The Role of R&D Factors in Economic Growth

by

Lorenz Ekerdt
U.S. Census Bureau

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Abstract

This paper studies factor usage in the R&D sector. I show that the usage of non-labor inputs in R&D is significant, and that their usage has grown much more rapidly than the R&D workforce. Using a standard growth decomposition applied to the aggregate idea production function, I estimate that at least 77% of idea growth since the early 1960s can be attributed to the growth of non-labor inputs in R&D. I demonstrate that a similar pattern would hold on the balanced growth path of a standard semi-endogenous growth model, and thus that the decomposition is not simply a by-product of rising research intensity. I then show that combining long-running differences in factor growth rates with non-unitary elasticities of substitution in idea production leads to a slowdown in idea growth whenever labor and capital are complementary. I conclude by estimating this elasticity of substitution and demonstrate that the results favor complimentarities.

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1 Introduction

That ideas affect the production of future ideas is a central tenet of modern theories of endogenous growth. This intertemporal dependence is most often argued to stem from knowledge spillovers, as captured by Newton's famous aphorism that researchers are "standing on the shoulders of giants". The canonical case is one of inspiration, in which an idea reveals some flaw in contemporary understanding and thereby begets more ideas. This focus obscures that research may also progress in more prosaic ways. Consider the discovery of the structure of DNA by Francis Crick and James Watson, a breakthrough with applications too numerous to name here. As is now commonly accepted, their work relied crucially on X-ray diffraction images of DNA produced by Rosalind Franklin. However, X-ray diffraction imaging itself was only possible due to the existence of a controllable source of X-rays, and, presumably, its availability at non-prohibitive prices.

Such anecdotes naturally motivate a thorough understanding of the shape of the aggregate idea production function. Existing models hew to one of two poles: either ideas are produced using labor as the sole factor of production, or only the expenditure function for ideas is modeled, generally implicitly assuming that idea production can be accomplished using only final output.[1] Of course neither assumption can bear much empirical scrutiny: idea production, as proxied via research and development (R&D) activities, makes use of a wide range of factors of production. What is lost by abstracting away from these patterns? What can we learn about growth by studying factor usage in R&D?

In this paper, I take first steps towards answering these questions. First, I show that R&D expenditure on non-labor inputs is significant, and that their usage has grown much more rapidly than the R&D workforce. As a direct consequence, however the output of R&D is measured, the majority of its historical growth can be attributed to the growth of non-labor inputs. Second, I demonstrate that this is not simply an unpacking of rising research intensity: faster relative growth of non-labor inputs is a natural characteristic of balanced growth paths that results simply from more output leading to more non-labor input usage in R&D.

Third, I show that combining faster relative growth of non-labor inputs with flexible elasticities of substitution in the idea production function can lead to a form of Baumol's cost disease in the R&D sector. Along a growth path, the faster relative growth of non-labor inputs can lead to substitution towards labor within the R&D sector; because the R&D workforce grows more slowly than non-labor inputs, this leads to a slowdown in growth. This dynamic is present whenever

[1] Recent examples of the former approach include Jones (2022a) and Peters (2022); recent examples of the latter include Akcigit and Ates (2023) and Sui (2022).

labor is complementary to the aggregate of other inputs in R&D.

To assess whether the above mechanism is empirically relevant, I estimate the R&D elasticity of substitution between capital and labor using variation in innovating firms' responses to a change in tax policy. I find evidence for complimentarities between labor and capital in R&D: my baseline estimates reject an elasticity of substitution greater than 1 at the 10% significance level.

My results rely on the National Science Foundation's (NSF) R&D Survey, which collects and tabulates information on the R&D activities of firms. Included among these data are the number of scientists and engineers employed in R&D, which I treat as a measure of the number of researchers following Jones (2002). I construct a series for non-labor inputs in R&D by subtracting wage and salary payments to researchers from total R&D expenditure, deflating using the aggregate investment deflator.

I use the resulting R&D factor series to decompose the aggregate idea production function. The contribution of a factor to idea growth can be measured using only data on expenditure shares and factor growth rates - this result is standard in growth decompositions and follows simply from mild restrictions on the aggregate idea production function together with the assumption of competitive factor markets. The key result of the decomposition is that the growth of non-labor inputs in R&D can account for at least 77% of idea growth between 1963 and 2020. This finding simply follows from the much faster growth rate of non-labor inputs relative to that of total researchers, together with a high non-labor expenditure share.

I next establish that this pattern of factor contributions is not only a by-product of rising research intensity, but is also what should obtain on the balanced growth path of a standard semi-endogenous growth model. To show this, I use a variant of the model in Jones (2022b) modified to allow for the usage of non-labor inputs in idea production. In the balanced growth path of this model, the growth rate of non-labor inputs in idea production always exceeds that of total researchers. This result simply follows because the growth rate of non-labor inputs is a weighted average of both the growth rate of the labor force and the growth rate of total output; the result is thus nearly by definition of a growth path.

Having argued that the growth of non-labor inputs in idea production exceeds the growth rate of total researchers both empirically and on a balanced growth path, I next examine the implications of long-running differences in factor growth rates under the assumption of a non-unitary elasticity of substitution. I do this by analyzing a constant elasticity of substitution version of the aggregate idea production function. Because a balanced growth path does not in general exist under this assumption, I instead analyze a constant growth path i.e. one in which the growth rate of ideas is constant but other aggregates need not be growing in proportion.

The key result of this analysis is a version of Baumol's cost disease in idea production, which holds whenever labor is complementary to the aggregate of other inputs used in idea production. The logic behind this result is simple: along a growth path, the faster relative growth rate of non-labor inputs, which as argued above holds nearly by definition of per capita income growth, leads to substitution towards labor within idea production. Because the growth of ideas on a constant growth path is simply a weighted average of the growth rate of inputs, this substitution naturally leads to a decline in idea growth unless offset by rising research intensity.

To examine the empirical plausibility of this mechanism, I estimate the elasticity of substitution between labor and capital in R&D. My estimate leverages variation in investment prices across firms created by the introduction of the bonus depreciation policy in the U.S. This policy allowed firms to immediately deduct some portion of investment expenses from their tax burden, thereby reducing the relative price of investment. In turn, because investment depreciation schedules for the purpose of tax write offs are asset-specific, the pre-policy asset mix of firms' investment creates firm-level variation in the magnitude of this reduction.

I use a difference-in-differences empirical design based on the pre-policy asset mix of firms' investment to examine the response of R&D employment and total R&D expenditure to the introduction of bonus depreciation. I then convert these estimates into a measure of the R&D elasticity of substitution between labor and capital using a variant of the method developed in Curtis, Garrett, Ohrn, Roberts, and Serrato (2021). The resulting estimates indicate that labor and capital are complements in idea production; my baseline estimate is -0.3, and none of the estimates are greater than 0.[2] The estimation rejects values of the elasticity of substitution greater than or equal to 1 at the 10% significance level. These results suggest that the earlier proposed mechanism, i.e. slowdown in idea growth due to substitution towards labor in idea production, is likely active. I leave a quantification of said mechanism to future work.

This paper proceeds as follows. Section 2 describes the data used and how I define R&D factors of production, section 3 uses the resulting series to decompose idea growth, section 4 discusses implications for growth, section 5 describes the estimation of the R&D elasticity of substitution between labor and capital, and 6 concludes.

Related literature: The distinction between a labor-only idea production approach and one based on only modeling the expenditure function was introduced by Rivera-Batiz and Romer (1991). They term the former the "knowledge-driven" specification, and the latter the "lab equip-

[2]Because a negative estimate of the elasticity of substitution is not compatible with a two factor CES production function due to a violation of second-order sufficiency conditions for cost minimization, I show that results extend to the three factor case in the appendix.

ment" specification. Little attention has been paid to the distinction since its introduction - to the best of my knowledge, this paper represents the first attempt to estimate an elasticity of substitution for idea production. An exception is Atkeson and Burstein (2019), who show that a higher capital share (or final output share) in idea production leads to slower transitions in a class of semi-endogeneous growth models. However, they restrict to a unitary elasticity of substitution and thus abstract away from factor substitution in idea production.

My empirical approach to estimating the idea production elasticity of substitution is closely related to the method used in Curtis et al. (2021), which estimates elasticities of substitution between labor and capital in production. In turn, the difference-in-differences estimation equation allowing for estimation of the elasticity of substitution originates in Zwick and Mahon (2017). These two papers are part of a rich literature studying the effect of accelerated depreciation policies on firms' investment behavior; other recent examples include Maffini, Xing, and Devereux (2019) and Ohrn (2019). Relative to this literature, my focus on the response of firms' R&D is unique.

