country-specific estimates for non-EMU countries. Results remain qualitatively the same.

Table 7: Inflation Targeting and Confidence

	(1)	(2)	(3) Country Specific
Not Confident	1.025***	-0.433***	-0.373***
	(0.0543)	(0.0755)	(0.0965)
Gov. Mistrust	0.178***	0.126**	0.084*
	(0.0548)	(0.0494)	(0.0485)
Inflation Target		-0.025	-0.100
		(0.1675)	(0.1694)
Not Confident × Inflation Target		0.009	0.030
		(0.0821)	(0.0993)
Below		-1.231***	-1.139***
		(0.0801)	(0.0684)
Not Confident × Below		-0.047	0.023
		(0.1483)	(0.0909)
Inflation Target × Below		-0.122	0.124*
		(0.0841)	(0.0728)
Not Confident × Inflation Target × Below		-0.079	-0.343***
		(0.1528)	(0.0964)
Above		4.172***	4.008***
		(0.1107)	(0.1254)
Not Confident × Above		1.344***	1.194***
		(0.1695)	(0.1707)
Inflation Target × Above		-0.823***	-0.662***
		(0.1446)	(0.1584)
Not Confident × Inflation Target × Above		0.168	0.257
		(0.1891)	(0.1918)
Observations	45,396	45,396	45,396
R-Squared	0.073	0.277	0.299
MacroVars	Yes	Yes	Yes
Socio	Yes	Yes	Yes
Region FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes

Notes: Robust standard errors in parentheses. Above represents dummy variable denoting a threshold of 2.5 percent and Below a threshold of 1.5 percent respectively for columns (1) and (3).
Medium-run expectations are 5 years ahead. All regressions include region and time fixed effects. FE denotes fixed effects. Socio stands for control variables on socioeconomic characteristics and MacroVar for macroeconomic control variables. *** $p < 0.01$ ** $p < 0.05$ * $p < 0.1$

4.5. Uncertainty and Inflation/Deflationary Bias

Could uncertainty or rounding explain our results regarding inflation and deflationary bias? Perhaps either households that are not confident that the central bank will achieve its price stability objective more often report 0 percent inflation or that non-confident households are more uncertain and thus more often report rounded responses (Binder, 2017). We investigate these conjectures and find little support for them. Specifically, the share of respondents who reported a round answer (0, 2, 5, and 10) is actually higher among those who reported confidence in the price stability objective (21.0 percent) compared to those who reported that they are not confident (15.4 percent), possibly implying that those who are not confident may regard inflation as an important issue, or being less uncertain, they more often report a non-rounded response. Focusing only on those that reported 0 percent inflation leads to similar results, confirming that there is no evidence that our results for deflationary bias are driven by uncertainty as described in Binder (2017).[35]

5. Testing the Mechanism

This section sheds more light on the relevance of the mechanism identified in the model for the formation of inflation expectations. Specifically, we study the causal effect of perceptions of the relative importance of the output gap in the objective function of the ECB and the effect of ZLB expectations via confidence in the price stability objective on inflation expectations in Germany. To do so we fielded questions eliciting these preferences in August 2022 as part of the Bundesbank Online Panel of Households (BOP-HH), which is a monthly survey consisting of several questions capturing macroeconomic expectations, preferences, and opinions. Using this survey, we had the opportunity to include questions that are better linked to the main mechanisms we explore in the model and do not require additional assumptions regarding identification. We also included questions that have exactly the same wording as in the M&G YouGov survey to maintain the comparability between surveys. The questions that pertain to the main mechanism in the model are: "How important is, in your opinion, the price stability objective relative to economic growth for the European Central Bank (ECB)" with answers on a scale of 0-10, and furthermore "How high is the probability, in your opinion, that the interest rates set by the European Central Bank (ECB) will be close to 0% in five years from now?" with possible answers in the range of 0 to 100 percent.

With these two additional questions we can test whether both the perception of the weight on deviations from the target level of the output gap and the probability of the ZLB matter for inflation expectations. The survey questions give us a direct measure of λ and $p_{H,i}$ as described in the model.

In the first step of the analysis in this section, we replicate our standard specification—with the same variables we used for the M&G YouGov survey—using the August 2022 BOP-HH dataset. Next we provide evidence that both variables significantly influence the degree of confidence in the monetary policy objective, and lastly, we show that using these variables we can explain our results in the first step. Table F.2 shows the estimation results of our standard specification for this survey wave. We can see that confidence in monetary policy and government policies matters, as we have seen already for the M&G YouGov survey; that is losing confidence increases short- and medium-run inflation expectations. The interaction term inflation expectations above 2.5 percent reveals a positive bias for short- and medium-run expectations. However, we see little statistical support for the lower bound interaction term. One reason might be the inflation surge observed in

[35] We also study this question using a regression analysis by controlling for other potentially relevant characteristics. Results, which are available upon request, confirm the results obtained using summary statistics. Furthermore, the share of those who reported negative expectations is about 0.8 percent of respondents who are not confident and 0.4 percent of those who are confident.

Germany at that time, with inflation rates about 10 percent followed by a sharp increase in interest rates. Consequently, only very few people expect that interest rates might return to levels close to the ZLB in the medium term.

Table 8: Inflation Expectations and Confidence in Price Stability Objective of the ECB: IV Regression with BOP-HH Data

	short-run	short-run	medium-run	medium-run
Not Confident	0.469***	0.008	0.729***	-0.299
	(0.163)	(0.375)	(0.206)	(0.222)
Gov. Mistrust	0.176**	0.278***	0.107	0.332***
	(0.082)	(0.031)	(0.102)	(0.048)
above		4.545*		-0.336
		(2.632)		(1.721)
above×Not Confident		0.236		0.750***
		(0.395)		(0.272)
below		-1.333		-3.199
		(2.995)		(2.740)
below×Not Confident		-0.472		0.034
		(0.464)		(0.440)
Observations	2037	2045	2022	2021
F-Stat First Stage	60.38	63.17	61.03	65.55
R-Squared	0.118	0.192	0.086	0.297

Notes: In the IV regressions we instrument "not confident" with the variables "weight econ" and "ZLB Probability." Robust standard errors in parentheses. Above represents dummy variable denoting a threshold of 2.5 percent and Below a threshold of 1.5 percent respectively. Medium-run expectations are 5 years ahead. *** p<0.01, **p<0.05,*p<0.1

In Table F.4 we explain our "not confident" variable with the probability of the ZLB and the weight respondents believe the ECB will put on the output gap objective. As we can observe, the variable capturing the weight of the output gap is highly significant and has the expected sign. More weight on the growth objective should reduce the confidence of consumers that the ECB will be able to maintain price stability in the medium run. Regarding the ZLB expectations we find no statistically significant effect. This is not surprising given that we conducted the survey in a high inflation environment.

Finally, we re-estimate our standard specification using the two variables (ZLB and perceived weight on the output growth target) as instruments for the confidence variable. With this approach we find strong support for our key mechanism in our theoretical model. The ZLB and the perceived weight on the output gap affect confidence in the ECB to fulfill its price stability objective and consequently induce a bias toward inflation expectations. Results are presented in Table 8.[36]

We can observe that our instruments are relevant, since they comfortably exceed the Olea and Pflueger (2013) thresholds and that results remain qualitatively identical to the standard specifica-

[36]The full set of tables, including summary statistics, the replication, first-stage results, and the IV estimation, is available in Appendix F

tion. Using the instrumental variable approach, we find statistical support for the upward bias in medium- and short-run inflation expectations, caused by a higher relative weight on the output gap in the perceived loss function of the ECB. Hence, we are able to provide clear evidence that confidence in the central bank's ability to meet the inflation target in the medium run is driven by the probability of reaching the zero lower bound of interest rates and the perception of the importance of the output target, which ultimately leads to upward and downward biases in inflation expectations. This inflation bias in on average present in the whole distribution of inflation expectations and is larger for medium-run than for short-run inflation expectations, as in the M&G YouGov survey's evidence. When we split distribution into three regions, the coefficient estimate is significant for the "above" group in the medium-run expectations regression.

6. Conclusion

In this paper, we provide evidence for the (co-)existence of inflation and deflationary biases in medium-run inflation expectations when agents lose confidence in the price stability objective of the central bank. We first build a model with heterogeneous perceptions of an occasionally binding zero-lower-bound constraint and of monetary policy objectives. We show that, in this environment, heterogeneous inflation expectations can arise from having either an inflation bias or a deflationary bias. Second, we test several implications of the model using individual-level data for nine countries on the relationship between losing confidence in a central bank's ability to achieve the specified price stability objective and inflation expectations. This novel survey design allows us to directly test for inflation and deflationary biases in a sample period that covers countries close to the zero lower bound where policymakers have been concerned with inflation below target levels.

In line with the model predictions, our empirical results suggest that both inflation and deflationary biases are present in the survey data. Both biases are sizable: Losing confidence is associated with an inflation bias of a little over 1 percentage point and a deflationary bias of -0.54 percentage point for medium-run inflation expectations. For our sample of countries, we also find that countries that pursue inflation targeting have lower inflation expectations and lower dispersion of inflation expectations as well. However, we do not find that inflation targeting mitigates inflation and deflationary biases: Our results indicate that in our sample the deflationary bias may even become larger under an inflation targeting regime. Thus, we show that there exists a sizable heterogeneity of medium-run inflation expectations due to perceptions of monetary policy objectives. These perceptions, together with the ZLB, cause both biases and, in turn, increase the disagreement of medium-run inflation expectations, imposing greater challenges for the central bank in steering expectations. While the exact size of the biases found in this paper could be specific to the time period analyzed, the heterogeneity of medium-run inflation expectations due to perceptions of monetary policy objectives of is likely to be present also in samples that do not pose a high risk of deflation. In fact, for Germany we can directly test the presence of the mechanism outlined in the model that causes inflation bias and show—using an instrumental variables approach—that higher perceptions of the relative importance of the output gap in the objective function of the ECB increase inflation expectations.

Furthermore, our model allows us to identify the average differences in the perceptions of the European Central Bank's objective function across euro-area countries in our sample. This is particularly interesting, since the EMU countries share the exact same experience in terms of the ECB's monetary policy. The empirical results show quite remarkable differences in terms of the size of the bias and which bias is dominating within each country. These results indicate that the ECB faces an ongoing challenge in convincing households of its objectives. To homogenize the different perceptions, the ECB could think about employing a more targeted communication strategy, tailoring it to the communication needs of each member country. More specifically, Italy would need communication addressing inflationary fears, while Spain would require statements mitigating deflationary concerns.

Overall, our results highlight the (co-)existence and economic relevance of both biases. Central banks have to design policies that would minimize the impact of these biases, particularly in a situation where both biases may arise simultaneously.[37] Besides adjustments in policymaking, these biases may impose additional challenges for central bank communication.

[37] Theoretically, appointing a conservative central banker may be a straightforward solution. Another option, however, would be to introduce a flexible average inflation target and "employment shortfalls" objective such as the FOMC announced in 2020. One could argue that an "employment shortfalls" objective is establishing an inflation bias (see Ruge-Murcia, 2003b) that would balance the deflationary bias due to the presence of the ZLB.

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Appendix A. Trust, Confidence, Reputation, and Credibility

When inflation or deflationary bias is discussed several descriptions of the relationship between the central bank and economic agents are often put forward. Note that it is important to clarify the difference between credibility, reputation, confidence, and trust in the context of this discussion. While all these concepts are inherently related, the game theoretic literature distinguishes between them. To clarify the objective of the question asked in the survey, it relates to the public confidence that the central bank is currently pursuing appropriate policies to achieve price stability's over the medium term. The main difference among credibility (reputation), trust, and confidence is that credibility (reputation) consists of the characteristics of the institution or individual (one-sided), while trust and confidence are inherently two-sided relationships, since they are characterized by the preferences of both agents involved in this *game* (relationship).[38] In economic terms, trust can be defined as "the belief or perception by one party (for example, a principal) that the other party (for example, an agent) to a particular transaction will not cheat" (Knack, 2001). It is more difficult to disentangle the difference between trust and confidence since they are strongly related. Potentially, trust could be a broader concept than confidence, because one could argue that confidence is based on trust, or that it is a perception of trust. In a game theoretic setup, a trust game embeds moral hazard due to uncertainty of which action will be implemented, while a reputation game is characterized by asymmetric information on the type of agent.

Appendix B. Proof of Propositions

B.1. Proof of Proposition 1

We first prove the economy-wide conditions and then discuss individual household conditions. The *standard* Markov perfect equilibrium is given by a vector $y_H, \pi_H, i_H, y_L, \pi_L, i_L$ that solves the following system of linear equations:

$$y_H = [(1-p_H)y_H + p_H y_L] + \sigma[(1-p_H)\pi_H + p_H\pi_L - i_H + r^*] + \tau_H, \tag{24}$$

$$\pi_H = \kappa y_H + \beta[(1-p_H)\pi_H + p_H\pi_L], \tag{25}$$

$$0 = \lambda(y_H - y^*) + \kappa\pi_H, \tag{26}$$

$$y_L = [(1-p_L)y_H + p_L y_L] + \sigma[(1-p_L)\pi_H + p_L\pi_L - i_L + r^*] + \tau_L, \tag{27}$$

$$\pi_L = \kappa y_L + \beta[(1-p_L)\pi_H + p_L\pi_L], \tag{28}$$

and

[38] Credibility and reputation depend solely on the actions and characteristics of the institution or individual. Reputation involves learning based on past experience; in other words, repeated credible actions and achieved targets lead to a certain reputation and uncertainty regarding the *type* of agents slowly dissipates. Generally, trust in institutions and making policy is a wider concept than reputation, since it is the nexus of both preferences of the trustee and the trustor (see, Bursian and Faia, 2018).

$$i_L = 0, \tag{29}$$

and satisfies the non-negativity of the nominal interest rate in the high state

$$i_H > 0, \tag{30}$$

ϕ_L denotes the Lagrange multiplier on the ZLB constraint in the low state:

$$\phi_L := \lambda(y_L - y^*) + \kappa\pi_L. \tag{31}$$

We will prove the four preliminary propositions (propositions 1.A-1.D), and use these propositions to prove the main proposition (proposition 1) on the necessary and sufficient conditions for the existence of the standard Markov perfect equilibrium.