2 Data and measurement

2.1 Data

Data on firm's R&D activities comes from the National Science Foundation's (NSF) R&D survey, the Business Enterprise Research and Development (BERD) Survey.[3] The BERD Survey provides a representative sample of the activities of R&D performing firms, and is the primary source of information used to construct aggregate R&D statistics. Though principally cross-sectional, the BERD Survey samples firms above a threshold of total R&D expenditure with certainty — it can thus be used as a panel for the subset of those firms.[4] Key variables used below are total R&D expenditure, number of scientists and engineers employed in R&D, and total wage and salary payments to R&D personnel.

Other firm-level variables are taken from the Longitudinal Business Database (LBD), which tracks the universe of private, non-farm establishments using administrative records produced by business tax filings.[5] To aggregate establishment-level industry codes from the LBD to the firm-level, I use the North American Industrial Classification System (NAICS) code accounting

[3]This survey has been renamed a number of times - in its previous incarnations it was known as the Business Research and Development Innovation Survey (BRDIS) and the Survey of Industrial Research and Development (SIRD). I will refer to all these surveys as the BERD Survey (the most recent name) for brevity's sake.

[4]See Foster, Grim, and Zolas (2020) for a comparison of these firms to the survey as a whole.

[5]See Chow, Fort, Goetz, Goldschlag, Lawrence, Perlman, Stinson, and White (2021) for a description of these data.

for the highest share of the firm's total payroll.

2.2 Measuring R&D factors

Data on labor used in R&D is taken directly from the published tables produced from the BERD Survey, which asks firms to report the number of scientists and engineers employed in R&D. To construct an aggregate series for the use of non-labor inputs in R&D, I deflate non-labor R&D expenditure, defined as the difference between total R&D expenditure and wage payments to R&D personnel, using the aggregate investment deflator.[6]

3 Decomposing idea growth

3.1 Framework

Consider an economy with three sectors: a final goods sector, an idea or R&D sector, and a sector producing non-labor inputs for use in the R&D sector. The production functions in these three sectors are:

$$\text{Final goods production: } Y_t = A_t L_{pt}^{\alpha} K_{pt}^{1-\alpha} \tag{1}$$

$$\text{Ideas: } \dot{A}_t = F_t(L_{rt}, X_t) A_t^{\phi} \tag{2}$$

$$\text{R\&D capital production: } X_t = G_t(Y_{xt}, K_{xt}). \tag{3}$$

This model is a generalized version of the semi-endogenous growth model in Jones (2022b). Final goods production Y is a Cobb-Douglas aggregate of production labor L_p and production capital K_p. Ideas A are produced using a constant returns to scale technology F_t over researchers L_r and and an aggregate of non-labor inputs X_r which I will refer to interchangeably as R&D capital.[7] To generate endogenous growth, the stock of ideas is assumed to directly enter the final goods production function in the form of total factor productivity. Idea production is subject to an intertemporal knowledge spillover parameterized by $\phi < 1$. R&D capital is produced using a constant returns to scale technology G_t over final goods Y_x and capital K_x.[8]

[6] See fred.stlouisfed.org/series/INVDEF

[7] Note that the assumption that F_t has constant returns to scale does not constrain the overall returns to scale of the idea production function, which is equal to ϕ.

[8] That labor does not enter R&D capital production is inessential to what follows; I make this restriction only for the sake of parsimony.

The allocation of factors of production in this economy is summarized by:

$$\text{Allocation of labor: } L_{rt} = s_{lt} L_t \tag{4}$$

$$\text{Allocation of final goods: } Y_{xt} = s_{yt} Y_t \tag{5}$$

$$\text{Allocation of capital: } K_{xt} = s_{kt} K_t. \tag{6}$$

I do not directly model the determination of the sectoral factor shares s_{lt}, s_{yt}, s_{kt}; in what follows, I will either work directly with observed levels L_r, X_r or analyze a BGP, in which case s_{lt}, s_{yt}, and s_{kt} must all be constant and thus do not affect any of the results.

The rest of the economy is standard: an aggregate resource constraint, labor market clearing, and capital market clearing must all hold; labor force growth is constant and equal to g_L; and capital accumulates subject to (instantaneous) depreciation rate δ.

3.2 Decomposition

Taking logs of the idea production function (2) and deriving w.r.t. to time, we obtain the following decomposition of the growth rate of ideas:

$$g_A(t) = \frac{1}{1-\phi}\left[\varepsilon_{F,L_r}(t) g_{L_r}(t) + \varepsilon_{F,X}(t) g_X(t) - \frac{\dot{g}_A(t)}{g_A(t)}\right], \tag{7}$$

with $g_Z(t)$ denoting the growth rate of variable Z at time t and $\varepsilon_{F,x}(t)$ denoting the elasticity of F w.r.t. input x at time t (equal to the output elasticity of the flow of ideas w.r.t input x).

To measure the contribution of R&D capital to idea growth, I proceed as follows: first, denote the counterfactual idea growth rate at t that would obtain if only R&D capital grew as $\tilde{g}_A(t)$; from equation 7, it is immediate that:

$$\tilde{g}_A(t) = \frac{1}{1-\phi}\left(\varepsilon_{F,X}(t) g_X(t) - \frac{\dot{\tilde{g}}_A(t)}{\tilde{g}_A(t)}\right) \tag{8}$$

Define the contribution of capital to idea growth as:

$$c_X(t) = \frac{\varepsilon_{F,X}(t) g_X(t) - \frac{\dot{\tilde{g}}_A(t)}{\tilde{g}_A(t)}}{\varepsilon_{F,L}(t) g_{L_r}(t) + \varepsilon_{F,X}(t) g_X(t) - \frac{\dot{g}_A(t)}{g_A(t)}} \tag{9}$$

In words, $c_X(t)$ simply captures how much smaller idea growth would have been if only R&D capital grew, expressed in units of the realized idea growth rate $g_A(t)$. However, calculating this expression relies on a value of ϕ; this is because the counterfactual growth path in which only R&D capital grows entails a different path of intertemporal knowledge spillovers as captured by $\dot{\tilde{g}}_A(t)/\tilde{g}_A(t)$. Because there is considerable uncertainty around the true value of ϕ (see e.g.

Bloom, Jones, Van Reenen, and Webb (2020)), I instead measure the contribution of R&D capital to growth as:

$$\hat{c}_X(t) = \frac{\varepsilon_{F,X}(t)g_X(t)}{\varepsilon_{F,L}(t)g_{L_r}(t) + \varepsilon_{F,X}(t)g_X(t)}. \tag{10}$$

Because $\hat{c}_X(t) < c_X(t)\ \forall\phi < 1$, we can interpret $\hat{c}_X(t)$ as a lower bound for the contribution of R&D capital to growth.[9] Note that this expression does not rely on any knowledge of the realized idea growth rate $g_A(t)$ and will thus hold no matter what series is used as a proxy for the output of R&D.

To take equation 10 to the data, assume that L_r and X correspond to labor and real non-wage expenditure used for research and development (R&D), and that factor markets are competitive. In this case $\varepsilon_{f,X} = s_X(t)$, the expenditure share of capital at time t, and we can directly measure $\hat{c}_X(t)$ from aggregate trends. Figure 1 shows these trends. The left panel shows the growth rate of the two R&D factors. Note that the growth rate of R&D capital is persistently higher than the growth rate of R&D labor: the former averages around 6.1% annually, whereas the latter averages around 2.4% annually. The right panel shows the time-series of the cost share of labor in R&D expenditure. The average over this time period is .55, with a slight upward trend.[10]

Figure 2 shows the contribution of R&D capital to idea growth; to aid interpretation, I replace equation 10 above with:

$$\overline{c}_X(t) = \frac{\sum_{t_0}^{t} \varepsilon_{F,X}(t)g_X(t)}{\sum_{t_0}^{T} \varepsilon_{F,L}(t)g_{L_r}(t) + \varepsilon_{F,X}(t)g_X(t)}, \tag{11}$$

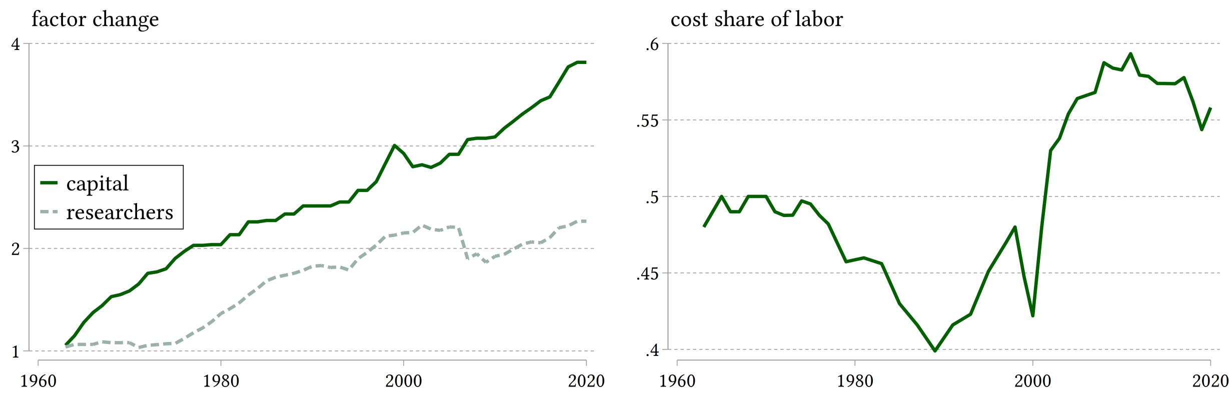
with t_0 the base period. Each point on the graph is thus equal to a lower bound of the idea growth rate that would obtain if only R&D capital grew, expressed as a share of the cumulative idea growth rate over the entire sample period. I weight factor growth rates using the average of the R&D labor share across the two adjoining periods i.e. $\varepsilon_{F,X}(t) = \frac{s_X(t)+s_X(t+1)}{2}$.[11] The growth of R&D capital accounts for the majority of idea growth over the sample period: the final point of the graph indicates that cumulative idea growth would have been at least 77% of its actual value if only R&D capital had grown. This result follows from much faster growth of R&D capital relative to researchers, together with a high and largely stable cost share of capital in R&D.

[9] This lower bound is close to the true value if ϕ is close to 1.

[10] A part of the large increase in 2001 is likely due to a survey redefinition of payments to labor: prior to 2001, this series included only wages and salaries of researchers, whereas after 2001 it also includes benefits. The publicly-available tables published by the NSF do not separately report wages/salaries and benefits.

[11] This can be justified in the usual manner as a discrete approximation to the ideal (Divisia) index; in practice it makes little difference because the R&D labor share does not vary much.

Figure 1: Aggregate trends in R&D factor usage



Notes: The left panel shows the factor change in inputs used in R&D relative to 1963. The number of researchers is defined as the number of scientists and engineers employed in R&D as tabulated by the BERD Survey. Capital is defined as the difference between total R&D expenditure and R&D expenditure on wages and salaries, deflated by the aggregate investment deflator. The right panel shows the cost-share of labor in R&D expenditure, defined as R&D expenditure on wages and salaries divided by total R&D. Data source: BERD.

3.3 R&D factors without rising research intensity

It has been elsewhere emphasized (e.g.Jones (2022b)) that much of idea growth can be attributed to an increasing share of resources devoted to R&D, i.e. rising aggregate research intensity, as opposed to growth in the overall scale of the economy as would result in the balanced growth path (BGP) of a standard semi-endogeneous growth model. Is the above result simply an unpacking of rising research intensity? Restated, is the faster growth rate of R&D capital relative to R&D labor underlying the results of the above decomposition incidental to rising research intensity?

To make progress towards answering these questions, I next consider the R&D factor growth rates that would obtain in the balanced growth path of a version of the economy laid out in section 3.1. Assume first that the input aggregators F and G in the R&D and R&D capital sectors, respectively, are Cobb-Douglas:

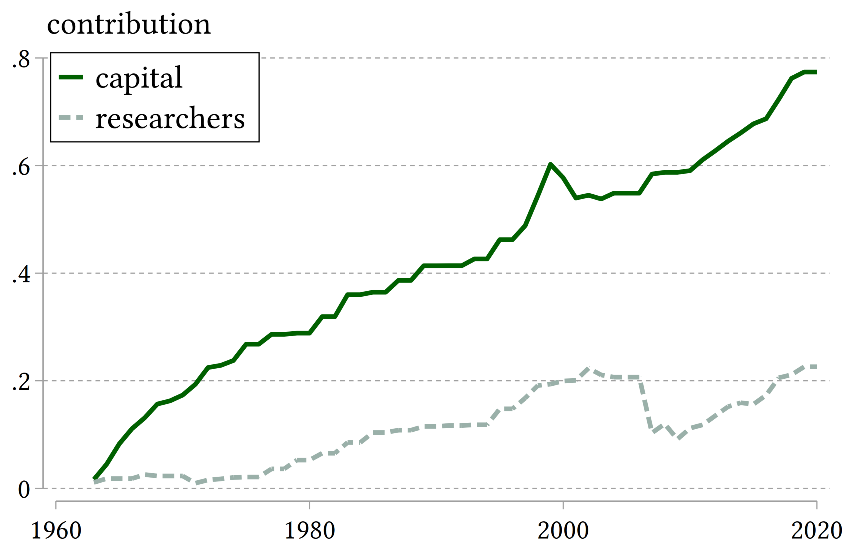
$$\text{Ideas: } \dot{A}_t = L_{rt}^{\gamma} X_t^{1-\gamma} A_t^{\phi} \tag{12}$$

$$\text{R\&D capital production: } X_t = Y_{xt}^{\eta} K_{xt}^{1-\eta}. \tag{13}$$

It then follows directly from equation (13) that, along a BGP, the growth of ideas satisfies

$$g_A = \frac{1}{1-\phi}\left[\gamma g_L + (1-\gamma) g_X\right], \tag{14}$$

Figure 2: Factor decomposition of idea growth



Notes: This figure shows the relative contribution of each factor to total idea growth, as defined in equation 11. Each point on a line represents a lower bound on the cumulative idea growth rate that would obtain if only the relevant factor grew, expressed relative to total cumulative idea growth over the sample period. The number of researchers is defined as the number of scientists and engineers employed in R&D as tabulated by BERD Survey. Capital is defined as the difference between total R&D expenditure and R&D expenditure on wages and salaries, deflated by the aggregate investment deflator. Data source: BERD.

where we have simply made use of the fact that, in a BGP, we must have that $s_{Lt} = \bar{s}_L$, $s_{Yt} = \bar{s}_Y$ for some constants $\bar{s}_L, \bar{s}_Y$. Expressing the production function for R&D capital in terms of growth rates, we have

$$g_X = \eta g_Y + (1 - \eta) g_K. \tag{15}$$

In a BGP the capital-output ratio is constant i.e. $g_Y = g_K$; it thus follows that $g_X = g_Y$. Moreover, because this economy is growing, we have $g_Y > g_L$ and therefore $g_X > g_L$; the growth rate of R&D capital is permanently higher than the growth rate of researchers along a BGP. The empirical trends in R&D factor usage are thus not simply an unpacking of rising research intensity; we should expect the same patterns to hold if research intensity was constant.

3.4 Discussion

The previous section has established 1) that the growth of R&D capital accounts for the majority of historical idea growth, 2) that this is due to the faster relative growth of R&D capital, and 3) that this pattern of R&D factor growth rates is not simply a byproduct of rising research intensity. What are the implications of these findings for growth? I next explore this question along two dimensions. First, I compare BGP growth rates across economies with different R&D capital intensities, abstracting away from substitution among R&D factors. Second, I allow for said substitution margin to be active along a constant (as opposed to balanced) growth path.

4 Implications for growth

4.1 Comparative statics across balanced growth paths

Let us first return to the balanced growth path described above. We have shown that

$$g_A = \frac{1}{1 - \phi} \left[\gamma g_L + (1 - \gamma) g_X \right] \tag{16}$$

$$g_X = g_Y. \tag{17}$$

Expressing the aggregate production function in terms of growth rates, we have

$$g_Y = g_A + \alpha g_L + (1 - \alpha) g_K \tag{18}$$

$$= g_A + \alpha g_L + (1 - \alpha) g_Y, \tag{19}$$

where the second line follows from a constant capital-output ratio. Combining these expressions and solving for g_A, we obtain

$$g_A = \frac{\alpha}{\alpha(1 - \phi) - (1 - \gamma)} g_L. \tag{20}$$

This is the usual semi-endogenous balanced growth relationship, stating that the growth of ideas is proportional to the growth rate of the labor force. However, the growth rate of ideas (and thus the growth rate of per capita output) is increasing in the share of capital expenditure in total R&D, given by $1-\gamma$. This follows directly from $g_X = g_Y > g_L$; decreasing γ (increasing the share of capital in R&D expenditure) increases the weight of a faster growing input in the production of ideas, and thereby increases the growth rate of ideas.