Let

$$A(\lambda) := -\beta\lambda p_H, \tag{32}$$

$$B(\lambda) := \kappa^2 + \lambda(1 - \beta(1 - p_H)), \tag{33}$$

$$C := \frac{(1-p_L)}{\sigma\kappa}(1 - \beta p_L + \beta p_H) - p_L, \tag{34}$$

$$D := -\frac{(1-p_L)}{\sigma\kappa}(1 - \beta p_L + \beta p_H) - (1 - p_L) = -1 - C, \tag{35}$$

and

$$E(\lambda) := A(\lambda)D - B(\lambda)C. \tag{36}$$

Assumption 1.A: $E(\lambda) \neq 0$.

Throughout the rest of this proof, we will assume that Assumption 1.A holds.

Proposition 1.A: There exists a vector $y_H, \pi_H, i_H, y_L, \pi_L, i_L$ that solves (24)-(29).

We can rearrange the system of equations (24)-(29) and eliminate y_H and y_L.
Using (26) we have:

$$y_H = y^* - \frac{\kappa}{\lambda}\pi_H$$

We substitute this value for y_H into equation (25):

$$\begin{aligned}
\pi_H &= \kappa y_H + \beta[(1-p_H)\pi_H + p_H\pi_L] \\
&= \kappa[y^* - \frac{\kappa}{\lambda}\pi_H] + \beta[(1-p_H)\pi_H + p_H\pi_L] \\
\beta p_H\pi_L + \kappa y^* &= \pi_H + \frac{\kappa^2}{\lambda}\pi_H - \beta(1-p_H)\pi_H
\end{aligned}$$

if we multiply this expression by λ:

$$[\kappa^2 + \lambda(1 - \beta(1 - p_H))]\pi_H - \beta\lambda p_H \pi_L = \kappa\lambda y^* \tag{37}$$

When we solve for y_H in equation 25 and y_L in equation 28 we have:

$$y_H = \frac{1}{\kappa}\pi_H - \frac{1}{\kappa}\beta[(1 - p_H)\pi_H + p_H\pi_L] \tag{38}$$

$$y_L = \frac{1}{\kappa}\pi_L - \frac{1}{\kappa}\beta[(1 - p_L)\pi_H + p_L\pi_L] \tag{39}$$

We substitute these values for y_H and y_L into equation 27:

$$\begin{aligned}(1 - p_L)\left[\frac{1}{\kappa}\pi_L - \frac{1}{\kappa}\beta[(1 - p_L)\pi_H + p_L\pi_L]\right] = (1 - p_L)\left[\frac{1}{\kappa}\pi_H - \frac{1}{\kappa}\beta[(1 - p_H)\pi_H + p_H\pi_L]\right] \\ + \sigma[(1 - p_L)\pi_H + p_L\pi_L + r^*] + \tau_L\end{aligned}$$

$$\left[\frac{(1 - p_L)}{\kappa}(1 - \beta p_L + \beta p_H) - \sigma p_L\right]\pi_L - \left[\frac{(1 - p_L)}{\kappa}(1 + \beta p_H - \beta p_L) + \sigma(1 - p_L)\right]\pi_H = \sigma r^* + \tau_L \tag{40}$$

Therefore we have two unknowns, π_H and π_L, and two equations:

$$\begin{aligned}\begin{bmatrix} A(\lambda) & B(\lambda) \\ C & D \end{bmatrix}\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} &= \begin{bmatrix} \kappa\lambda y^* \\ r_L \end{bmatrix} \\ \Rightarrow \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} &= \frac{1}{A(\lambda)D - B(\lambda)C}\begin{bmatrix} D & -B(\lambda) \\ -C & A(\lambda) \end{bmatrix}\begin{bmatrix} \kappa\lambda y^* \\ r_L \end{bmatrix}\end{aligned} \tag{41}$$

where $r_L = r^* + \frac{1}{\sigma}\tau_L$.

Therefore, we have:

$$\pi_H = \frac{A(\lambda)}{E(\lambda)}r_L - \frac{C}{E(\lambda)}\kappa\lambda y^* \tag{42}$$

and

$$\pi_L = \frac{-B(\lambda)}{E(\lambda)}r_L + \frac{D}{E(\lambda)}\kappa\lambda y^* \tag{43}$$

This gives us the following Phillips curves in both states:

$$
\begin{aligned}
y_H &= y^* - \frac{\kappa}{\lambda}\pi_H \\
y_H &= y^* - \frac{\kappa}{\lambda}\left[\frac{A(\lambda)}{E(\lambda)}r_L - \frac{C}{E(\lambda)}\kappa\lambda y^*\right] \\
y_H &= \frac{\beta\kappa p_H}{E(\lambda)}r_L + \left(1 + \frac{C}{E(\lambda)}\kappa^2\right)y^* \qquad (44)
\end{aligned}
$$

$$
\begin{aligned}
y_L &= \frac{1}{\kappa}\left[\pi_L - \beta\left[(1-p_L)\pi_H + p_L\pi_L\right]\right] \\
y_L &= -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)}r_L - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\frac{y^*}{E(\lambda)} \qquad (45)
\end{aligned}
$$

Household local conditions are given by:

$$
\pi_{H,i} = \frac{A_i(\lambda_i)}{E_i(\lambda_i)}r_L - \frac{C_i}{E_i(\lambda_i)}\kappa\lambda_i y_i^* \qquad (46)
$$

$$
\pi_{L,i} = \frac{-B_i(\lambda_i)}{E_i(\lambda_i)}r_L + \frac{D}{E_i(\lambda_i)}\kappa\lambda_i y_i^* \qquad (47)
$$

$$
y_{H,i} = \frac{\beta\kappa p_{H,i}}{E_i(\lambda_i)}r_L + \left(1 + \frac{C_i}{E_i(\lambda_i)}\kappa^2\right)y_i^* \qquad (48)
$$

$$
y_{L,i} = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_{H,i} - \beta p_L)\lambda_i}{\kappa E_i(\lambda_i)}r_L - \lambda_i\left[(1-\beta)C_i + (1-\beta p_L)\right]\frac{y_i^*}{E_i(\lambda_i)} \qquad (49)
$$

Proposition 1.B: Suppose (24)-(29) are satisfied. Then $\phi_L < 0$ for any $\lambda, y^* \geqslant 0$ if and only if $E(\lambda) < 0$

Proof: Notice that

$$
\begin{aligned}
\phi_L &= (\lambda y_L - y^*) + \kappa\pi_L \\
&= \lambda\left[-\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)}r_L - y^*\right] \\
&\quad - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\frac{\lambda y^*}{E(\lambda)} \\
&\quad + \kappa\left[\frac{-B(\lambda)}{E(\lambda)}r_L + \frac{D}{E(\lambda)}\kappa\lambda y^*\right]
\end{aligned} \qquad (50)
$$

Group terms:

$$\phi_L = -\left[\frac{\lambda}{\kappa}\left[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\right] + \kappa B(\lambda)\right]\frac{r_L}{E(\lambda)} + \left[\kappa^2 D - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\right]\frac{\lambda y^*}{E(\lambda)} - \lambda y^* \tag{51}$$

Now simplify the second term:

$$\begin{aligned} &= \left[\kappa^2 D - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\right]\frac{\lambda y^*}{E(\lambda)} \\ &= \left[-\left[\kappa^2 + \lambda(1-\beta)\right]\frac{(1-\beta p_L + \beta p_H)}{\sigma\kappa}(1-p_L) - [\kappa^2+\lambda](1-p_L)\right]\frac{\lambda y^*}{E(\lambda)} \end{aligned} \tag{52}$$

Putting all the terms together we have:

$$\phi_L = -\left[\frac{\lambda}{\kappa}\left[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\right] + \kappa B(\lambda)\right]\frac{r_L}{E(\lambda)} - (1-p_L)\left[\left[\kappa^2 + \lambda(1-\beta)\right]\frac{(1-\beta p_L + \beta p_H)}{\sigma\kappa} + \left[\kappa^2+\lambda\right]\right]\frac{\lambda y^*}{E(\lambda)} - \lambda y^* \tag{53}$$

Thus $(1-p_L)\left[\frac{(\lambda - \lambda\beta + \kappa^2)}{\sigma\kappa}(1-\beta p_L + \beta p_H) + \left[\kappa^2 + \lambda\right]\right] \leqslant 0$ if $p_L \leqslant 1$.

We have that $\tau_L < 0, (1-\beta p_L)\kappa^2 > 0, (1-\beta)(1+\beta p_H - \beta p_L)\lambda \geqslant 0$, and $\kappa B(\lambda) \geqslant 0$. Also, if $E(\lambda) < 0$, then $\phi_L < 0$.

However, if $y^* > 0$, given that:

$$-\left[\frac{\lambda}{\kappa}\left[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\right] + \kappa B(\lambda)\right] r_L > (p_L - 1)\left[\frac{(\lambda - \lambda\beta + \kappa^2)}{\sigma\kappa}(1-\beta p_L + \beta p_H) + (\kappa^2 + \lambda\right.$$

then we come to the same conclusion as above that if $\phi_L < 0$for any $\lambda, y^* \geqslant 0$, then we have to have $E(\lambda) < 0$ and also if $E(\lambda) < 0$, then $\phi_L < 0$. The intuition is that when either λ or y^* is higher, a less negative r_L is needed to hit the ZLB.

Proposition 1.C: $E(\lambda) < 0$ **if and only if** $p_L^* < (\Theta_{-p_L})$
Proof: Let $E(\cdot)$ be a function of p_H and p_L for this purpose.

$$
\begin{aligned}
E(p_H,p_L) =& A(p_H,p_L)D - B(p_H,p_L)C \quad (54)\\
=& -\beta p_H\lambda(-1-C) - \left[\kappa^2 + \lambda\left(1-\beta(1-p_H)\right)\right]C\\
=& \beta p_H\lambda - \left[\kappa^2 + \lambda\left(1-\beta\right)\right]\left[\frac{(1-p_L)}{\sigma\kappa}\left(1-\beta p_L+\beta p_H\right) - p_L\right]
\end{aligned} \tag{55}
$$

Let $\Gamma = \kappa^2 + \lambda\left(1-\beta\right)$.

$$
\begin{aligned}
E(p_H,p_L) =&\\
=& \beta p_H\lambda - \Gamma\left[\frac{(1-p_L)}{\sigma\kappa}\left(1-\beta p_L+\beta p_H\right) - p_L\right]\\
=& -\Gamma\beta\frac{1}{\sigma\kappa}p_L^2 + \Gamma\left[\frac{1}{\sigma\kappa}\left(1+\beta+\beta p_H\right)+1\right]p_L + \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}\left(1+\beta p_H\right)\\
:=& q_2 p_L^2 + q_1 p_L + q_0
\end{aligned} \tag{56}
$$

Where we have that:

$$
q_0 := \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}\left(1+\beta p_H\right) \tag{57}
$$

$$
q_1 := \Gamma\left[\frac{1}{\sigma\kappa}\left(1+\beta+\beta p_H\right)+1\right] \tag{58}
$$

$$
q_2 := -\Gamma\beta\frac{1}{\sigma\kappa} \tag{59}
$$

The function, $E(\cdot,\cdot)$, has the following two properties:

Property 1: $E(p_H,1) > 0$ **for any** $0 \leqslant p_H \leqslant 1$**.**

$$
\begin{aligned}
E(p_H,p_L) =& -\Gamma\beta\frac{1}{\sigma\kappa} + \Gamma\left[\frac{1}{\sigma\kappa}\left(1+\beta+\beta p_H\right)+1\right] + \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}\left(1+\beta p_H\right)\\
=& \Gamma + \beta\lambda p_H > 0
\end{aligned} \tag{60}
$$

Property 2: $E(p_H,p_L)$ **is maximized at** $p_L > 1$ **for any** $0 \leqslant p_H \leqslant 1$**.**

$$\begin{aligned}
\frac{\partial E(p_H, p_L)}{\partial p_L} =& 2q_2 p_L^* + q_1 = 0 \\
\leftrightarrow p_L^* =& -\frac{q_1}{2q_2} \\
=& \frac{\Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right]}{2\Gamma\beta\frac{1}{\sigma\kappa}} \\
=& \frac{\left[\frac{1}{\sigma\kappa}(2\beta+(1-\beta)+\beta p_H)+1\right]}{2\beta\frac{1}{\sigma\kappa}} > 1 \qquad (61)
\end{aligned}$$

(62)

Property 1 and property 2 imply together that i) one root of $E(\cdot, p_L)$ is below 1 and ii) $E(\cdot, p_L) < 0$ below this root. We will call this root $p_L^*(\Theta_{-p_L})$

$$p_L^*(\Theta^1_{-p_L}) := \frac{-q_1 - \sqrt{q_1^2 - 4q_2q_0}}{2q_2}. \qquad (63)$$

Based on the properties outlined above, if $E(\lambda) < 0$, then $p_L < p_L^*(\Theta_{-p_L})$. Likewise, if $p_L < p_L^*(\Theta_{-p_L})$, then $E(\lambda) < 0$. This completes the proof of proposition 1.C. Proposition 1.C. holds regardless of whether the system of linear equations (24)-(29) is satisfied or not.

Proposition 1.D: Suppose (24)-(29) are satisfied and $E(\lambda) < 0$. Then $i_H > 0$ if and only if $p_H < p_H^*(\Theta_{-p_H})$.

Proof: i_H comes from rearranging y_H

$$y_H = [(1-p_H)y_H + p_H y_L] + \sigma[(1-p_H)\pi_H + p_H\pi_L - i_H + r^*] + \tau_H \qquad (64)$$

We multiply by $\frac{1}{\sigma}$:

$$\begin{aligned}
i_H &= \frac{1}{\sigma}[(1-p_H)y_H + p_H y_L] + (1-p_H)\pi_H + p_H\pi_L + r^* + \frac{1}{\sigma}\tau_H - \frac{1}{\sigma}y_H \\
&= \frac{1}{\sigma}[-p_H y_H + p_H y_L] + (1-p_H)\pi_H + p_H\pi_L + r_H
\end{aligned}$$

where $r_H = r^* + \frac{1}{\sigma}\tau_H$.

We have that i_H is equal to the following:

$$i_H = \frac{1}{\sigma}[-p_H y_H + p_H y_L] + (1-p_H)\pi_H + p_H\pi_L + r_H$$

where $r_H = r^* + \frac{1}{\sigma}\tau_H$.