4.2 Substitution along a constant growth path

To examine factor substitution along a growth path, suppose now that the idea production function takes the constant elasticity of substitution (CES) form:

$$\dot{A}(t) = \left(\gamma L_{rt}^{\eta} + (1-\gamma) X_{rt}^{\eta}\right)^{\frac{1}{\eta}} A_t^{\phi}. \tag{21}$$

In this case we have $\varepsilon_{F,x}(t) = \frac{(1-\gamma)X(t)^{\eta}}{\gamma L_r(t)^{\eta} + (1-\gamma)X^{\eta}}$. Note that a BGP requires $\varepsilon_{F,X}(t)$ constant over t, which can only happen if $g_X(t) = g_L(t)$. However, as shown above, this cannot hold as long as the economy is growing, and thus a BGP generically does not exist once the idea production function is CES. In what follows, I thus instead focus on a constant growth path, i.e. one in which $\dot{g}_A \approx 0$.

Expressing the idea production function in growth rates, we have:

$$g_A(t) \approx \frac{1}{1-\phi}\left[\varepsilon_{F,L} g_{L_r}(t) + \varepsilon_{F,X} g_X(t)\right] \tag{22}$$

$$g_A(t) \approx \frac{1}{1-\phi}\left[g_{L_r}(t) + \frac{(1-\gamma)X(t)^{\eta}}{\gamma L_r(t)^{\eta} + (1-\gamma)X(t)^{\eta}}(g_X(t) - g_{L_r}(t))\right] \tag{23}$$

$$g_A(t) \approx \frac{1}{1-\phi}\left[g_{L_r}(t) + \frac{1-\gamma}{1-\gamma+\gamma\left(\frac{X_t}{L_{rt}}\right)^{-\eta}}(g_X(t) - g_{L_r}(t))\right], \tag{24}$$

where the second line follows from Euler's homogeneous function theorem expressed in terms of elasticities, and the third follows by rearranging terms. We can use this equation directly to infer two key properties of this model. First, noting that $\frac{X_t}{L_{rt}}$ is increasing in t over some time period if and only if $g_X(t) > g_L(t)$ over that same time period, the growth impact of growth in R&D capital per researcher is determined by η and thus by the elasticity of substitution between labor and capital.[12] If $\eta < 0$, i.e. if labor and capital are (gross) complements, then $\varepsilon_{F,x} < 1-\gamma$ as long as $\frac{X_t}{L_{rt}} > 1$, and is smaller the greater is $\frac{X_t}{L_{rt}}$.

[12]The elasticity of substitution $\sigma = \frac{1}{1-\eta}$.

Second, if $g_X(t) > g_L(t)$ for any t, then $\lim_{t\to\infty} \varepsilon_{F,X}(t) = 0$ and thus

$$\lim_{t\to\infty} g_A(t) = \frac{g_{L_r}(t)}{1-\phi}. \tag{25}$$

Note that this is exactly the idea growth rate that would obtain in a model in which the only input used in the production of ideas is labor.[13] Restated, if labor and goods expenditure are gross complements in the production of ideas, then the contribution to the growth of ideas of the growth of capital per researcher becomes increasingly muted, and vanishes in the limit. If research intensity does not compensate for this effect, then idea growth slows down.[14] This is analogous to Baumol's cost disease applied within the R&D sector: as the economy grows, the faster relative growth rate of capital causes substitution towards the less plentiful input, researchers; because more resources are devoted to the more slowly growing input, idea growth slows down.

4.3 Discussion

This section has derived two key implications of differences in relative factor growth rates along a growth path. First, the level of idea growth in a balanced growth path (provided it exists) is increasing in the share of R&D expenditure allocated to capital. Second, if labor and non-labor inputs are complements in idea production, idea growth is subject to a decelerating force stemming from substitution towards the more slowly growing input, labor. To determine whether this mechanism is operative requires an estimate of the elasticity of substitution between labor and capital in idea growth. The next section uses the R&D response of innovating firms to a change in tax policy to provide this estimate.

5 Estimating the idea production function elasticity of substitution

5.1 Empirical design

Estimating the idea production function elasticity of substitution requires variation in input prices. As the source of this variation, I make use of the introduction of bonus depreciation in the U.S. In brief, firms can write off investment expenses for the purposes of reducing their tax burden; however, they cannot deduct all expenses contemporaneous to incurring them but

[13] This follows from setting $\gamma = 1$ in equation 14.

[14] Note that, because the above is derived using equations that hold on a constant growth path, I am implicitly assuming that this compensating effect is taking place. This is for ease of exposition rather than by necessity; see Appendix Section A.3 for the proof of the result that idea growth is declining whenever research intensity does not compensate for the reduction in idea growth stemming from R&D capital.

must instead deduct them over a specified depreciation schedule. This depreciation schedule varies across asset classes: assets such as heavy machinery have very long depreciation schedules, whereas others, such as computers, can be written off much faster. Bonus depreciation is a federal policy implemented in 2001 that allows firms to write off some set percentage of investment expenses immediately, regardless of asset class.

How does the introduction of this policy create variation in input prices? Bonus depreciation allows firms to shift future deductions into the present, thereby decreasing the (after-tax) relative price of investment. In turn, how much the relative price of investment falls depends on the difference between the net present value of investment deductions before and after the introduction of bonus depreciation. Due to time-discounting, firms whose investment is particularly intensive in asset classes with long depreciation schedules have low present values of investment deduction prior to bonus depreciation; it follows that the relative price of investment falls more for these firms. My empirical design will thus be based on categorizing firms according to their net present value of investment deductions prior to the introduction of bonus policy.

To do this, I use the net present values of investment deductions calculated at the NAICS-4 industry-level from Zwick and Mahon (2017). I assign firms to high net present value (>70th percentile) or low net present value (<30th percentile) groups. I then compare the firm-level responses of R&D expenditure and employment of researchers to the introduction of bonus depreciation using the following difference-in-differences equation:

$$Y_{it} = \beta_0 + \beta_{Y,bonus}\left[\text{High NPV}_i \times \mathbb{1}\{t \geq 2001\}\right] + \mathbf{X}_{it} + \varepsilon_{it} \tag{26}$$

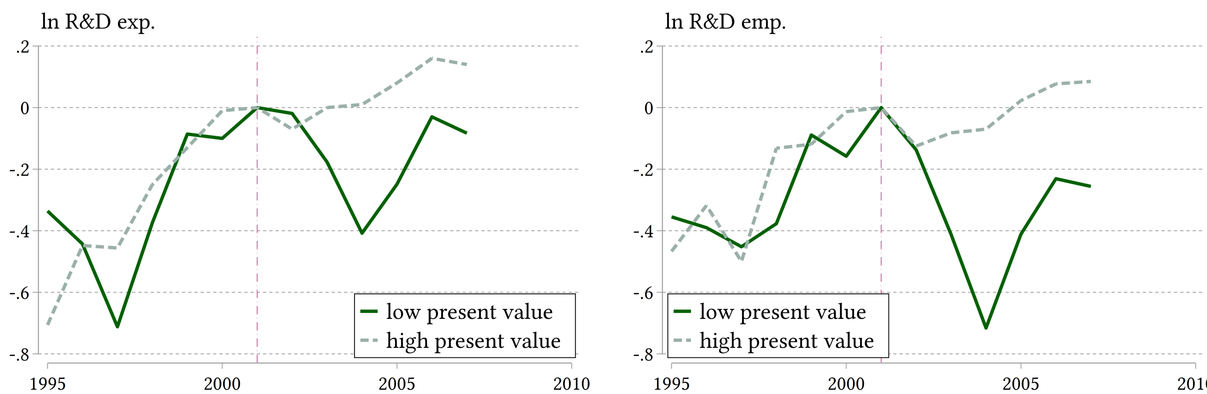
with High NPV an indicator for whether or not a firm is in the high net present value of investment deductions group and $\mathbf{X}_{it}$ denoting a vector of controls. The coefficient of interest is $\beta_{Y,bonus}$: the effect of the bonus depreciation policy on firms' R&D. The baseline specification includes firm- and year-fixed effects; I will also test robustness to using a year by binned firm-size fixed effect using the distribution of employment in 1995.

The identifying assumption is that there are no confounding group-level shocks, so that the difference between the group-level responses reflects only the effect of bonus depreciation. To increase the likelihood that this assumption holds, I make the following two sample restrictions: first, I restrict to a balanced panel, i.e. to firms who are continuously sampled over the analysis period. The SIRD underwent dramatic changes in its sampling frame in the early 2000s; to the extent that survey entry and exit are correlated with group assignment, this could bias my estimation of the coefficient of interest in an unbalanced panel.

Secondly, I restrict to firms whose primary industry is in the manufacturing sector (NAICS codes 31-33). This ensures that any industry-level shocks that affect manufacturing and nonmanufacturing R&D performers differently will be netted out in my estimation. I see this restriction as essential towards identification, first because the manufacturing sector was declining over this time period, and secondly because the introduction of bonus depreciation (2001) occurred nearly contemporaneously with the bursting of the dot-com bubble (2000-2002).

Figure 3 shows averages of log R&D expenditure and log R&D employment across the two groups, normalized by their values in the year in which the bonus depreciation policy was introduced. The group-specific trends do not differ significantly prior to the introduction of bonus depreciation, lending plausibility to the identifying assumption that differences appearing after the introduction of the policy do not reflect confounding group-level shocks.

Figure 3: Trends of dependent variables across treatment groups



Notes: The left panel shows the sample-weighted averages of log R&D expenditure across the control ("low present value") and treatment ("high present value") groups, normalized by their values in 2001. The right panel shows the same for log R&D employment. The vertical lines denote the introduction of the bonus depreciation policy. Firms are assigned to groups based on the value of their net present discounted value of investment deductions as calculated by Zwick and Mahon (2017). Data source: BERD.

5.2 Estimation results

Table 1 shows the results of estimating equation 26. Column 1 controls for year and firm fixed effects, whereas column 2 replaces the year fixed effects with year-by-binned firm-size fixed effects using the distribution of employment in 1995. The top panel shows results using the natural log of R&D employment as the dependent variable, whereas the bottom panel shows results using the natural log R&D expenditure. All estimates are significant at the 95% confidence level.[15]

[15] Coefficient estimates in the first column are significant at the 99% confidence level.

The coefficient interpretation e.g. for the top panel, column 1, is that firms with high net present values of investment deductions expanded their R&D employment by 0.235 log points relative to firms with low net present values of investment deductions. The effect of bonus depreciation is stronger on R&D employment than R&D expenditure in both specifications. Including year-by-binned-firm-size fixed effects decreases both coefficients, but increases the gap between them.

Table 1: Effect of bonus depreciation on firms' R&D

Specification	(1)	(2)
Dependent variable	ln R&D emp.	
$\beta_{\text{R\&D emp., bonus}}$	0.235	0.199
	(0.096)	(0.101)
Dependent variable	ln R&D exp.	
$\beta_{\text{R\&D exp., bonus}}$	0.209	0.152
	(0.072)	(0.076)
Year FE	X	
Firm FE	X	X
Year × size FE		X
Observations	800	

Notes: This table shows the results of estimating equation 26. Coefficient estimates and standard errors are only shown for the variable of interest, which represents the average differential response between firms in the low- and high-present-value of investment deductions groups. The top panel uses the log of R&D employment as the dependent variable, whereas the bottom panel uses the log of R&D expenditure. The first column is the baseline specification; the second column replaces year fixed effects by year-by-binned-size fixed effects using firms' position in the 1995 distribution of employment. Observation counts are rounded to the nearest hundred in accordance with Census disclosure guidelines. Data sources: BERD, LBD.

5.3 Recovering the elasticity of substitution

To move from the estimated effects of bonus depreciation on firm-level R&D activities to an idea production function elasticity of substitution, I make use of cross-equation restrictions coming from firms' conditional factor demands. This technique closely parallels that used in Curtis et al. (2021), with the difference being that I am unable to observe the response of inputs other than labor and must thus use the responses of total R&D expenditure to disentangle scale and substitution effects. Note that the estimation does not impose that firms' conditional factor demand functions are consistent with the model presented in Section 3.2.

Let I denote a firm's total innovation and $C(I)$ denote the cost function of innovation. Write conditional (innovation) factor demands as $l(r, I), k(r, I)$, where l refers to the total number of researchers, k refers to R&D capital, and r refers to the relative price of R&D capital. Taking a total derivative of $l(r, I)$ we obtain

$$dl = \frac{\partial l}{\partial r}\frac{\partial r}{\partial \text{bonus}} d\text{bonus} + \frac{\partial l}{\partial I}\frac{\partial I}{\partial \text{bonus}} d\text{bonus} \tag{27}$$

$$\frac{dl}{d\text{bonus}} = \frac{\partial l}{\partial r}\frac{\partial r}{\partial bonus} + \frac{\partial l}{\partial I}\frac{\partial I}{\partial \text{bonus}}. \tag{28}$$

$$\tag{29}$$

Defining $\overline{\varepsilon}_{y,x}$ to be the elasticity of y w.r.t. x holding constant total innovation and $\varepsilon_{y,x}$ the (unconditional) elasticity of y w.r.t. x, we can rewrite this in terms of elasticities as

$$\varepsilon_{l,\text{bonus}} = \overline{\varepsilon}_{l,r}\varepsilon_{r,\text{bonus}} + \varepsilon_{l,I}\varepsilon_{I,\text{bonus}}. \tag{30}$$

This equation simply decomposes the elasticity of labor with respect to a relative price movement into a substitution effect and a scale effect. Appendix Section A.2 shows that the substitution effect identifies the elasticity of substitution between capital and labor in idea production, whereas the scale effect can be inferred using the estimated effect of bonus depreciation on total R&D expenditure. Combining, the estimator of the elasticity of substitution is

$$\sigma = \frac{\varepsilon_{l,\text{bonus}} - \varepsilon_{C,\text{bonus}}}{s_k \times \varepsilon_{r,\text{bonus}}}, \tag{31}$$

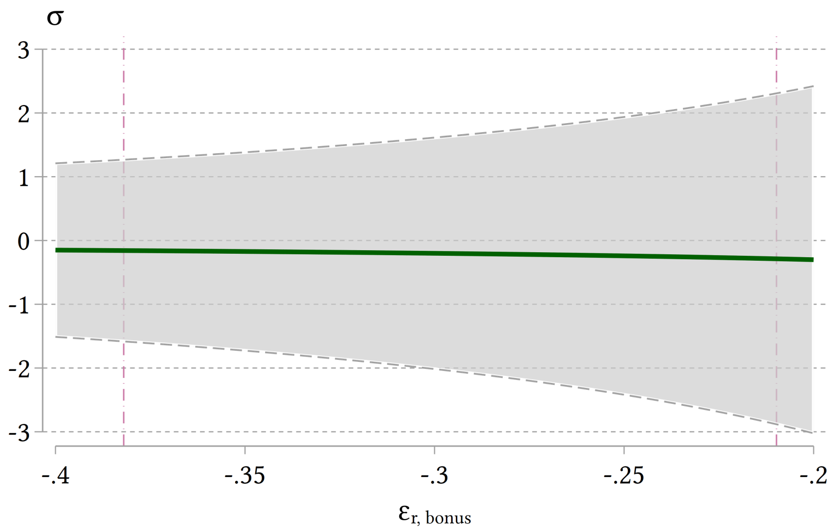
with s_k denoting the cost share of capital in R&D expenditure. The intuition for this expression is straightforward: subtracting the scale effect from the observed effect on labor leaves only the substitution effect; deflating this substitution effect by the cost share of capital in R&D times the effect of the bonus depreciation policy on the relative price of R&D capital recovers the elasticity of substitution.

To implement equation 31, I rely on the estimates of $\varepsilon_{r,\text{bonus}}$ from Curtis et al. (2021).[16] Before doing so, note that equation 31 already rules out estimates of σ significantly above 0 given that $\varepsilon_{r,\text{bonus}} < 0$.[17] This simply reflects the larger response to bonus depreciation of R&D employment relative to R&D expenditure, which can only be accommodated with $\sigma < 0$.

[16]These estimates are recovered from a structural model estimated using firm-level responses of investment and employment to the introduction of bonus depreciation.

[17]This is what one would expect given the structure of the policy, and indeed Curtis et al. (2021) find $\varepsilon_{r,\text{bonus}} < 0$ across a range of specifications.

Figure 4: Estimated elasticity of substitution between capital and labor in R&D



Notes: This figure shows the estimated R&D elasticity of substitution between capital and labor, denoted σ, implied by applying Equation 31 to the bonus depreciation results shown in column 1 of Table 1. Each point on the curve shows the estimated elasticity under a different value for $\varepsilon_{r,\text{bonus}}$, the elasticity of the relative price of capital with respect to the bonus depreciation rate. The solid line shows point estimates, whereas the dashed lines and shaded area depict the corresponding 95% confidence interval. The vertical dashed lines indicate the upper- and lower-bounds of the 95% confidence interval of $\varepsilon_{r,\text{bonus}}$ from Curtis et al. (2021).