Now we plug in for y_H, y_L, π_H, and π_L:

$$\begin{aligned} i_H = & \frac{-p_H}{\sigma}\left[\frac{\beta\kappa p_H}{E(\lambda)}r_L + \left(1+\frac{C}{E(\lambda)}\kappa^2\right)y^*\right] \\ & + \frac{p_H}{\sigma}\left[-\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)}r_L - \left[(1-\beta)C + (1-\beta p_L)\right]\frac{\lambda y^*}{E(\lambda)}\right] \\ & + (1-p_H)\left[\frac{A(\lambda)}{E(\lambda)}r_L - \frac{C}{E(\lambda)}\kappa\lambda y^*\right] \\ & + p_H\left[\frac{-B(\lambda)}{E(\lambda)}r_L + \frac{D}{E(\lambda)}\kappa\lambda y^*\right] + r_H \end{aligned} \tag{65}$$

Now we group the r_L and y^* terms:

$$\begin{aligned} i_H = & \left[-p_H\left(\left[\left(1+\kappa^2\frac{C}{E(\lambda)}\right) + \left[(1-\beta p_L)+(1-\beta)C\right]\frac{\lambda}{E(\lambda)}\right]\frac{1}{\sigma} - \frac{C\lambda\kappa}{E(\lambda)} - \frac{D\lambda\kappa}{E(\lambda)}\right) - \frac{C\lambda\kappa}{E(\lambda)}\right]y^* \\ & + \left[-p_H\left(\left[\kappa\beta p_H + \frac{(1-\beta p_L)\kappa^2 + \lambda(1-\beta)(1-\beta p_L+\beta p_H)}{\kappa}\right]\frac{1}{\sigma} + A(\lambda) + B(\lambda)\right) + A(\lambda)\right]\frac{r_L}{E(\lambda)} + r_H \end{aligned}$$

Now we look at the y^* term and simplify it:
First we pull $\frac{1}{E(\lambda)}$ out of the expression and group it with the y^* term:

$$\left[-p_H\left(\left[\left(E(\lambda)+\kappa^2 C\right) + \left[(1-\beta p_L)+(1-\beta)C\right]\lambda\right]\frac{1}{\sigma} - C\lambda\kappa - D\lambda\kappa\right) - C\lambda\kappa\right]\frac{y^*}{E(\lambda)}$$

First, we will look at the term multiplied by $\frac{1}{\sigma}$:

$$\begin{aligned} & \left[\left(E(\lambda)+\kappa^2 C\right) + \left[(1-\beta p_L)+(1-\beta)C\right]\lambda\right]\frac{1}{\sigma} \\ & \left[-\beta\lambda p_H D - [\kappa^2 + \lambda(1-\beta(1-p_H))]C + \kappa^2 C + \left[(1-\beta p_L)+(1-\beta)C\right]\lambda\right]\frac{1}{\sigma} \\ & \left[\beta\lambda p_H + \lambda(1-\beta p_L)\right]\frac{1}{\sigma} \end{aligned} \tag{66}$$

Then we look at the term multiplied by $-p_H$

$$\begin{aligned} & -p_H\left(\left[\beta\lambda p_H + \lambda(1-\beta p_L)\right]\frac{1}{\sigma} - C\lambda\kappa - D\lambda\kappa\right) \\ & -p_H\left(\left[\beta\lambda p_H + \lambda(1-\beta p_L)\right]\frac{1}{\sigma} + \lambda\kappa\right) \end{aligned} \tag{67}$$

So our y^* term is:

$$\left[-p_H\left([\beta\lambda p_H+\lambda(1-\beta p_L)]\frac{1}{\sigma}+\lambda\kappa\right)-C\lambda\kappa\right]\frac{y^*}{E(\lambda)}$$
$$\left[\left(-\beta\lambda p_H^2-\lambda p_H+\lambda\beta p_L p_H-(1-p_L)\lambda(1-\beta p_L)-(1-p_L)\lambda\beta p_H\right)\frac{1}{\sigma}-p_H\lambda\kappa+p_L\lambda\kappa\right]\frac{y^*}{E(\lambda)} \tag{68}$$

Now we group our y^* term by power of p_H:

$$\left[\frac{-\beta\lambda}{\sigma}p_H^2+\left(\frac{1}{\sigma}\left[\beta p_L\lambda-\lambda-\lambda(1-p_L)\beta\right]-\lambda\kappa\right)p_H-\frac{\lambda(1-p_L)(1-\beta p_L)}{\sigma}+p_L\lambda\kappa\right]\frac{y^*}{E(\lambda)} \tag{69}$$

So our full simplified y^* term is:

$$\left[\frac{-\beta\lambda}{\sigma}p_H^2-\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma}+\kappa\right)p_H-\lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma}-p_L\kappa\right)\right]\frac{y^*}{E(\lambda)}$$

Next, we group the r_L terms and simplify the expression:

$$\left[-p_H\left(\left[\kappa\beta p_H+\frac{(1-\beta p_L)\kappa^2+\lambda(1-\beta)(1-\beta p_L+\beta p_H)}{\kappa}\right]\frac{1}{\sigma}+A(\lambda)+B(\lambda)\right)+A(\lambda)\right]\frac{r_L}{E(\lambda)}$$
$$\left[-p_H\left(\frac{[\kappa^2+\lambda(1-\beta)][\beta p_H+(1-\beta p_L)]}{\kappa\sigma}+\kappa^2+\lambda(1-\beta)]\right)-\beta\lambda p_H\right]\frac{r_L}{E(\lambda)} \tag{70}$$

We know that $\Gamma=\kappa^2+\lambda(1-\beta)$, so we have:

$$\left[-p_H\frac{\Gamma[\beta p_H+(1-\beta p_L)]}{\kappa\sigma}-p_H\kappa^2-p_H\lambda+p_H\lambda\beta)-\beta\lambda p_H\right]\frac{r_L}{E(\lambda)}$$
$$\left[-p_H\frac{\Gamma[\beta p_H+(1-\beta p_L)]}{\kappa\sigma}-p_H\kappa^2-p_H\lambda\right]\frac{r_L}{E(\lambda)} \tag{71}$$

Now we group our r_L term by the power of p_H:

$$\left[\frac{-\Gamma\beta}{\kappa\sigma}p_H^2-\left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma}+\kappa^2+\lambda\right)p_H\right]\frac{r_L}{E(\lambda)} \tag{72}$$

With the simplification of the y^* term and r_L term, our full expression for i_H becomes:

$$\begin{aligned} i_H = &\left[\frac{-\beta\lambda}{\sigma}p_H^2 - \lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)p_H - \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)\right]\frac{y^*}{E(\lambda)} \\ &+ \left[\frac{-\Gamma\beta}{\kappa\sigma}p_H^2 - \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)p_H\right]\frac{r_L}{E(\lambda)} + r_H \end{aligned} \tag{73}$$

The final expression for i_H grouped by the power of p_H:

$$\begin{aligned} i_H = &\left[\frac{-\beta\lambda}{\sigma}\frac{y^*}{E(\lambda)} - \frac{\Gamma\beta}{\kappa\sigma}\frac{r_L}{E(\lambda)}\right]p_H^2 \\ &- \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{E(\lambda)} + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)\frac{r_L}{E(\lambda)}\right]p_H \\ &- \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)\frac{y^*}{E(\lambda)} + r_H \end{aligned}$$

We want to show that $i_H > 0$ when $E(\lambda) < 0$:

We will multiply the expression by $-E(\lambda)$:

$$\left[\frac{\beta\lambda}{\sigma}y^* + \frac{\Gamma\beta}{\kappa\sigma}r_L\right]p_H^2 + \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)y^* + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)r_L\right]p_H + \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p\right.$$

$$\begin{aligned} &\left[\frac{\beta\lambda}{\sigma}y^* + \frac{\Gamma\beta}{\kappa\sigma}r_L\right]p_H^2 + \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)y^* + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)r_L - r_H\beta\lambda + r_H\Gamma\beta\frac{1-p_L}{\kappa\sigma}\right]p_H \\ &+ \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)y^* + r_H\Gamma\left(\frac{(1-p_L)}{\kappa\sigma}(1-\beta p_L) - p_L\right) > 0 \end{aligned}$$

Now we divide by Γ and $-r_L$:

$$\begin{aligned} &\left[\frac{-\beta\lambda}{\sigma\Gamma r_L}y^* - \frac{\beta}{\kappa\sigma}\right]p_H^2 - \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{\Gamma r_L} + \frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H}{r_L}}{\kappa\sigma} + \frac{\kappa^2 + \lambda(1-\beta\frac{r_H}{r_L})}{\Gamma}\right]p_H \\ &- \frac{\lambda}{\Gamma r_L}\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)y^* - \frac{r_H}{r_L}\left(\frac{(1-p_L)}{\kappa\sigma}(1-\beta p_L) - p_L\right) > 0 \end{aligned}$$

Let

$$P(p_H) = \phi_2 p_H^2 + \phi_1 p_H + \phi_0 \tag{74}$$

where

$$\phi_0 := -\frac{\lambda}{\Gamma r_L}\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right) y^* - \frac{r_H}{r_L}\left(\frac{(1-p_L)}{\kappa\sigma}(1-\beta p_L) - p_L\right) \tag{75}$$

$$\phi_1 := \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{\Gamma r_L} + \frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H}{r_L}}{\kappa\sigma} + \frac{\kappa^2 + \lambda(1-\beta\frac{r_H}{r_L})}{\Gamma}\right] \tag{76}$$

$$\phi_2 := \left[\frac{-\beta\lambda}{\sigma\Gamma r_L} y^* - \frac{\beta}{\kappa\sigma}\right] \tag{77}$$

$$\tag{78}$$

Property 1: $\phi_0 > 0$

$$\begin{aligned} i_H = &\left[\frac{-\beta\lambda}{\sigma}\frac{y^*}{E(\lambda)} - \frac{\Gamma\beta}{\kappa\sigma}\frac{r_L}{E(\lambda)}\right] p_H^2 \\ &- \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{E(\lambda)} + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)\frac{r_L}{E(\lambda)}\right] p_H \\ &- \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)\frac{y^*}{E(\lambda)} + r_H \end{aligned}$$

If we have that $p_H = 0$, then the expression reduces to:

$$i_H = -\lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)\frac{y^*}{E(\lambda)} + r_H \tag{79}$$

When $p_H = 0$ we have $i_H = -\lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)\frac{y^*}{E(\lambda)} + r_H$. $\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa > 0$, when $p_L < p_L^*(\Theta^2_{-p_L}) := \frac{\beta\kappa\sigma+1-\sqrt{(\beta\kappa\sigma+1)^2-4\beta}}{2\beta}$.

This completes the proof of property 1.

Property 2: $\phi_2 < 0$

In order for $\phi_2 < 0$, we must have that:

$$\left[\frac{-\beta\lambda}{\sigma\Gamma r_L} y^* - \frac{\beta}{\kappa\sigma}\right] < 0$$
$$\frac{-\beta\lambda}{\sigma\Gamma r_L} y^* < \frac{\beta}{\kappa\sigma}$$

We will multiply by $-r_L$ since we have by assumption that $r_L < 0$

$$\frac{\beta\lambda}{\sigma\Gamma}y^* < -r_L\frac{\beta}{\kappa\sigma}$$
$$\frac{\lambda}{\Gamma}y^* < -r_L\frac{1}{\kappa}$$
$$y^* < -r_L\frac{\Gamma}{\kappa\lambda}$$
$$y^* < -r_L\frac{\kappa^2+\lambda(1-\beta)}{\kappa\lambda} \qquad r_L < -y^*\frac{\kappa\lambda}{\kappa^2+\lambda(1-\beta)} \tag{80}$$

As long as $r_L < -y^*\frac{\kappa\lambda}{\kappa^2+\lambda(1-\beta)}$, then $\phi_2 < 0$. This completes the proof of Property 2.

Property 1 and property 2, $\phi_0 > 0$ and $\phi_2 < 0$, imply that one root of (74) is non-negative and $i_H > 0$ if and only if p_H is below this non-negative root, given by

$$p_H^*(\Theta_{-p_H}) := \frac{-\phi_1 - \sqrt{\phi_1^2 - 4\phi_0\phi_2}}{2\phi_2}. \tag{81}$$

This completes the proof of proposition 1.D.

With these four preliminary propositions (1.A -1.D), we have what we need to prove proposition 1 for the economy-wide equilibrium.

Proposition 1: There exists a vector $\{y_H, \pi_H, ß_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (24)-(29) and satisfies $\phi_L < 0$ and $i_H > 0$ if and only if $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$, where $p_L^*(\Theta_{-p_L}) = min\{p_L^*(\Theta^1_{-p_L}), p_L^*(\Theta^2_{-p_L})\}$.[39]

Proof of "if" part: Suppose that $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$. According to propositions 1.A there exists a vector $\{y_H, \pi_H, ß_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (24)-(29). According to propositions 1.B and 1.C, $E(\lambda) < 0$ and $\phi_L < 0$. According to proposition 1.D and the fact that $E(\lambda) < 0$, $i_H > 0$. This completes the "if" part of the proof.

Proof of "only if" part: Suppose that $\phi_L < 0$ and $i_H > 0$. According to proposition 1.A there exists a vector $\{y_H, \pi_H, ß_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (24)-(29). According to proposition 1.B and 1.C, $E(\lambda) < 0$ and $p_L < p_L^*(\Theta_{-p_L})$. According to proposition 1.D and the fact that $E(\lambda) < 0$, $p_H < p_H^*(\Theta_{-p_H})$. This completes the "only if" part of the proof for the economy-wide equilibrium.

The proof is exactly the same for each individual household, as $E(\lambda_i) < 0$ and $\phi_L < 0$ holds for every i given proposition 1.B. We just need that for all i $p^*_{H,i} < p^*_{H,i}(\Theta_{-p_{H,i}})$. This is assured if the following expression holds for each i:

[39] It is straightforward to show that the sufficient (but not necessary) condition for $p_L^*(\Theta^1_{-p_L}) < p_L^*(\Theta^2_{-p_L})$ is that $\kappa\sigma < 2/\beta$.

$$p^*_{H,i}(\Theta_{-p_{H,i}}) := \frac{-\phi_{1,i} - \sqrt{\phi^2_{1,i} - 4\phi_{0,i}\phi_{2,i}}}{2\phi_{2,i}}. \tag{82}$$

where:

$$\phi_{0,i} := -\frac{\lambda_i}{\Gamma_i r_L}\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right) y^*_i - \frac{r_H}{r_L}\left(\frac{(1-p_L)}{\kappa\sigma}(1-\beta p_L) - p_L\right) \tag{83}$$

$$\phi_{1,i} := \left[\lambda_i\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*_i}{\Gamma_i r_L} + \frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H}{r_L}}{\kappa\sigma} + \frac{\kappa^2 + \lambda_i(1-\beta\frac{r_H}{r_L})}{\Gamma_i}\right] \tag{84}$$

$$\phi_{2,i} := \left[\frac{-\beta\lambda_i}{\sigma\Gamma_i r_L}y^*_i - \frac{\beta}{\kappa\sigma}\right]. \tag{85}$$

$$\tag{86}$$

B.2. Proof of Proposition 2

We characterize the sign of inflation and output depending on whether it is in a low or high state. We will use the restriction regarding r_L that guarantees us the existence and $E(\lambda) < 0$, the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, that when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

When $y^* = 0$, we get, as in Nakata and Schmidt (2019), that $\pi_H \leqslant 0$, $\pi_L < 0$, $y_H > 0$ and $y_L < 0$.

However, when y^* does not equal zero, our equations are augmented.

$$\pi_H = \frac{A(\lambda)}{E(\lambda)}r_L - \frac{C}{E(\lambda)}\kappa\lambda y^*$$

Given that $-\frac{C}{E(\lambda)}\kappa\lambda y^*$ is a positive number, it is possible under certain conditions for π_H to be positive. Whenever, $y^* > \frac{A(\lambda)r_L}{C\kappa\lambda}$ then $\pi_H > 0$. Under the assumption that restrictions for the existence of equilibrium are satisfied (proposition 1) we can conclude that:

$$\pi_H = \begin{cases} \frac{A(\lambda)}{E(\lambda)}r_L - \frac{C}{E(\lambda)}\kappa\lambda y^* \leqslant 0, & \text{iff } y^* \leqslant -\frac{\beta p_H}{\kappa C}r_L \\ \frac{A(\lambda)}{E(\lambda)}r_L - \frac{C}{E(\lambda)}\kappa\lambda y^* > 0, & \text{iff otherwise} \end{cases}$$

Note that $-\frac{\beta p_H}{\kappa C}r_L = \frac{A(\lambda)r_L}{C\kappa\lambda}$.

$$\pi_L = \frac{-B(\lambda)}{E(\lambda)}r_L + \frac{D}{E(\lambda)}\kappa\lambda y^*$$

Given that $\frac{D}{E(\lambda)}\kappa\lambda y^* > 0$, it is possible under certain conditions for π_L to be positive. Whenever $y^* > \frac{B(\lambda)r_L}{D\kappa\lambda}$, then $\pi_H > 0$.

$$\pi_L = \begin{cases} \frac{-B(\lambda)}{E(\lambda)} r_L + \frac{D}{E(\lambda)} \kappa\lambda y^* < 0, & \text{iff } y^* < \frac{B(\lambda) r_L}{D\kappa\lambda} \\ \frac{-B(\lambda)}{E(\lambda)} r_L + \frac{D}{E(\lambda)} \kappa\lambda y^* \geqslant 0, & \text{iff otherwise} \end{cases}$$

$$y_H = \frac{\beta\kappa p_H}{E(\lambda)} r_L + \left(1 + \frac{C}{E(\lambda)}\kappa^2\right) y^*$$

$y_H > 0$ as long as $\frac{\beta\lambda p_H}{E(\lambda)} r_L + \left(1 + \frac{C}{E(\lambda)}\kappa^2\right) y^* > 0$

We will multiply by $-E(\lambda)$:

$$\begin{aligned} &-\beta\kappa p_H r_L + \left(-E(\lambda) - C\kappa^2\right) y^* > 0 \\ &-\beta\kappa p_H r_L > \lambda\left(\beta p_H - (1-\beta)C\right) y^* \end{aligned} \tag{87}$$

If $(\beta p_H - (1-\beta)C) < 0$, then we have:

$$\frac{-\beta\kappa p_H}{\lambda\left(\beta p_H - (1-\beta)C\right)} r_L < y^* \tag{88}$$

Since both the numerator and the denominator are negative, and given that $r_L < 0$, $\frac{-\beta\kappa p_H}{\lambda(\beta p_H - (1-\beta)C)} r_L < 0$. $y^* \geqslant 0$, this equality will always hold. Thus, $y_H > 0$ when $\beta p_H - (1-\beta)C < 0$. Solving this inequality leads us to the condition $p_L < p_L^*(\Theta^3_{-p_L})$. Now we can straightforwardly show that $p_L^*(\Theta^3_{-p_L}) > p_L^*(\Theta^1_{-p_L})$ from proposition 1 (existence of the equilibria), and thus for all values of p_L that satisfy the existence conditions we have that $y_H > 0$.[40]

$$y_L = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L - \lambda\left[(1-\beta)C + (1-\beta p_L)\right] \frac{y^*}{E(\lambda)}$$

Given that $-\lambda\left[(1-\beta)C + (1-\beta p_L)\right] \frac{y^*}{E(\lambda)} > 0$, it is possible under certain conditions for y_L to be positive. If $y^* > \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{-\lambda[(1-\beta)C + (1-\beta p_L)]\kappa} r_L$, then y_L is positive.

$$y_L = \begin{cases} -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L - \lambda\left[(1-\beta)C + (1-\beta p_L)\right] \frac{y^*}{E(\lambda)} < 0, & \text{iff } y^* < \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)}{-\lambda[(1-\beta)C + (1-\beta p_L)]\kappa} \\ -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L - \lambda\left[(1-\beta)C + (1-\beta p_L)\right] \frac{y^*}{E(\lambda)} \geqslant 0, & \text{iff otherwise} \end{cases}$$

Inflation expectations in the high state are equal to:

$$E_H \pi = (1 - p_H)\pi_H + p_H \pi_L$$

[40] By comparing $p_L^*(\Theta^3_{-p_L})$ and $p_L^*(\Theta^1_{-p_L})$, one can easily show that $p_L^*(\Theta^3_{-p_L}) > p_L^*(\Theta^1_{-p_L})$ as long as $-\kappa^2\beta p_H < 0$, which is always true.

Thus $E_H\pi$ can be written as:

$$E_H\pi = -\frac{p_H(\kappa^2+\lambda)}{E(\lambda)}r_L - \frac{C+p_H}{E(\lambda)}\kappa\lambda y^* \tag{89}$$

and

$$E_H\pi = \begin{cases} -\frac{p_H(\kappa^2+\lambda)}{E(\lambda)}r_L - \frac{C+p_H}{E(\lambda)}\kappa\lambda y^* \leqslant 0, & \text{iff } y^* \leqslant -\frac{p_H(\kappa^2+\lambda)}{(C+p_H)\kappa\lambda}r_L \\ -\frac{p_H(\kappa^2+\lambda)}{E(\lambda)}r_L - \frac{C+p_H}{E(\lambda)}\kappa\lambda y^* > 0, & \text{iff otherwise} \end{cases}$$

Inflation expectations in the low state are:

$$E_L\pi = (1-p_L)\pi_H + p_L\pi_L$$

Thus $E_L\pi$ can be written as:

$$E_L\pi = \frac{\beta\lambda(p_H-p_L) - p_L(\kappa^2+\lambda)}{E(\lambda)}r_L - \frac{C+p_L}{E(\lambda)}\kappa\lambda y^* \tag{90}$$

and

$$E_L\pi = \begin{cases} \frac{\beta\lambda(p_H-p_L)-p_L(\kappa^2+\lambda)}{E(\lambda)}r_L - \frac{C+p_L}{E(\lambda)}\kappa\lambda y^* \leqslant 0, & \text{iff } y^* \leqslant -\frac{\beta\lambda(p_H-p_L)-p_L(\kappa^2+\lambda)}{(C+p_L)\kappa\lambda}r_L \\ \frac{\beta\lambda(p_H-p_L)-p_L(\kappa^2+\lambda)}{E(\lambda)}r_L - \frac{C+p_L}{E(\lambda)}\kappa\lambda y^* > 0, & \text{iff otherwise} \end{cases}$$

B.3. Proof of Proposition 3

In this proposition we characterize how λ affects inflation and output in the low and high state. We will use the restriction that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$\begin{aligned}
\frac{\partial \pi_H}{\partial \lambda} =& \frac{A'(\lambda)E(\lambda) - A(\lambda)E'(\lambda)}{E(\lambda)^2}r_L - \frac{E(\lambda)-\lambda E'(\lambda)}{E(\lambda)^2}C\kappa y^* \\
=& \frac{A'(\lambda)\left[-A(\lambda)-A(\lambda)C-B(\lambda)C\right] - A(\lambda)\left[-A'(\lambda)-A'(\lambda)C-B'(\lambda)C\right]}{E(\lambda)^2}r_L \\
&- \frac{\left[-A(\lambda)-A(\lambda)C-B(\lambda)C\right] - \lambda\left[-A'(\lambda)-A'(\lambda)C-B'(\lambda)C\right]}{E(\lambda)^2}C\kappa y^* \\
=& \frac{-\beta p_H\lambda\left(1-\beta+\beta p_H\right) + \beta p_H\left(\kappa^2 + \left(1-\beta+\beta p_H\right)\lambda\right)}{E(\lambda)^2}Cr_L \\
&- \frac{\left[\beta\lambda p_H + \beta\lambda p_H C - \left[\kappa^2 + \lambda\left(1-\beta(1-p_H)\right)\right]C\right] - \lambda\left[\beta p_H + \beta p_H C - \left(1-\beta(1-p_H)\right)C\right]}{E(\lambda)^2}C\kappa y^* \\
=& \frac{\beta p_H\kappa^2}{E(\lambda)^2}Cr_L + \frac{\kappa^2 C}{E(\lambda)^2}C\kappa y^*
\end{aligned} \tag{91}$$

(92)

$$
\begin{aligned}
\frac{\partial \pi_L}{\partial \lambda} &= \frac{-B'(\lambda)E(\lambda) + E'(\lambda)B(\lambda)}{E(\lambda)^2} r_L + \frac{E(\lambda) - \lambda E'(\lambda)}{E(\lambda)^2} D\kappa y^* \\
&= \frac{A'(\lambda)B(\lambda) - A(\lambda)B'(\lambda)}{E(\lambda)^2} D r_L - \frac{\kappa^2 C}{E(\lambda)^2} D\kappa y^* \\
&= -\frac{\beta p_H \kappa^2}{E(\lambda)^2} D r_L - \frac{\kappa^2 C}{E(\lambda)^2} D\kappa y^*
\end{aligned} \tag{93}
$$

$$
\begin{aligned}
\frac{\partial y_H}{\partial \lambda} &= \frac{-E(\lambda)}{E(\lambda)^2} \beta\kappa p_H r_L + \frac{-E(\lambda)}{E(\lambda)^2} C\kappa^2 y^* \\
&= \frac{-[\beta p_H - (1-\beta)C]}{E(\lambda)^2} \beta\kappa p_H r_L + \frac{-[\beta p_H - (1-\beta)C]}{E(\lambda)^2} C\kappa^2 y^* \\
&= \frac{-[\beta p_H - (1-\beta)C]}{E(\lambda)^2} \left[\beta\kappa p_H r_L + C\kappa^2 y^*\right]
\end{aligned}
$$

As shown in proposition 2, $(\beta p_H - (1-\beta)C) < 0$ for any plausible value of p_L that satisfies the existence of equilibria (proposition 1) then $\frac{\partial y_H}{\partial \lambda} > 0$ iff $y^* > \frac{-\beta p_H}{C\kappa} r_L$ and $\frac{\partial y_H}{\partial \lambda} \leqslant 0$ iff $y^* \leqslant \frac{-\beta p_H}{C\kappa} r_L$.

$$
\begin{aligned}
\frac{\partial y_L}{\partial \lambda} &= \frac{(1-\beta)(1-\beta p_L + \beta p_H)E(\lambda) - \left((1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta p_H)\lambda\right) E'(\lambda)}{\kappa E(\lambda)^2} r_L \\
&\quad - \frac{E(\lambda) - \lambda E'(\lambda)}{E(\lambda)^2} \left[(1-\beta)C + (1-\beta p_L)\right] y^* \\
&= \left[\frac{(1-\beta)(1-\beta p_L + \beta p_H)\left[A(\lambda)D - B(\lambda)C\right]}{\kappa E(\lambda)^2}\right] r_L \\
&\quad - \left[\frac{\left((1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta p_H)\lambda\right)\left[A'(\lambda)D - B'(\lambda)C\right]}{\kappa E(\lambda)^2}\right] r_L + \frac{\kappa^2 C}{E(\lambda)^2} \left[(1-\beta)C + (1-\beta p_L)\right] y^* \\
&= \frac{\beta\kappa p_H}{E(\lambda)^2} \left[(1-\beta)C + (1-\beta p_L)\right] r_L + \frac{\kappa^2 C}{E(\lambda)^2} \left[(1-\beta)C + (1-\beta p_L)\right] y^*
\end{aligned} \tag{94}
$$

$$
\begin{aligned}
\frac{E_H \pi}{\partial \lambda} &= -\left[\frac{E(\lambda)p_H - p_H(\kappa^2 + \lambda)E'(\lambda)}{E(\lambda)^2}\right] r_L - \left[\frac{E(\lambda)(C + p_H)\kappa - (C + p_H)\kappa\lambda E'(\lambda)}{E(\lambda)^2}\right] y^* \\
&\left[\frac{p_H \kappa^2 \beta(p_H + C)}{E(\lambda)^2}\right] r_L + \left[\frac{(C + p_H)\kappa^3 C)}{E(\lambda)^2}\right] y^* \\
&\frac{(p_H + C)\kappa^2}{E(\lambda)^2} \left[\beta p_H r_L + \kappa C y^*\right]
\end{aligned} \tag{95}
$$

Similarly to the condition in proposition 3, we have that $\frac{\partial E_H \pi}{\partial \lambda} > 0$ iff $y^* > \frac{-\beta p_H}{C\kappa} r_L$ and $\frac{\partial E_H \pi}{\partial \lambda} \leqslant 0$ iff $y^* \leqslant \frac{-\beta p_H}{C\kappa} r_L$. Actually, since $E_H \pi$ is a convex combination of π_H and π_L, the thresholds for π_H and π_L are the same for both. Then for $E_H \pi$ it also has to hold in the same manner, as we have shown above. Thus, since the same also holds for $E_L\pi$ (just with different weights), the same thresholds must also hold for $E_L\pi$.