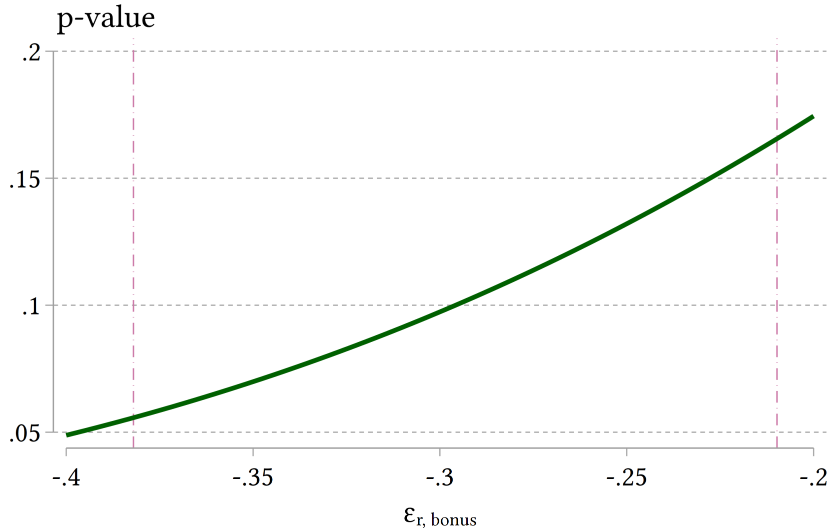
Figure 4 shows estimates of the idea production function elasticity of substitution implied by a range of plausible values for $\varepsilon_{r,\text{bonus}}$.[18] The point estimates of σ range between approximately -.15 and -.3 over this interval. Because the estimator is nonlinear in $\varepsilon_{r,\text{bonus}}$, relatively small standard errors in the estimation of $\varepsilon_{l,\text{bonus}}$ and $\varepsilon_{C,\text{bonus}}$ create considerable uncertainty in the value of σ, particularly for less negative values of $\varepsilon_{r,\text{bonus}}$.[19]

Recall that Section 4.2 showed that the contribution of R&D capital to idea growth is declining over time if the elasticity of substitution is less than 1. Accordingly, Figure 5 shows p-values for the null hypothesis that $\sigma \geq 1$ under different values for $\varepsilon_{r,\text{bonus}}$. The null hypothesis is rejected at significance levels less than or equal to 10% for values of $\varepsilon_{r,\text{bonus}}$ less than approximately -.26.

[18] This figure uses the baseline specification for the bonus depreciation difference-in-differences equation i.e. column 1 in Table 1. See Appendix Figure 6 for results using the alternate specification.

[19] The range of values of $\varepsilon_{r,\text{bonus}}$ resulting from different specifications of the estimation in Curtis et al. (2021) is quite broad; to avoid obscuring the scale, Figure 4 only plots values for the 95% confidence interval of one of these specifications. See Appendix Figure 8 for results over the entire range of values reported in Curtis et al. (2021).

Figure 5: P-values for elasticity of substitution greater than or equal to 1



Notes: This figure shows p-values for $\mathbb{H}_0 : \sigma \geq 1$ under different values for $\varepsilon_{r,\text{bonus}}$, the elasticity of the relative price of capital with respect to the bonus depreciation rate. σ is estimated by applying Equation 31 to the bonus depreciation results shown in column 1 of Table 1. The vertical dashed lines indicate the upper- and lower-bounds of the 95% confidence interval of $\varepsilon_{r,\text{bonus}}$ from Curtis et al. (2021).

5.4 Discussion

The key takeaway from the above is that the estimated elasticity of substitution in idea production points towards capital and labor being gross complements in the production of ideas. Following the discussion in section 4, this implies that, along a constant growth path, we should expect the contribution of R&D capital per researcher to idea growth to be declining. Moreover, because expenditure is being reallocated towards the more slowly growing input (labor), idea growth as a whole slows down unless there is a compensating increase in research intensity. Note that this prediction accords with the time path of the cost share of labor in R&D expenditure shown in figure 1 which, though noisy, appears to be trending upwards.

A subtlety of the above estimation is that the confidence interval of the estimated elasticity of substitution contains negative values. Applied directly to the two-factor CES idea production function in section 4.2, this result violates second-order sufficiency conditions for cost minimization.[20] We can resolve this tension by introducing a third factor of production. Appendix section A.1 shows that the results from section 4.2 extend to the three-factor case. Moreover, the conditions required for the results to hold are exactly those implied by the estimated elasticity of substitution together with second-order sufficiency conditions for cost minimization.

6 Conclusion

This article emphasizes the importance of non-labor input usage in idea production. Non-labor inputs represent a significant share of total R&D expenditure; moreover, their usage has grown at a much faster rate than the number of researchers employed in R&D. I show that these two observations together imply that at least 77% of idea growth since the early 1960s can be attributed to the growth of non-labor inputs. I then demonstrate that is pattern of relative factor growth rates in R&D is not simply a by-product of the rapid expansion in aggregate research intensity; instead, we would expect the same to obtain along the balanced growth path of a standard semi-endogenous growth model.

To assess the implications of long-running differences in relative factor growth rates in R&D, I analyze the constant-growth path of a semi-endogenous growth model in the elasticity of substitution between capital and labor in idea production is not necessarily unity. The key result is that a form of Baumol's cost disease in R&D obtains whenever said elasticity is less than 1; in this case the faster relative growth of non-labor inputs leads to substitution towards the more slowly

[20] In the two factor case this condition is just $\eta < 1$; however, the elasticity of substitution $\sigma = \frac{1}{1-\eta}$ and thus negative estimates of σ violate this condition. See appendix section A.4.1 for the derivation of this result.

growing labor input, thereby decreasing the idea growth rate.

Having demonstrated the importance of the elasticity of substitution between labor and capital in idea production, I next turn towards estimating this parameter. To do this, I make use of firm-level variation in the relative price of investment created by the introduction of bonus depreciation in the U.S. in 2001. My estimates point towards complimentarities between labor and capital in idea production: I can reject values of the elasticity of substitution above 1 for most specifications.

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A Theory appendix

A.1 Idea growth with three factors

Suppose now that there is some third factor of production M. The idea production function is as follows:[21]

$$\dot{A}(t) = \left[\gamma M_t^{\eta_1} + (1-\gamma)\left(\alpha X_t^{\eta_2} + (1-\alpha) L_{rt}^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}\right]^{\frac{1}{\eta_1}} A_t^{\phi}.$$

We can thus write

$$g_A \approx \frac{1}{1-\phi}\left[\varepsilon_{F,L_r}(t) g_{L_r} + \varepsilon_{F,X}(t) g_X + \varepsilon_{F,M}(t) g_M\right],$$

along a constant growth path. Assume $g_X = g_M$, then we can rewrite the above as

$$g_A \approx \frac{1}{1-\phi}\left[g_{L_r} + (\varepsilon_{F,X}(t) + \varepsilon_{F,M}(t))(g_X - g_{L_r})\right].$$

We can derive that

$$\varepsilon_{F,M} = \frac{\gamma M^{\eta_1}}{\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha) L_r^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}} \tag{32}$$

$$\varepsilon_{F,X} = \frac{(1-\alpha)\left(\alpha L^{\eta_2} + (1-\alpha) X^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}-1}(1-\gamma) X^{\eta_2}}{\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha) L_r^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}} \tag{33}$$

It can be shown that a sufficient condition for $\frac{\partial(\varepsilon_{F,X}(t) - \varepsilon_{F,M}(t))}{\partial t} < 0$ is:

$$\frac{\alpha L_0^{\eta_2} g_L^{1+\eta_2 t} + (1-\alpha) X_0^{\eta_2} g_X^{1+\eta_2 t}}{\alpha L_0^{\eta_2} g_L^{\eta_2 t} + (1-\alpha) g_X^{\eta_2 t}} - \alpha \eta_1 M_0^{\eta_1} g_M^{1+\eta_1 t} > 0.$$

This in turn holds for t large enough if $\eta_2 > 0, \eta_1 < 0$. Moreover, under these conditions, $\lim_{t\to\infty} \varepsilon_{F,M}(t) = \lim_{t\to\infty} \varepsilon_{F,X}(t) = 0$, and thus the three factor model also converges to the idea

[21] Of course an alternative way of writing this production function is separately nesting L and M instead of L and X; however, in this case the estimated elasticity of substitution violates second-order sufficiency conditions for cost minimization because it implies two out of three elasticities of substitution are negative.

growth rate of a labor-only model. That $\varepsilon_{F,M}(t)$ converges to 0 in the limit is clear. To see that $\varepsilon_{F,X}(t)$ also converges to 0, rewrite it as:

$$\varepsilon_{F,X}(t) = \frac{1-\alpha}{\frac{\gamma M^{\eta_1}}{(\alpha L^{\eta_2}+(1-\alpha)X^{\eta_2})^{\eta_1/\eta_2}} + 1 - \gamma} \; \frac{(1-\gamma)X^{\eta_2}}{\alpha X^{\eta_2} + (1-\alpha)L_r^{\eta_2}}.$$

The first term converges to 0; the second converges to a constant, and thus the entire expression converges to 0.