B.4. Proof of Proposition 4

In this proposition we characterize how y^* affects inflation and output in the low and high state. We will use the restrictions that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$\frac{\partial \pi_H}{\partial y^*} = -\frac{C}{E(\lambda)}\kappa\lambda > 0$$

$$\frac{\partial \pi_L}{\partial y^*} = \frac{D}{E(\lambda)}\kappa\lambda > 0$$

$$\begin{aligned}
\frac{\partial y_H}{\partial y^*} &= \left(1 + \frac{C}{E(\lambda)}\kappa^2\right) \\
&= -E(\lambda) - C\kappa^2 \\
&= -\big[\beta\lambda p_H - (\kappa^2 + \lambda[1-\beta])C\big] - C\kappa^2 \\
&= -\beta\lambda p_H + \lambda(1-\beta)C \\
&= -\lambda(\beta p_H + (1-\beta)C)
\end{aligned}$$

Since in our earlier propositions we've imposed the condition that $(\beta p_H + (1-\beta)C) < 0$, then we have that $\frac{\partial y_H}{\partial y^*} > 0$

$$\frac{\partial y_L}{\partial y^*} = \frac{-\lambda[(1-\beta) + (1-\beta)p_L]}{E(\lambda)} > 0$$

$$\frac{E_H\pi}{\partial y^*} = -\frac{C + p_H}{E(\lambda)}\kappa\lambda > 0$$

$$\frac{E_L\pi}{\partial y^*} = -\frac{C + p_L}{E(\lambda)}\kappa\lambda > 0$$

B.5. Proof of Proposition 5

In this proposition we first characterize how p_H affects inflation and output in the low and high state. We will use the restriction that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$\begin{aligned}
E(\lambda) =& -\beta\lambda p_H\left[-1-C\right]-\left[\kappa^2+\lambda(1-\beta(1-p_H))\right]C \\
E(\lambda) =& \beta\lambda p_H-\left[\frac{1-p_L}{\sigma\kappa}(1-\beta p_L+\beta p_H)-p_L\right]\left[\kappa^2+\lambda(1-\beta)\right]
\end{aligned}$$

$$\frac{\partial E(\lambda)}{\partial p_H}=\beta\lambda-\frac{1-p_L}{\sigma\kappa}\beta[\kappa^2+\lambda(1-\beta)]\frac{\partial C}{\partial p_H}=\frac{1-p_L}{\sigma\kappa}\beta\frac{\partial A}{\partial p_H}=-\beta\lambda$$

$$\begin{aligned}
\frac{\partial \pi_H}{\partial p_H} =& \frac{\frac{\partial A(\lambda)}{\partial p_H}E(\lambda)-A(\lambda)\frac{\partial E(\lambda)}{\partial p_H}}{E(\lambda)^2}r_L-\frac{\frac{\partial C}{\partial p_H}E(\lambda)-C\frac{\partial E(\lambda)}{\partial p_H}}{E(\lambda)^2}\kappa\lambda y^* \\
\frac{\partial \pi_H}{\partial p_H} =& \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L\right)\beta\lambda\left[\frac{[\kappa^2+\lambda(1-\beta)]}{E(\lambda)^2}r_L+\frac{\lambda\kappa}{E(\lambda)^2}y^*\right] \qquad (96)
\end{aligned}$$

If we want to examine when $\frac{\partial \pi_H}{\partial p_H} < 0$:

$$\frac{\partial \pi_H}{\partial p_H}=\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L\right)\beta\lambda\left[\frac{[\kappa^2+\lambda(1-\beta)]}{E(\lambda)^2}r_L+\frac{\lambda\kappa}{E(\lambda)^2}y^*\right]<0 \qquad (97)$$

We thus need to show that $\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L\right)\beta\lambda\left[[\kappa^2+\lambda(1-\beta)]r_L+\lambda\kappa y^*\right]<0$.

We know that $\beta\lambda > 0$ and that $\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L\right) > 0$, as the negative root of this equation is equivalent to $p_L^*(\Theta^2_{-p_L})$, which is needed for the existence of the equilibria. Thus, our expression becomes:

$$\begin{aligned}
&\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L\right)\beta\lambda\left[[\kappa^2+\lambda(1-\beta)]r_L+\lambda\kappa y^*\right]<0 \qquad (98)\\
&\left[[\kappa^2+\lambda(1-\beta)]r_L+\lambda\kappa y^*\right]<0 \qquad (99)\\
&\lambda\kappa y^*<-[\kappa^2+\lambda(1-\beta)]r_L \qquad (100)
\end{aligned}$$

Therefore, $\frac{\partial \pi_H}{\partial p_H} < 0$ iff $r_L < -\frac{\kappa\lambda}{[\kappa^2+\lambda(1-\beta)]}y^*$. We showed that this condition must hold for property 2 of the proof of existence.

$$\frac{E_H\pi}{\partial p_H} = -\left[\frac{E(\lambda)(\kappa^2+\lambda) - p_H(\kappa^2+\lambda)\frac{E(\lambda)}{\partial p_H}}{E(\lambda)^2}\right] r_L - \left[\frac{E(\lambda)(\frac{C}{\partial p_H}+1) - (C+p_H)\frac{E(\lambda)}{\partial p_H}}{E(\lambda)^2}\right]\kappa\lambda y^*$$

$$\left[\frac{(\kappa^2+\lambda)[\kappa^2+\lambda(1-\beta)](\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L)}{E(\lambda)^2}\right] r_L + \left[\frac{(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L)\left[\kappa^2+\lambda\right]}{E(\lambda)^2}\right]\kappa\lambda y^*$$

$$\frac{(\kappa^2+\lambda)}{E(\lambda)^2}\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L\right)\left[[\kappa^2+\lambda(1-\beta)]r_L+\kappa\lambda y^*\right] \quad (101)$$

From above, we have that $\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L)-p_L\right) > 0$ so that $\frac{E_H\pi}{\partial p_H} < 0$ iff $r_L < -\frac{\kappa\lambda}{[\kappa^2+\lambda(1-\beta)]}y^*$. Once again, this is the condition that holds for property 2 of the proof of existence.

Appendix C. Estimation Procedure

We use simulated methods of moments to estimate nine parameters of the models. Note that the likelihood function is so non-linear that SMM is the most efficient method to estimate this model. We focus on estimating the following nine parameters $p_{H,H}, p_{H,L}, y^*_H, y^*_L, \lambda_H, \lambda_L, \omega, \xi$, and ζ. Because estimating these parameters varies both the eight point estimates of inflation expectations and their distribution weights, we proceed as follows:

1. For a given combination of the above parameters —if they satisfy the existence criteria— we obtain model values of inflation expectations and the associated weights.

2. Then we compute the midpoints between these inflation expectations, so that we can obtain the eights regions for the distribution of inflation expectations with their associated weights.[41]

3. The next step involves discretizing the empirical distribution to obtain the empirical weights for the same distribution intervals as in the model.

4. We then compute the loss function constituting two parts. The first set of components penalizes the square difference between the empirical and model weights. The second set of components of the loss function penalizes the deviation of the model distribution moments from the empirical distribution moments. In this part we use the square deviations from the mode, the mean, a measure of variance, and a measure of skewness.

5. After computing different components of the loss function, we apply a weighting function that is an identity matrix for the first set of components and a scaled vector (adjusted for different units) for the second set of components. This ensures that both sets of components matter approximately equally.

6. We sum all of the components of the loss function.

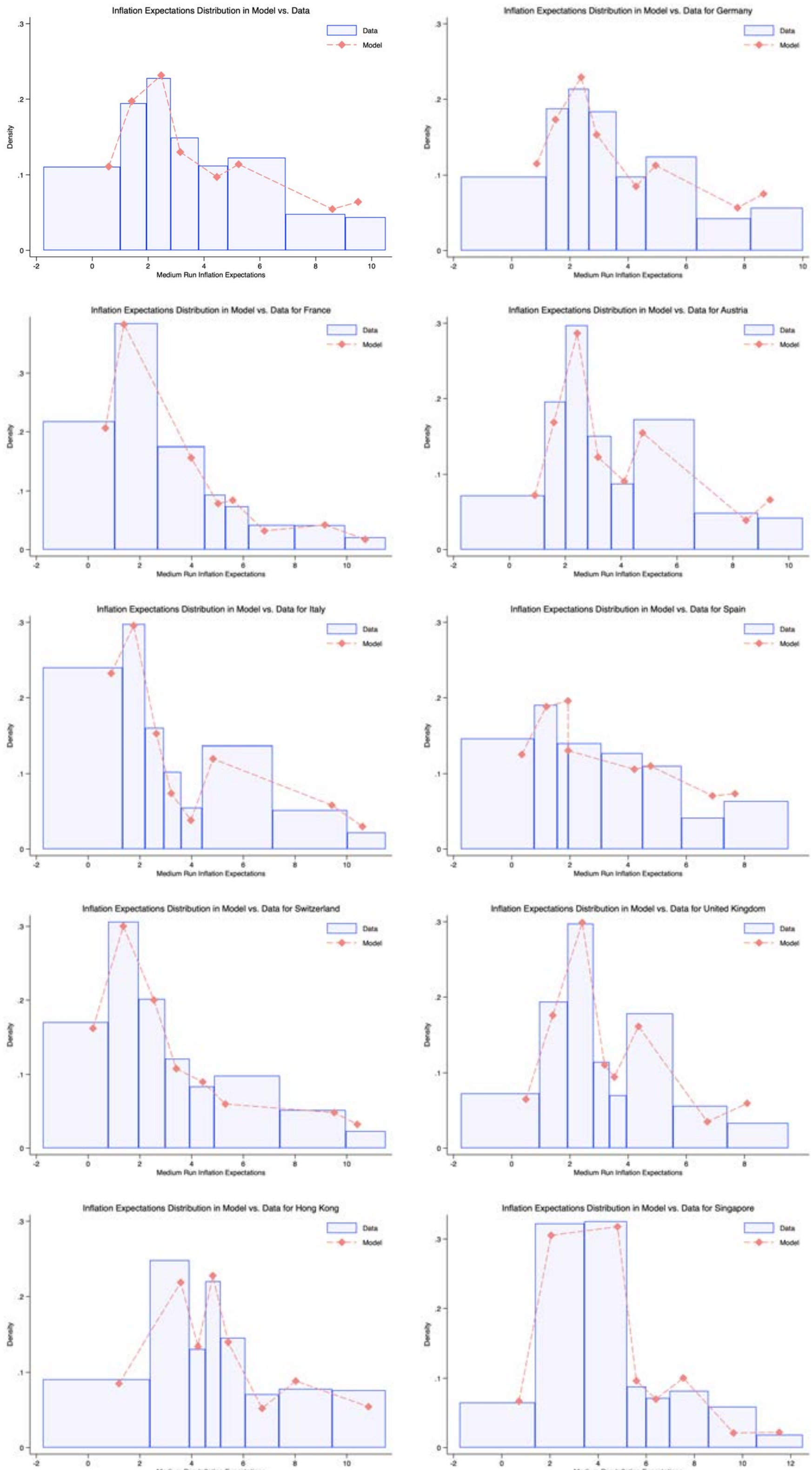
7. We pick the set of parameters that minimize the loss function.

[41] We assume that the inflation target is 2 percent in all countries but Singapore (3 percent) and Hong Kong (4 percent). Note that model estimates for Hong Kong and Singapore only make sense when the inflation target is higher than 2 percent.

Table C.1: Calibrated Parameters

	Calibrated Parameters
β	0.991
α	0.8
η	1.5
σ	1
θ	9
p_L	0.6
τ_h	0.08
τ_l	-0.12

Figure C.1. Distribution of Inflation Expectations in the Model and Data



Appendix D. Summary Statistics

We provide first evidence on the link between confidence in the central bank's price stability objective and inflation expectations. Roughly 60 percent of the survey population is not confident with respect to the central bank in their country meeting its inflation target.[42] This is not unexpected, since this period is dominated by very low inflation rates in Europe. Furthermore, the values are comparable to the EU Eurobarometer survey. The Eurobarometer survey asks: "For each of the following institutions, please tell me if you tend to trust it or tend not to trust it or don't know?". During that period roughly 30 percent of the respondents trusted the ECB, which is very close to the average of 28 percent we report. Interestingly, we can observe that we have some country heterogeneity. While in most of the EMU countries, the majority of respondents are not confident, in countries such as the United Kingdom or Switzerland, the majority are confident that the central bank will meet its inflation objective. Interestingly, only 11.1 percent of the respondents to this survey answered "Don't know" to the question about confidence in the central bank's price stability objective, while "Don't know" answers are much more common for inflation expectations questions: 35.3 percent for short-run inflation expectations and 39.3 percent for medium-run inflation expectations.

To get a first indication of the existence of inflation or deflationary bias, we look at the overall mean for both short- and medium-run inflation expectations, separating the sample into confident and not confident respondents. Using a t-test to compare the mean inflation rate and a Kruskal-Wallis equality of populations rank test to compare the median inflation rate across confidence levels, we find that the mean and median inflation expectations in both the short and the medium run are statistically significantly different at the 99 percent confidence level when compared between different levels of confidence.[43] We furthermore observe that individuals who are not confident in the central bank achieving its price objective have, on average, short-run inflation expectations that are 1 percent higher and almost 1.5 percent higher in the medium run.

From these observations, so far, we can infer that the majority tend to have higher inflation expectations when they lose confidence in achieving price stability. Losing confidence, under certain conditions, may lead to higher inflation expectations for some or to lower inflation expectations for others. Hence we need to find a way to elicit whether hidden behind the majority's response showing an inflation bias, there is a group that responds with lower inflation expectations.