What does the estimated elasticity of substitution from the bonus depreciation results imply about η_1, η_2? Treating the bonus depreciation results as estimating $\sigma_{X,L}$, we have $\sigma_{X,L} = \frac{1}{1-\eta_2}$. Because we estimate that $\sigma_{X,L} < 1$, we have $\eta_2 > 0$; in turn, $\eta_1 < 0$ is required in order for the production function to satisfy second-order sufficiency conditions for cost minimization.[22]

A.2 Derivation of the elasticity of substitution estimator

The main text has shown that

$$\varepsilon_{l,\text{bonus}} = \overline{\varepsilon}_{l,r}\varepsilon_{r,\text{bonus}} + \varepsilon_{l,I}\varepsilon_{I,\text{bonus}}. \tag{34}$$

Because I is homogeneous of degree 1, we know that $C(I)$ is also homogeneous of degree 1. It can thus be written as: $I \times c = C(I)$ with $c \equiv C(1)$ the unit cost of innovation. Taking logs and differentiating w.r.t bonus, we obtain $\varepsilon_{C,\text{bonus}} = \varepsilon_{I,\text{bonus}}$. Similar, and again because I is homogeneous of degree 1, we know that $l(r, I)$ is homogeneous of degree 1 in I, and thus, by Euler's homogeneous function theorem, we have $\varepsilon_{l,I} = 1$. Combining, we can express the above equation as:

$$\varepsilon_{l,\text{bonus}} = \overline{\varepsilon}_{l,r}\varepsilon_{r,\text{bonus}} + \varepsilon_{C,\text{bonus}} \tag{35}$$

To derive an expression for $\overline{\varepsilon}_{l,r}$, note that by Shephard's lemma we can write:

$$l = C_w \tag{36}$$

$$l_r = C_{wr} \tag{37}$$

$$\frac{l_r r}{l} = \frac{C_{wr} r}{C_w} \tag{38}$$

$$\overline{\varepsilon}_{l,r} = \frac{rC_r}{C}\frac{C_{wr}}{C_w C_r}C \tag{39}$$

$$\overline{\varepsilon}_{l,r} = s_k \times \sigma \tag{40}$$

[22] See appendix section A.4.2 for the derivation of this result.

with s_k denoting the cost share of k in total R&D expenditure, σ denoting the elasticity of substitution between k and l, and equation (31) following from equation (30) by another application of Shephard's lemma. Combining, we have:

$$\varepsilon_{l,\text{bonus}} = s_k \times \sigma \varepsilon_{r,\text{bonus}} + \varepsilon_{C,\text{bonus}} \tag{41}$$

$$\sigma = \frac{\varepsilon_{l,\text{bonus}} - \varepsilon_{C,\text{bonus}}}{s_k \times \varepsilon_{r,\text{bonus}}} \tag{42}$$

A.3 Further decomposition results

Recall that a CES idea production function implies the following:

$$g_A(t) + \frac{1}{1-\phi}\frac{\dot{g}_A(t)}{g_A(t)} = \frac{1}{1-\phi}\left[g_{L_r}(t) + \frac{1-\gamma}{1-\gamma+\gamma\left(\frac{X_t}{L_{rt}}\right)^{-\eta}}(g_X(t) - g_L(t))\right]. \tag{43}$$

The main text shows that the contribution of R&D capital to idea growth is declining in t provided $\dot{g}_A = 0$, $\eta < 0$, and $g_X(t) > g_{L_r}(t)$. I show here that this declining contribution results in declining idea growth whenever it is not offset by rising research intensity. An instructive case is $g_{L_r}(t) = g_{L_r} < g_X = g_X(t)$ for some constants g_X, g_{L_r}. In this case, the RHS of equation 43 is declining in t whenever $\eta < 0$. The time derivative of the LHS is

$$\dot{g}_A(t) + \frac{1}{1-\phi}\left[\frac{g_A(t)\ddot{g}_A(t) - \dot{g}_A(t^2)}{g_A(t)^2}\right] \tag{44}$$

$$\frac{\dot{g}_A(t)}{g_A(t)}\left[g_A(t) - \frac{1}{1-\phi}\frac{\dot{g}_A(t)}{g_A(t)}\right] + \frac{1}{1-\phi}\frac{\ddot{g}_A(t)}{g_A(t)}. \tag{45}$$

The first term has the sign of $\dot{g}_A(t)$; to see that the second also has the sign of $\dot{g}_A(t)$ note that, on any growth path, $g_A(t) > 0$ and thus $\frac{1}{1-\phi}\frac{\ddot{g}_A(t)}{g_A(t)} < 0 \iff \ddot{g}_A(t) < 0$. This, in turn, can only be true if $\dot{g}_A(t) < 0$. The sign of the LHS is thus declining if and only if $\dot{g}_A(t) < 0$.[23] More generally, we can say that research intensity is not compensating for the decline of idea growth stemming from the lower contribution of R&D capital whenever the time derivative of the RHS of equation 43 is less than 0.

A.4 Second-order sufficiency conditions for CES

A.4.1 Two factor case

The two factor CES production function is

$$F(L, X) = (\gamma L^\eta + (1-\gamma)X^\eta)^{\frac{1}{\eta}}. \tag{46}$$

[23] This can be shown by taking an integral of both sides of $\frac{1}{1-\phi}\frac{\ddot{g}_A(t)}{g_A(t)} < 0$.

The second-order sufficiency conditions for the associated cost minimization problem is that the following matrix has a negative determinant:

$$\begin{bmatrix} 0 & -F_L & -F_X \\ -F_L & -F_{LL} & -F_{LX} \\ -F_X & -F_{LX} & -F_{XX} \end{bmatrix},$$

where F_y denotes the first partial derivative of F with respect to y and F_{yz} denotes the partial derivative of F_y with respect to z. This determinant is

$$F_L^2 F_{XX} + F_X^2 F_{LL} - 2F_L F_X F_{LX}$$

which is negative over the entire domain of (X, L) iff $F_{LL}, F_{XX} < 0$, $F_{LX} > 0$. We can write these partial derivatives as

$$\begin{aligned} F_{LL} &= \gamma L^{\eta-2} \left(\gamma L^\eta + (1-\gamma)X^\eta\right)^{\frac{1}{\eta}-1} (\eta - 1) \left[1 - \frac{\gamma L^\eta}{\gamma L^\eta + (1-\gamma)X^\eta}\right] \\ F_{XX} &= \gamma X^{\eta-2} \left(\gamma L^\eta + (1-\gamma)X^\eta\right)^{\frac{1}{\eta}-1} (\eta - 1) \left[1 - \frac{(1-\gamma)X^\eta}{\gamma L^\eta + (1-\gamma)X^\eta}\right] \\ F_{LX} &= (1-\eta)\gamma L^{\eta-1}(1-\gamma)X^{\eta-1} \left(\gamma L^\eta + (1-\gamma)X^\eta\right)^{\frac{1}{\eta}-2} \end{aligned}$$

and thus $F_{LL}, F_{XX} < 0$, $F_{LX} > 0$ iff $\eta < 1$.

A.4.2 Three factor case

The three factor CES production function is

$$F(M, X, L) = \left[\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}\right]^{\frac{1}{\eta_1}}$$

The second-order sufficiency condition for the associated cost minimization problem is that the third and fourth leading principal minors of the following matrix have negative determinants:

$$\begin{bmatrix} 0 & -F_M & -F_X & -F_L \\ -F_M & -F_{MM} & -F_{XM} & -F_{LM} \\ -F_X & -F_{XM} & -F_{XX} & -F_{XL} \\ -F_L & -F_{LM} & -F_{LX} & -F_{LL} \end{bmatrix}.$$

Instead of deriving the sufficiency conditions explicitly, I will show that, if $\eta_2 > 1$ as implied by the estimated elasticity of substitution,[24] then the sufficiency condition can only hold if $\eta_1 < 0$.

[24] $\sigma_{X,L} = \frac{1}{1-\eta_2}$.

The third leading principal minor is

$$\begin{bmatrix} 0 & -F_M & -F_X \\ -F_M & -F_{MM} & -F_{XM} \\ -F_X & -F_{XM} & -F_{XX} \end{bmatrix}.$$

which has negative determinant iff

$$F_M^2 F_{XX} + F_X^2 F_{MM} - 2F_M F_X F_{MX} < 0,$$

which in turn can only hold if $F_{MM}, F_{XX} < 0$, $F_{MX} > 0$. We can express these partial derivatives as

$$F_{MM} = \gamma M^{\eta_1 - 2}(\eta_1 - 1)$$
$$\left[\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}\right]^{\frac{1}{\eta_1}-1}\left[1 - \frac{\gamma M^{\eta_1}}{\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}}\right]$$

$$F_{MX} = (1-\eta_1)(1-\gamma)\left[\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}\right]^{\frac{1}{\eta_1}-2}\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}-1} X^{\eta_2 - 1} M^{\eta_1}$$

$$F_{XX} = (1-\gamma)\alpha(\eta_2 - 1)X^{\eta_2 - 2}\left[\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}\right]^{\frac{1}{\eta_1}-1}\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}-1}(\eta_2 - 1)$$
$$\left[1 + \frac{\eta_1 - \eta_2}{\eta_2 - 1}\frac{\alpha X^{\eta_2}}{\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}} + \frac{1-\eta_1}{\eta_2 - 1}\frac{\alpha X^{\eta_2}}{\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}}\frac{\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}}{\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}}\right]$$

First, note that F_{MM} has the sign of $(\eta_1 - 1)$ and that F_{MX} has the sign of $(1 - \eta_1)$; thus, $\eta_1 < 1$ is sufficient to ensure $F_{MM} < 0, F_{MX} > 0$. Second, note that the sign of F_{XX} only depends on the second bracketed term. We thus require:

$$\left[1 + \frac{\eta_1 - \eta_2}{\eta_2 - 1}\frac{\alpha X^{\eta_2}}{\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}} + \frac{1-\eta_1}{\eta_2 - 1}\frac{\alpha X^{\eta_2}}{\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}}\frac{\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}}{\gamma M^{\eta_1} + (1-\gamma)\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}}\right] < 0$$

Rewrite this as

$$0 < 1 - \eta_2 + \frac{\alpha}{\alpha + (1-\alpha)\left(\frac{L}{X}\right)^{\eta_2}}(\eta_2 - \eta_1) + (\eta_1 - 1)\frac{\alpha}{\alpha + (1-\alpha)\left(\frac{L}{X}\right)^{\eta_2}}\frac{1-\gamma}{\frac{\gamma M^{\eta_1}}{\left(\alpha X^{\eta_2} + (1-\alpha)L^{\eta_2}\right)^{\frac{\eta_1}{\eta_2}}} + 1 - \gamma} \equiv \Gamma$$

If $\eta_1 > 0$ then

$$\lim_{M\to\infty} \Gamma = 1 - \eta_2 + \frac{\alpha}{\alpha + (1-\alpha)\left(\frac{L}{X}\right)^{\eta_2}}\left[\eta_2 - \eta_1\right]$$

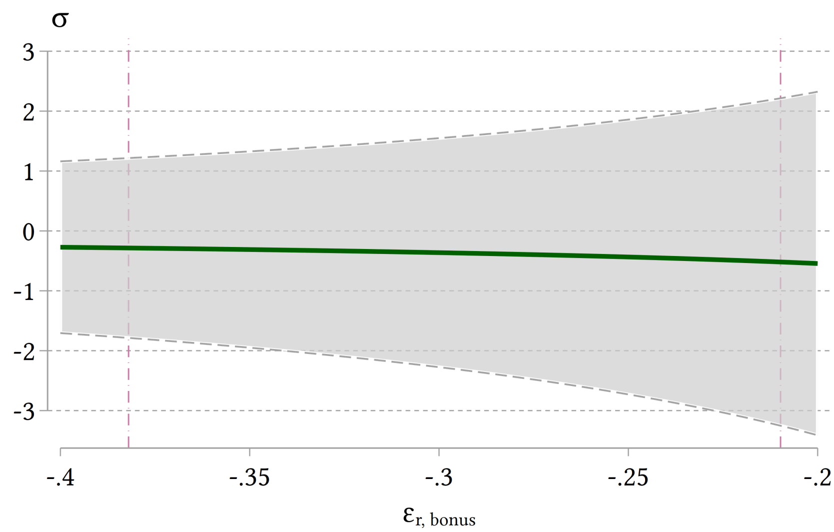
and thus

$$\lim_{X\to\infty}\lim_{M\to\infty} \Gamma = 1 - \eta_2$$

but $1 - \eta_2 < 0$ and therefore the sufficiency conditions cannot be met.

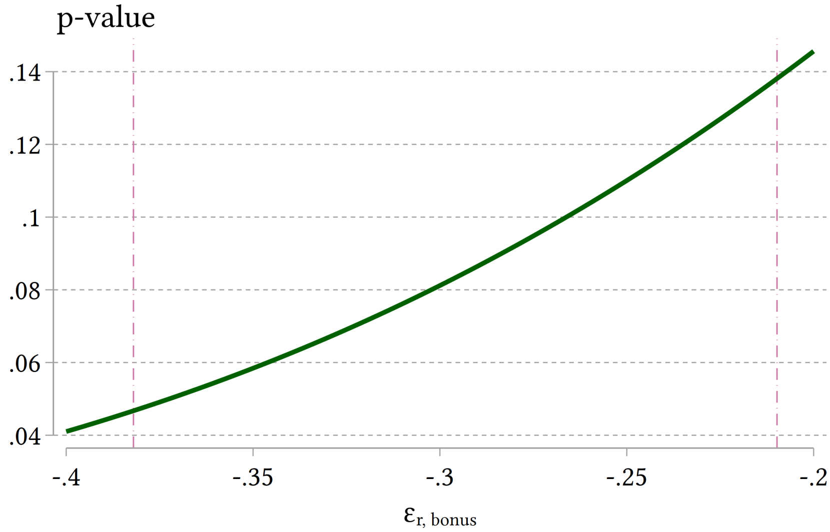
B Appendix Figures

Figure 6: Estimated elasticity of substitution between capital and labor in R&D, alternate specification



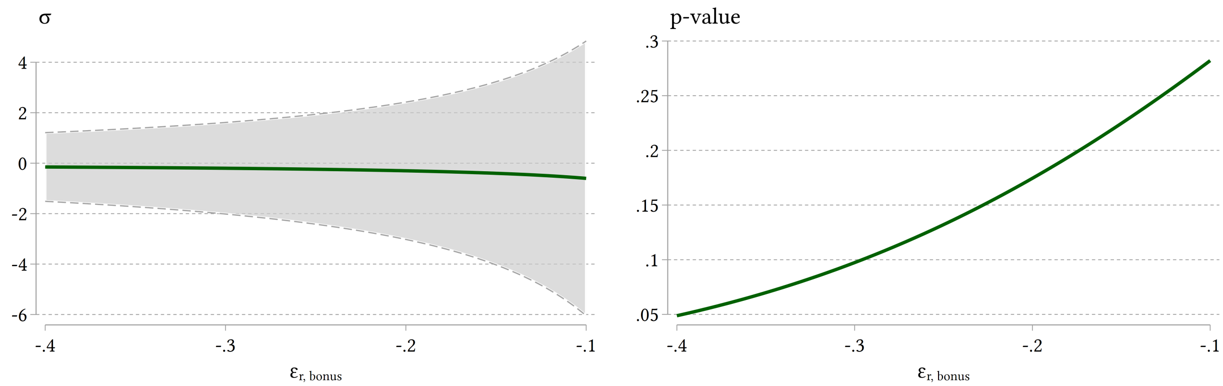
Notes: This figure shows the estimated R&D elasticity of substitution between capital and labor, denoted σ, implied by applying Equation 31 to the bonus depreciation results shown in column 2 of Table 1. Each point on the curve shows the estimated elasticity under a different value for $\varepsilon_{r,\text{bonus}}$, the elasticity of the relative price of capital with respect to the bonus depreciation rate. The solid line shows point estimates, whereas the dashed lines and shaded area depict the corresponding 95% confidence interval. The vertical dashed lines indicate the upper- and lower-bounds of the 95% confidence interval of $\varepsilon_{r,\text{bonus}}$ from Curtis et al. (2021).

Figure 7: P-values for elasticity of substitution greater than or equal to 1, alternate specification



Notes: This figure shows p-values for $\mathbb{H}_0 : \sigma \geq 1$ under different values for $\varepsilon_{r,\text{bonus}}$, the elasticity of the relative price of capital with respect to the bonus depreciation rate. σ is estimated by applying Equation 31 to the bonus depreciation results shown in column 2 of Table 1. The vertical dashed lines indicate the upper- and lower-bounds of the 95% confidence interval of $\varepsilon_{r,\text{bonus}}$ from Curtis et al. (2021).

Figure 8: Results over expanded domain



Notes: The left plot shows the estimated R&D elasticity of substitution between capital and labor, denoted σ, implied by applying Equation 31 to the bonus depreciation results shown in column 1 of Table 1. Each point on the curve shows the estimated elasticity under a different value for $\varepsilon_{r,\text{bonus}}$, the elasticity of the relative price of capital with respect to the bonus depreciation rate. The solid line shows point estimates, whereas the dashed lines and shaded area depict the corresponding 95% confidence interval. This figure shows p-values for $\mathbb{H}_0 : \sigma \geq 1$ under different values for $\varepsilon_{r,\text{bonus}}$. The domain is chosen to cover all values of $\varepsilon_{r,\text{bonus}}$ reported in Curtis et al. (2021).