To shed some light on the effect across the distribution, we take three approaches resulting in two graphs and one table. First, we look at the share of "not confident" individuals across the spectrum of inflation expectations, and second, we compare how the distribution of inflation expectations changes depending on whether people are confident or not. Finally, following the empirical strategy based on our model summarized in Table 3, we split the sample into three groups according to the level of inflation expectations (in line with the inflation target, above the target, and below the target) and investigate the mean difference of losing confidence in each of the three groups. For the

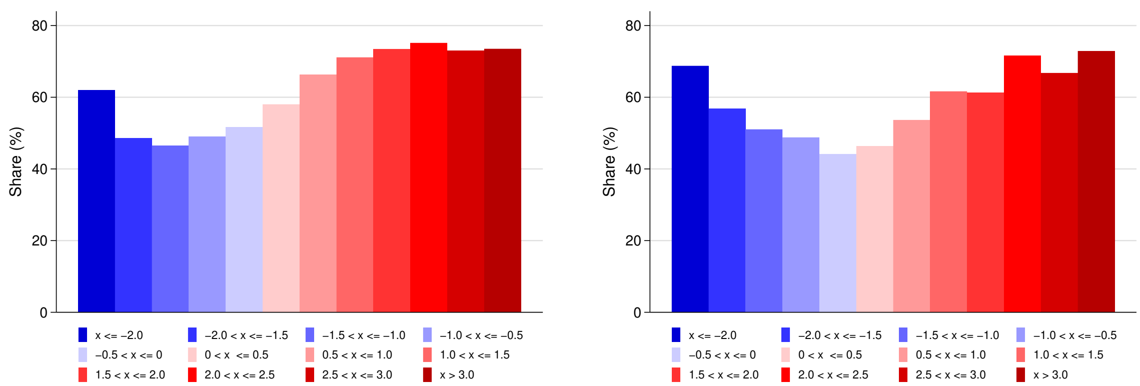
[42] See Table D.3 for details. It contains the shares of consumers who are confident overall and per specific characteristics (age, gender, etc.), countries, and over time.

[43] In Table D.2, we compare the means for short-run inflation expectations and medium-run inflation expectations by the range of confidence in the central bank and compare these means across political orientation, inflation targeting countries, trust in government, countries, gender, age, and survey wave.

first approach, we calculate the share of people who are confident in the price stability objective for intervals of inflation expectations. If we plot the share of confidence across different bins of inflation expectations when there is no deflation bias, we should expect the share of people who are not confident to increase with rising inflation expectations. However, if both biases exist, we should expect a u-shaped relationship. Close to the target inflation rate, the shares of confident respondents should be higher. If we move away from this area, there should be fewer respondents who are confident. Most respondents who have no confidence in the central bank's price stability objective could expect either higher inflation or lower inflation.

Figure C.2 shows the resulting distribution for the share of respondents who are not confident across different levels of inflation expectations for short- and medium-run inflation expectations. We can clearly observe the u-shaped pattern. Most people who are confident have inflation expectations around 1.0 percent to 2.0 percent. Hence, losing confidence is associated with either having high expectations (>2.5 percent) or very low expectations (<0 percent), which clearly indicates the potential of generating both an inflation and a deflationary bias.

Figure C.2. Confidence Level Shares by Distance from Inflation Target



(a) Short Run

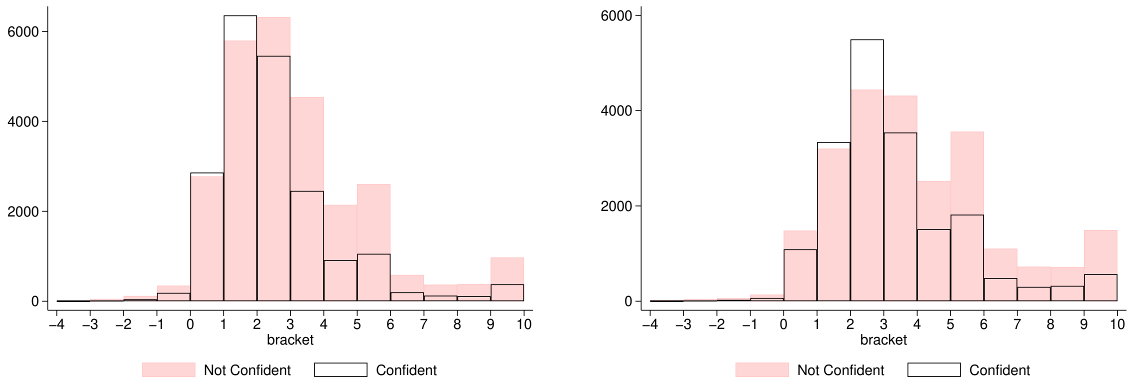
(b) Medium Run

Notes: Bars show the share of people who are not confident in the central bank. Brackets denote the reported inflation expectation minus the inflation target.

Figure C.3 shows the distribution of expectations of confident and not confident individuals for short- and medium-run expectations. Not confident individuals are presented in the shaded (pink) columns with no border outline. We can observe that losing confidence moves the distribution substantially to the right and causes an inflation bias. On closer inspection, however, we can observe that there is a movement to lower inflation expectations as well. This movement is particularly visible for medium-run inflation expectations in the 0-1 percent bracket.

In both figures we observe that there is a substantial inflation bias of not confident individuals, since there is a higher number of people with higher expectations. However, of particular interest and in line with our model predictions, we also observe an increasing amount of individuals who

Figure C.3. Distribution of Short- and Medium-run Inflation Expectations of Confident and Non-Confident Individuals



(a) Short Run

(b) Medium Run

Notes: The black outlined clear bars show the distribution of confident individuals' inflation expectations, while the shaded (pink) columns with no border outline show the distribution of expectations of not confident individuals. Short-run expectations are one year ahead. Medium-run expectations are five years ahead.

have lower inflation expectations as a response to losing confidence and hence have a deflationary bias. This deflationary bias is particularly strong for medium-term inflation expectations.

Table D.1: Summary Statistics for Macroeconomic Variables

	Austria	France	Germany	Hong Kong	Italy	Singapore	Spain	Switzerland	UK	All
Inflation Rate										
Mean	1.36	0.50	0.94	3.95	0.39	0.68	0.01	-0.47	1.24	0.95
	(0.52)	(0.40)	(0.49)	(1.02)	(0.44)	(1.13)	(0.87)	(0.58)	(1.07)	(1.41)
Min, Max	0.64, 2.27	-0.21, 1.04	0.02, 1.58	2.30, 5.33	-0.13, 1.21	-0.73, 2.16	-1.08, 1.87	-1.40, 0.12	-0.02, 2.73	-1.40, 5.33
Interest Rate										
Mean	0.20	0.20	0.20	0.51	0.20	0.25	0.20	-0.28	0.50	0.22
	(0.19)	(0.19)	(0.19)	(0.03)	(0.19)	(0.24)	(0.19)	(0.40)	(0.00)	(0.30)
Min, Max	0.05, 0.58	0.05, 0.58	0.05, 0.58	0.50, 0.58	0.05, 0.58	0.05, 0.79	0.05, 0.58	-0.84, 0.02	0.50, 0.50	-0.84, 0.79
Unemployment Rate										
Mean	5.57	10.32	4.92	3.28	12.26	1.93	24.01	4.79	6.26	8.15
	(0.22)	(0.12)	(0.26)	(0.08)	(0.41)	(0.09)	(1.80)	(0.22)	(0.94)	(6.46)
Min, Max	5.08, 5.88	10.14, 10.47	4.49, 5.27	3.13, 3.40	11.52, 12.74	1.80, 2.10	20.94, 26.19	4.44, 5.14	5.08, 7.75	1.80, 26.19
Output Gap, % of Potential GDP										
Mean	-0.51	-0.12	-0.28	-0.03	-0.92	0.09	-1.37	0.08	0.24	-0.31
	(0.34)	(0.17)	(0.22)	(4.22)	(0.32)	(0.58)	(1.23)	(0.38)	(0.30)	(1.52)
Min, Max	-1.05, 0.05	-0.38, 0.11	-0.51, 0.15	-5.29, 5.68	-1.42, -0.28	-0.92, 1.01	-2.58, 0.69	-0.27, 0.93	-0.45, 0.67	-5.29, 5.68
Data Source	OECD Economic Outlook No. 106[a]	OECD Economic Outlook No. 106[a]	OECD Economic Outlook No. 106[a]	Hong Kong Monetary Authority Statistical Bulletin	OECD Economic Outlook No. 106[a]	Dept. of Statistics Singapore	OECD Economic Outlook No. 106[b]	OECD Economic Outlook No. 106[a]	OECD Economic Outlook No. 106[c]	

Notes: The variable means over the sample period are presented with standard deviations in parentheses. Inflation rate is calculated as the changes in the CPI from same quarter one year ago to current quarter. Output gap is expressed as a percentage of potential GDP. Potential GDP is calculated as the trend component of HP filtered GDP. The interest rate for the ECB countries is the Main Refinancing Operations fixed rate. Since during this time period the SwiSS National Bank (SNB) set a target range around the three-month Swiss franc Libor, the interest rate for Switzerland is set at the three-month Swiss Franc Libor rate. The United Kingdom's interest rate is the official Bank Rate. The interest rate for Singapore is the Singapore overnight rate average. The interest rate for Hong Kong is the overnight discount window rate.

[a] With the exception that the interest rate is taken from ECB Statistical Data Warehouse.

[b] With the exception that the interest rate is taken from SNB data portal.

[c] With the exception that the interest rate is the official Bank Rate taken from the Bank of England database.

Table D.2: Summary Statistics for Expected Inflation for Combined Confidence Measure

			Political Orientation		Inflation Targeting		Gov. Trust		Country							
	No. Obs.	Total	Right	Left	No	Yes	Trust	Distrust	Austria	France	Germany	Hong Kong	Italy	Singapore	Spain	Switzerland	UK
Short-Run Inflation																	
Not Confident	29,233	3.84***	3.74***	3.84***	5.08***	3.56***	3.83***	3.85***	3.39***	3.09***	3.53***	5.37**$^{\diamond}$	4.68***	4.59***	3.65$^{\diamond\diamond\diamond}$	3.52***	2.75***
Confident	21,246	2.81	2.75	3.04	4.39	2.51	2.91	2.45	2.84	2.62	2.44	5.06	3.30	4.00	3.69	1.99	1.98
Medium-Run Inflation																	
Not Confident	27,204	5.34***	5.13***	5.08***	7.29***	4.88***	5.24***	5.39***	5.09***	4.66***	5.21***	7.82***	5.41***	6.40***	4.48$^{\diamond\diamond\diamond}$	5.39***	4.29***
Confident	19,928	3.84	4.06	3.65	5.71	3.45	3.92	3.53	3.59	3.62	3.47	6.56	3.69	5.22	4.36	3.33	3.07

		Gender		Age			Survey										
	No. Obs.	Male	Female	Young	Middle	Old	3	4	5	6	7	8	9	10	11	12	13
Short Run Inflation																	
Not Confident	29,233	3.33***	4.47***	4.59***	3.99***	3.37***	4.37***	4.07***	3.99***	4.10***	3.80***	3.84***	3.78***	3.45***	3.57***	3.66***	3.64***
Confident	21,246	2.53	3.31	3.82	3.11	2.26	3.58	3.22	3.28	2.92	3.18	2.80	2.79	2.53	2.39	2.56	2.41
Medium Run Inflation																	
Not Confident	27,204	4.95***	5.82***	6.11***	5.48***	4.84***	5.68***	5.52***	5.64***	5.59***	5.44***	5.18***	5.20***	5.14***	5.11***	5.08***	5.12***
Confident	19,928	3.55	4.35	4.81	4.03	3.32	4.40	4.14	4.19	4.19	3.94	3.79	3.85	3.58	3.61	3.64	3.49

Notes: The mean (median) of each group variable is compared between confident and not confident. For example, when testing Austria we are comparing the mean (median) of Not Confident to the mean (median) of Confident within Austria. $\star$,$\star\star$, $\star\star\star$ denote significance at the 10, 5, and 1 percent level, respectively, for the two sample t tests. $\diamond$,$\diamond\diamond$, $\diamond\diamond\diamond$ denote significance at the 10, 5, and 1 percent level, respectively, for the Kruskal-Wallis equality of populations rank test.

Table D.3: Share of Confidence for Combined for Confidence Measure

		Political Orientation		Inflation Targeting		Gov. Trust		Country									
	Total	Right	Left	No	Yes	Trust	Distrust	Austria	France	Germany	Italy	Spain	EU	Hong Kong	Singapore	Switzerland	UK
Not Confident	61.98***	57.35***	71.58***	60.78***	62.19***	43.65***	83.26***	73.55***	77.54***	66.54***	65.88***	77.37***	71.77***	72.54***	48.55**	37.02***	40.29***
Confident	38.02	42.65	28.42	39.22	37.81	56.35	16.74	26.45	22.46	33.46	34.12	22.63	28.23	27.46	51.45	62.98	59.71

		Gender		Age			Survey										
	Total	Male	Female	Young	Middle	Old	3	4	5	6	7	8	9	10	11	12	13
Not Confident	61.98***	56.86***	67.24***	62.55***	63.11***	61.32***	69.67***	65.28***	69.24***	65.40***	61.62***	61.04***	61.54***	57.71***	58.16***	59.23***	57.17***
Confident	38.02	43.14	32.76	37.45	36.89	38.68	30.33	34.72	30.76	34.60	38.38	38.96	38.46	42.29	41.84	40.77	42.83

*Notes:*The median of each group variable is compared between confident and not confident. For example, when testing Austria we are comparing the median of Not Confident to the median of Confident within Austria. *,**,*** denote significance at the 10, 5, and 1 percent level, respectively, for a one-sample test of proportion.

Appendix E. Robustness

In this section, we conduct several types of robustness checks to solidify our main results. Specifically, we check the robustness of the specification in Table 5 column (3). Table E.1 contains a set of robustness exercises we executed. For ease of reading, column (1) of Table E.1 replicates our main specification we show in column (3) of Table 5.

E.1. Short-Run Effects

Our main results focus on medium-run expectations. Nevertheless, it is interesting to check what the short-run (one-year-ahead) effect is of losing confidence in the price stability objective and whether we can observe the same asymmetry in the short run. One would expect that, in the short run, the deflationary and inflation biases are smaller. Column (2) of Table E.1 contains the estimation results for this time horizon. Overall, the results are in line with our expectations; we provide evidence for inflation and deflationary biases also in one-year-ahead expectations. In terms of size, the effects are lower than in the medium run. Losing confidence results in an inflation bias of 0.59 percent compared to 1.04 percent for the medium run. Deflationary bias is 0.42 in the short run, compared to 0.54 in the medium run. This is not unexpected, since one-year-ahead is a shorter horizon where the effect of losing confidence should not fully materialize.

E.2. Government Trust

One could argue that consumers may state that they are not confident in the central bank—not necessarily because of their belief in the efficacy of the central bank to maintain price stability, but because of a general lack of trust in government policy or public institutions in general. Usually this is hard to tackle, since one must compare the opinion toward other government bodies or entities with the opinion regarding the central bank. To account for this possibility, we decided to exclude all respondents who distrust the government. With this, we exclude everybody who distrusts the government and also has no confidence in the central bank, thereby accounting for a potential generally negative attitude toward public institutions. Estimation results are presented in column (3) of Table E.1. Again, our results hold, and we confirm the inflation and deflationary biases even in this substantially reduced sample. The inflation bias is 0.75 percent and the deflationary bias is 0.37 percent in this subsample.

E.3. Higher-Order Fixed Effects

While we include a set of variables to control for macroeconomic events, one could argue that we potentially miss some relevant variation in the data. To control for that, we estimate region times time fixed effects accounting for any variation in one quarter within one region (most countries in the sample are composed of several regions) that could drive our results. Estimation results are presented in column (4) of Table E.1 and again are qualitatively identical.

Table E.1: Robustness Checks

	(1) Main	(2) Short-Run	(3) Gov. Trust	(4) Higher Order FE	(5) Huber	(6) Bootstrap	(7) Heckman	(8) 1%-3%	(9) Country Specific	(10) Full Measure
Not Confident	-0.423***	-0.232***	-0.333***	-0.457***	-0.040*	-0.423***	-0.047***	-0.370***	-0.349***	
	(0.04)	(0.05)	(0.04)	(0.04)	(0.02)	(0.03)	(0.01)	(0.04)	(0.04)	
Not Confident - Full										-0.153***
										(0.01)
Gov. Mistrust	0.127***	-0.099**		0.117**	0.037***	0.127**	-0.015	0.105**	0.084*	0.058
	(0.05)	(0.04)		(0.05)	(0.01)	(0.05)	(0.02)	(0.05)	(0.05)	(0.05)
Inflation Target	-0.650***	-0.607***	-0.560***	-0.541**	-0.420***	-0.650***	-0.107	-0.391**	-0.502***	-0.660***
	(0.18)	(0.21)	(0.20)	(0.26)	(0.04)	(0.18)	(0.07)	(0.18)	(0.17)	(0.18)
Below	-1.337***	-0.797***	-1.346***	-1.361***	-1.053***	-1.337***	-2.775***	-1.608***	-1.031***	-1.394***
	(0.03)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)	(0.14)	(0.03)	(0.03)	(0.02)
Below × Not Confident	-0.123***	-0.186***	-0.042	-0.112***	-0.114***	-0.123***	-1.580***	-0.156***	-0.288***	-0.050***
	(0.03)	(0.06)	(0.05)	(0.04)	(0.03)	(0.04)	(0.20)	(0.04)	(0.03)	(0.01)
Above	3.439***	1.892***	3.500***	3.439***	1.683***	3.439***	1.143***	3.940***	3.445***	4.112***
	(0.08)	(0.07)	(0.09)	(0.09)	(0.02)	(0.08)	(0.03)	(0.10)	(0.09)	(0.06)
Above × Not Confident	1.462***	0.821***	1.082***	1.458***	0.407***	1.462***	0.247***	1.344***	1.387***	0.626***
	(0.08)	(0.08)	(0.10)	(0.08)	(0.03)	(0.07)	(0.02)	(0.08)	(0.08)	(0.03)
Observations	45,396	48,517	24,934	45,357	47,232	45,396	74,265	45,396	45,396	45,396
R-Squared	0.276	0.163	0.275	0.292	0.614	0.276		0.307	0.299	0.278
Region FE	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes
Macrovar	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
RegionTimeFE	No	No	No	Yes	No	No	No	No	No	No

If not otherwise specified in the column: Two-way (region time) clustered standard errors in parentheses. Above represents dummy variable denoting a threshold of 2.5 percent and Below a threshold of 1.5 percent in all columns but column (8) respectively. Medium-run inflation expectations are 5 years ahead. Short-run inflation expectations are 1 year ahead. All regressions include region and time fixed effects. Below × Not Confident and Above × Not Confident represent interaction terms between Not Confident and the Above or Below threshold. FE denotes fixed effects. Socio stands for control variables on socioeconomic characteristics and MacroVar for macroeconomic control variables. Not Confident - Full refers to measuring confidence on a scale between -2 to 2 while, Not Confident is a 0/1 variable. Higher values for both variables imply being less confident. The estimates of the interaction terms Below × Not Confident and Above × Not Confident in column (9) are based on the interaction between the full measure and the thresholds.

** p<0.01

** p<0.05

* p<0.1

E.4. Huber Estimation

In consumer survey data extreme values are regularly observed, and they may affect the estimation. As already indicated in the data section, various approaches have been taken to address this issue, ranging from truncating the sample to outlier robust estimation techniques. To make sure that our results are not driven by our truncation method, we follow Coibion et al. (2022) and employ Huber estimation. As can be observed in column (5) of Table E.1 our main results hold, as both inflation and deflationary biases remain statistically significant. In terms of changes in the magnitudes of the coefficient estimates, it seems that both biases become less pronounced which is not surprising as some larger values/changes have been re-weighted by this procedure.

E.5. Bootstrap Standard Errors

Our standard errors are clustered at a time times regional level and therefore account for joint variation in regions at one point in time. However, one could argue that we are not accounting for the possibility that the errors follow a different pattern. To capture this concern, we decided to bootstrap our errors at the level of our clustered standard errors. Results are presented in column (6) of Table E.1. Again our main results remain unaffected.

E.6. Heckman Selection Bias

Another issue to check is a potential selection bias. To account for that, we use a Heckman selection approach. In the selection equation we use the same set of variables plus an additional variable: the fear of inflation.[44] Results are presented in column (7) of Table E.1. Again our results remain qualitatively identical. Results suggest that the size of the inflation and deflationary biases may be partially influenced by the selection bias, as the size of the deflationary bias increases and the size of the inflation bias decreases. This may be because respondents who are more likely to report deflationary bias more often do not answer the questions regarding long-run inflation expectations.

E.7. Alternative Threshold of 1 Percent to 3 Percent

As already mentioned in the main text, the threshold we chose was ad hoc and results might hinge upon that. To strengthen our results further, we re-estimated the main specification for an alternative threshold of 1 percent to 3 percent. Results are presented in column (8) of Table E.1. Comparing column (8) to column (1), we can observe that we have identical qualitative results as well as results that are quantitatively very similar. The overall inflation bias is 1.04 in column (1) and is 0.97 column (8) of Table E.1. The Above×Not Confident coefficient is slightly lower at 1.35 compared to 1.46 in column (1). The deflationary bias is almost exactly the same at 0.52 in column (8) and 0.54 in column (1). The Below×Not Confident coefficient in column (8) is slightly more negative. Hence, changing the threshold does not affect our results.

An additional check for the thresholds is to utilize another question in the survey that asks the tendency to agree with the following statement: "Rising inflation is giving me and my family

[44] The fear of inflation question reads: "Rising inflation is giving me and my family cause for concern at the moment."

cause for concern at the moment." As for the questions regarding confidence in the price stability objective, the answers range from strongly agree to strongly disagree. Table E.2 shows the categories of being concerned about rising inflation against the corresponding average short- and medium-run inflation expectations in this category. As can be seen, the average responses of those who agree and disagree are broadly in line with the thresholds we consider, that is, either 1.5 percent and 2.5 percent or 1 percent and 3 percent.

Table E.2: Inflation Concern and Short- and Medium-Run Inflation Expectations

Concern	π_s^e	π_m^e
Strongly disagree	1	2
Tend to disagree	1.5	2.4
Neither agree nor disagree	2	3
Tend to agree	2.5	3.5
Strongly agree	3	4

Notes: π_s^e denotes averaged short-run expectations, while π_m^e represents averaged medium-run expectations in the corresponding "concern" category.

E.8. Alternative Country-Specific Thresholds

To provide yet another robustness test on the thresholds, we compute country-specific thresholds. Our goal is to find country-specific thresholds, where the reference group will have no inflation and no deflation biases. As we can see in column (1) of Table E.1, there is a deflationary bias in the reference group, as the coefficient on not confident is significantly negative: At least in some countries the reference group thresholds are potentially too low. To study this we implement a country-specific regression with a fixed reference group width (difference between above and below). We find that at least for Singapore and Hong Kong, the reference group is potentially set too low.[45] Results are presented in column (9) of Table E.1. Comparing this table to the main table, we can observe that we have qualitative identical results. As one would expect, by more correctly identifying the region with deflation bias, the estimate of deflationary bias slightly increases, by about 0.1 percent, while the estimate of inflation bias is virtually unchanged (1.04).

E.9. Full Measure of Confidence

For ease of exposition, we decided to work with a 0/1 measure of confidence in price stability. Our survey, however, captures confidence on an ordinal scale. To see whether this simplification might affect our results, we re-estimate the main table with this full (ordinal) measure, which ranges between -2 and 2, where 2 denotes individuals who are not at all confident and -2 respondents who are very confident in the central bank achieving price stability. Results are reported in column

[45] Specifically, we set the reference group width to 3-4 percent for Hong Kong; 2-3 percent for Singapore; 1-2 percent for Switzerland; 2-3 percent for Germany, the UK, and Italy; and 1.5-2.5 percent for the remaining countries.

(10) of Table E.1. Again, the results remain qualitatively identical. By imposing linearity on the marginal effect, the estimates of the full measure are very close to the coefficient estimates of our dichotomous measure of confidence.

E.10. Additional Socioeconomic Variables

One might argue that some socioeconomic controls are not accounted for in our main specification. To counter this argument, we use a substantially larger set of socioeconomic characteristics and compare the results to the results obtained from our main specification. Column (2) of Table E.3 shows the result for the UK with the set of socioeconomic characteristics we use in our main specification, while column (3) presents the results with the full set of socioeconomic characteristics. Considering the variable of interest, we see that the qualitative results remain the same. Even quantitatively the estimated coefficients are almost identical. Hence, we can conclude that our results are robust regarding the choice of the conditioning set of socioeconomic controls.

Table E.3: Additional Sociodemographic Controls

	(1) Full Sample	(2) UK	(3) UK
Not Confident	-0.416***	-0.191***	-0.238***
	(0.0359)	(0.0331)	(0.0405)
Gov. Mistrust	0.113**	0.175***	0.153***
	(0.0491)	(0.0430)	(0.0415)
Below	-1.337***	-1.024***	-1.053***
	(0.0259)	(0.0274)	(0.0292)
Below × Not Confident	-0.127***	-0.162**	-0.133*
	(0.0343)	(0.0643)	(0.0685)
Above	3.454***	2.024***	2.012***
	(0.0832)	(0.0517)	(0.0520)
Above × Not Confident	1.455***	1.252***	1.279***
	(0.0765)	(0.0872)	(0.0858)
Female	0.680***	0.437***	0.460***
	(0.0501)	(0.0598)	(0.0607)
Age: Young	0.633***	0.593***	0.601***
	(0.0668)	(0.1175)	(0.1287)
Age: Old	-0.356***	0.098	0.112
	(0.0524)	(0.0595)	(0.0700)
Income Changed	-0.145***	-0.163***	-0.141***
	(0.0312)	(0.0451)	(0.0473)
Social Grade: ABC1			-0.303***
			(0.0706)
Marital Status: Living as Married			-0.074
			(0.1207)
Marital Status: Married or Civil Partnership			-0.133
			(0.0881)
Marital Status: Separated or Divorced			0.017
			(0.1153)
Marital Status: Widowed			-0.383***
			(0.1378)
Marital Status: Refused			-0.154
			(0.3060)
Working Status: Working Part Time			0.094
			(0.1101)
Working Status: Unemployed			0.638**
			(0.2743)
Working Status: Not Working or Other			0.394**
			(0.1695)
Working Status: Retired			0.122
			(0.0781)
Working Status: Full Time Student			-0.246
			(0.2181)
Children in Household: One			0.199**
			(0.0941)
Children in Household: Two			0.163
			(0.0992)
Children in Household: Three or More			0.116
			(0.1353)
Children in Household: Don't Know or Refused			0.313
			(0.2840)
Observations	45,396	8,631	8,624
R-Squared	0.276	0.283	0.289
MacroVars	Yes	Yes	Yes
Socio	Yes	Yes	Yes
Region FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes

Notes: Baseline group is an individual with no children living in the household, never married and working full-time. Two-way (region time) clustered standard errors in parentheses. "Above" represents dummy variable denoting a threshold of 2.5 percent and "Below" a threshold of 1.5 percent respectively. Medium-run inflation expectations are 5 years ahead. All regressions include regional and year fixed effects. Errors are two-way clustered over time and region. Below × Not Confident and Above × Not Confident represent interaction terms between "Not Confident" and the "Above" or "Below" threshold. "FE" denotes fixed effects. "Socio" stands for control variables on socioeconomic characteristics and "MacroVar" for macroeconomic control variables.
*** $p < 0.01$
** $p < 0.05$
* $p < 0.1$

E.11. Country-Specific Estimates for Non-EMU Countries

While we have already discussed the country estimates for the EMU countries, this section provides the estimates of the remaining countries in our sample. Table E.4 contains the estimates for this group. For all countries in this group we confirm an inflation bias for inflation expectations above 2.5 percent. Only for the UK can we report a statistically significant coefficient estimate for the deflationary bias. For Singapore and Hong Kong the coefficient estimates are negative, but insignificant. The UK result is remarkable, as the UK economy in our sample period has had positive growth rates and inflation levels very close to the target level. Despite these economic conditions, we still find a substantial deflationary bias. Hence, this shows that deflationary bias is present when the inflation level is already very low and the probability of a negative shock is very high. It is no surprise that we find no significant effects for Hong Kong and Singapore, as observing higher levels of inflation and a positive output gap reduces the likelihood of a negative economic shock pushing the economy to the zero lower bound.

Table E.4: Non-EMU Countries Sample

	(1) Non-EMU	(2) UK	(3) Switzerland	(4) Singapore	(5) Hong Kong
Not Confident	-0.499***	-0.191***	-0.106	-0.364***	-0.504***
	(0.0631)	(0.0331)	(0.1100)	(0.0997)	(0.1405)
Gov. Mistrust	0.374***	0.175***	-0.048	1.091***	0.598***
	(0.0692)	(0.0430)	(0.2159)	(0.2516)	(0.1849)
Below	-1.360***	-1.024***	-1.205***	-1.150***	-1.328***
	(0.0458)	(0.0274)	(0.0552)	(0.0893)	(0.2147)
Below × Not Confident	0.076	-0.162**	0.023	-0.125	-0.202
	(0.0719)	(0.0643)	(0.1183)	(0.1634)	(0.2780)
Above	2.820***	2.024***	4.021***	3.965***	4.840***
	(0.0952)	(0.0517)	(0.1820)	(0.1103)	(0.1857)
Above × Not Confident	1.586***	1.252***	2.284***	0.945***	1.203***
	(0.1019)	(0.0872)	(0.3178)	(0.1799)	(0.2555)
Observations	19,719	8,631	2,969	3,812	4,307
R-Squared	0.261	0.283	0.367	0.156	0.081
MacroVars	Yes	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes

Notes: See notes to Table 4 in main text.

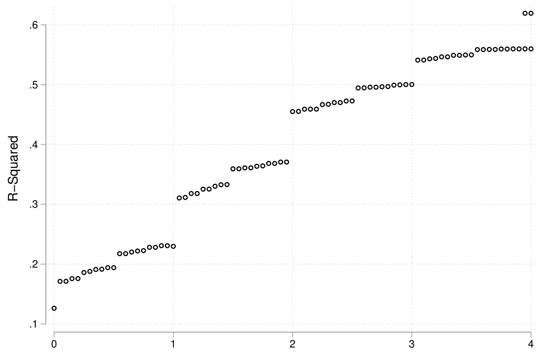
E.12. Robustness for the EMU Results

There is one additional possibility regarding differences in the perceptions of the ECB's objective function that is not explicitly modeled in eq. (10): differences with respect to the perceptions of the inflation target. Although the ECB clearly stated at that time that the objective is to keep inflation "close, but below 2 percent inflation," Paloviita et al. (2021) have shown that, in practice, this means that the inflation target is around 1.7 percent. To study the robustness of our results for the EMU we perform two exercises. In the first exercise we replicate Table 6 using a threshold of 1 percent to 2 percent that may be more appropriate given the price stability definition of the ECB. Results are reported in Table E.5: They are qualitatively very similar. To see how different the average household's perception is across the member states in our sample, we use the following approach in the second exercise to investigate whether the perceived inflation target is within the 1.5 percent and 2.5 percent range that we specify: We use our main regression, as in Table 6, with one small adjustment that introduces an additional dummy variable for expectations above 5 percent to investigate which range for the reference group maximizes the fit of the model.

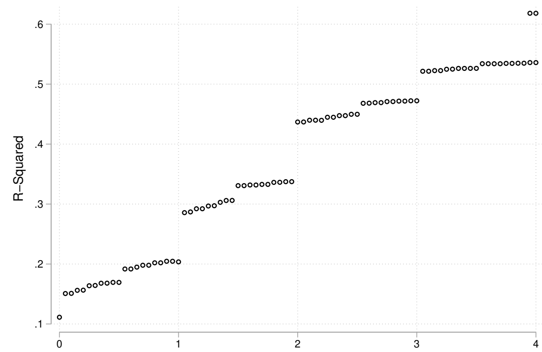
We run a grid search for each country separately with a constant 1 percentage-point spread between the lower bound and the upper bound that maximizes the overall fit of the model. We find that for the medium-run expectations, the best fit for all countries is roughly a range between 2 percent and 3 percent (see Figure E.1). Thus, it is considerably above the ECB's inflation objective, although the absolute differences in fit are particularly small for Spain and France, while they are larger in particular for Austria and Germany. This has a clear implication regarding the anchoring of inflation expectations, suggesting that the ECB still faces a challenge convincing households of the medium-run inflation objective. The more striking difference among countries actually emerges if we repeat the same exercise using short-run expectations. In that case, the fit for Spain, Italy, and France is maximized for the 1.5 percent to 2.5 percent range—in line with our main regressions. For Germany and Austria, it is maximized in the 2 percent to 3 percent range, similar to the medium-run expectations.

These results thus complement the results in the main text, where we observe that households' perceptions in Germany and Austria are in line with the ECB not putting enough weight on inflation developments (relative to the output gap) in its utility function. In Germany and Austria it is believed that the inflation target that the ECB is pursuing is slightly larger, both in the short run and in the medium run, compared to the publicly stated objective.

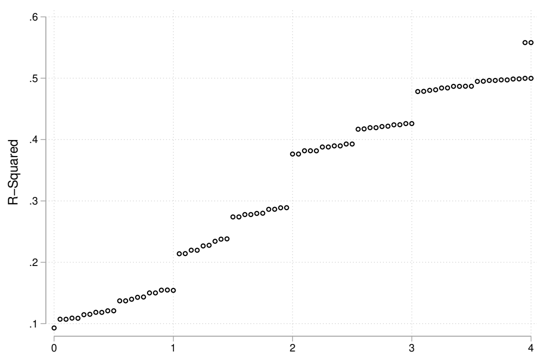
Figure E.1. Perception of ECB's inflation target in different euro-area countries.



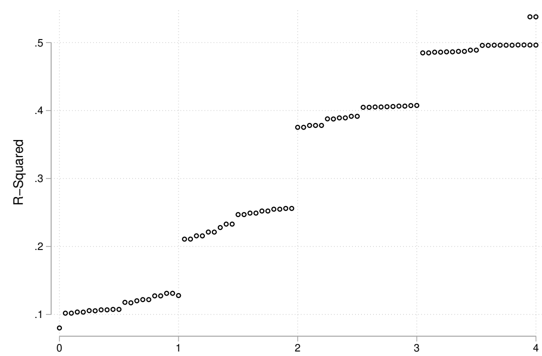
(a) Spain



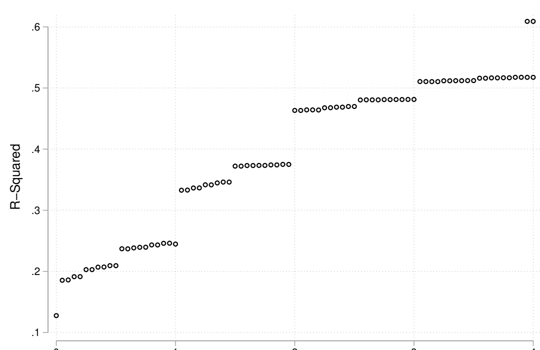
(b) Italy



(c) Germany



(d) Austria



(e) France

Table E.5: EMU with 1%-2%

	(1) EMU	(2) Germany	(3) Austria	(4) France	(5) Spain	(6) Italy
Not Confident	-0.293***	-0.539***	-0.177	-0.074	-0.195*	-0.105
	(0.0525)	(0.0867)	(0.1436)	(0.0794)	(0.1139)	(0.1354)
Gov. Mistrust	-0.019	-0.357***	0.018	-0.049	0.053	0.152**
	(0.0327)	(0.0630)	(0.1069)	(0.0621)	(0.0601)	(0.0673)
Below	-1.063***	-0.883***	-1.097***	-0.890***	-0.975***	-0.994***
	(0.0416)	(0.0741)	(0.1959)	(0.0753)	(0.0930)	(0.0910)
Below × Not Confident	-0.215***	-0.549***	-0.516*	-0.276***	-0.241**	-0.073
	(0.0550)	(0.1462)	(0.2782)	(0.0965)	(0.1108)	(0.1415)
Above	3.304***	2.562***	2.440***	3.681***	4.404***	3.372***
	(0.0846)	(0.1117)	(0.1920)	(0.1871)	(0.1902)	(0.1638)
Above × Not Confident	1.353***	1.583***	1.531***	1.261***	-0.002	1.814***
	(0.0924)	(0.1408)	(0.2063)	(0.2094)	(0.2176)	(0.2190)
Observations	25,677	6,340	2,726	4,401	6,073	6,137
R-Squared	0.194	0.154	0.128	0.245	0.229	0.204
MacroVars	Yes	Yes	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: See notes to Table 4 in main text.

Table E.6: Confidence and Dispersion

	(1) Short-Run	(2) Medium-Run	(3) Short-Run IQR	(4) Medium-Run IQR	(5) Short-Run Region	(6) Medium-Run Region
Not Confident	2.931***	2.746***	0.663*	1.939***	2.095***	1.828***
	(0.648)	(0.658)	(0.364)	(0.559)	(0.441)	(0.421)
Constant	2.443***	3.391***	1.440***	2.289***	2.839***	3.685***
	(0.442)	(0.457)	(0.248)	(0.351)	(0.286)	(0.276)
Observations	97	97	97	97	1,025	1,021
R-squared	0.189	0.201	0.033	0.114	0.027	0.022

Notes: Robust standard errors are in parenthesis. The dependent variable is the variance of medium-run inflation expectations within a country per time in columns (1) and (2). In columns (3) and (4) the dependent variable is the inter quartile range (IQR) and in columns (5) and (6) the dependent variable is the variance calculated at the regional level instead of the country level. "Not Confident" is the share of the population within a country that indicated being not confident in the inflation objective of the central bank. *** p<0.01 ** p<0.05 * p<0.1

E.13. Confidence and Dispersion

We also study whether the unconditional variance of inflation expectations (or disagreement of inflation expectations around the target level) is lower in countries where the level of confidence is higher (lower p_H and/or lower y^*). This examines the relevance of the inflation and deflationary biases for the distribution of inflation expectations, as studied in Corollary 2. We test this relationship using the following equation:

$$VAR(\pi^e_{j,t}) = \alpha + \beta NC_{j,t} + \mu_j + \nu_t + \varepsilon_{j,t};$$

where $VAR(\pi^e_{j,t})$ represents the dispersion of beliefs (the variance or the inter quartile range), and is regressed on the not confident variable plus country/region and time fixed effect (μ_j, ν_t), with $\varepsilon_{j,t}$ being an i.i.d error term. According to our model, being not confident should have implications for both the sample mean and the overall dispersion of beliefs, because a lack of confidence increases the heterogeneity of beliefs.

We also compare the variance and dispersion of inflation expectations against the share of people who are confident across countries and time. Our model predicts that with more people who are not confident in the price stability objective, both inflation and deflationary biases increase, implying a positive correlation. Table E.6 shows a simple bi-variate ordinary least squares regression between the variance of short- and medium-run inflation expectations (across countries and time) against the share of people being not confident. We provide evidence that a higher share of not confident respondents increases the dispersion of their inflation expectations. The medium-run effect is stronger and exerts a higher level of statistical significance as compared to the impact on short-run expectations. This result is robust to using alternative robust measures of dispersion such as the interquartile range (that is, the difference between the 25th and 75th quartiles), reported in columns (3) and (4). We further check the validity of these conclusions by computing the variance by region instead of by country. These results are reported in columns (5) and (6) and confirm our baseline estimation results using country-level variance.

E.14. Additional Tables

Table E.7: **Short-Run Inflation Expectations**

group	mean	median
Below Confidence	1.02	1
Below No Confidence	1.00	1
Mid Confidence	1.63	1.5
Mid No Confidence	1.83	1.8
Above Confidence	3.78	2.5
Above No Confidence	4.74	3
Total	3.40	2

Appendix F. Additional Tables for the BOP-HH results

Table F.1: Summary Statistics: BOP-HH Data

	Mean	**Median**	**Minimum**	**Maximum**	**Std. Dev.**	**Observations**
Short Run Infl Exp	8.88	8	-88	100	7.2	8874
Long Run Infl Exp	7.07	5	-30	87.5	7.14	3382
Not Confident	6.42	6.5	1	10	2.4	2178
Gov. Mistrust	6.87	6	1	11	2.52	2181
ZLB probability	29.3	10	0	100	33.3	2116
Age	55.8	58	16	80	16.3	8996
Gender	1.43	1	1	2	.495	8996
Education	4.29	5	1	8	1.62	8983

Table F.2: Replication of Standard Specification with the BOP-HH Data

	short-run	short-run	medium-run	medium-run
Not Confident	0.244***	-0.233**	0.310***	-0.147*
	(0.046)	(0.096)	(0.059)	(0.079)
Gov. Mistrust	0.258***	0.207***	0.295***	0.251***
	(0.047)	(0.038)	(0.058)	(0.053)
Above		3.718***		1.849***
		(0.719)		(0.499)
Not Confident×Above		0.442***		0.453***
		(0.099)		(0.091)
Below		-3.704***		-2.582***
		(0.933)		(0.787)
Not Confident×Below		0.057		0.019
		(0.143)		(0.120)
Observations	2084	2084	2051	2051
R-Squared	0.130	0.344	0.137	0.309

Table F.3: Explaining Expectations with ZLB and Perceptions about the Weights in the ECB's Loss Function

	short-run	medium-run
Weight econ	0.212***	0.212**
	(0.064)	(0.083)
ZLB Probability	0.008	-0.002
	(0.010)	(0.013)
Weight econ×ZLB Probability	-0.000	0.002
	(0.001)	(0.002)
Constant	4.729***	2.452**
	(1.062)	(1.048)
Observations	2039	2017
R-Squared	0.047	0.070

Table F.4: Explaining "Not Confident" in Achieving the Price Stability Objective of the ECB

	Not Confident	Not Confident
Gov. Mistrust	0.326***	0.336***
	(0.027)	(0.027)
Weight econ	0.290***	0.298***
	(0.040)	(0.039)
ZLB Probability	0.004	0.005
	(0.006)	(0.006)
Weight econ×ZLB Probability	-0.000	-0.000
	(0.001)	(0.001)
Short-run Exp	0.106***	
	(0.022)	
Medium-Run Exp		0.082***
		(0.018)
Observations	2037	2016
R-Squared	0.266	0.271